PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXXV.

PART I.

LONDON:

PRINTED BY RICHARD TAYLOR, RED LION COURT, FLEET STREET.

MDCCCXXXV.

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions,* take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions,* that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule the Society, to which they will always adhere, never to give their opinion, as a Body,

upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

ROYAL MEDALS.

HIS MAJESTY KING WILLIAM THE FOURTH, in restoring the Foundation of the Royal Medals, graciously Commanded a Letter, of which the following is an extract, to be addressed to the Royal Society, through His Royal Highness the Duke of Sussex, K.G., President:

"Windsor Castle, March 25, 1833.

"It is His Majesty's wish,—

"First, That the Two Gold Medals, value of Fifty Guineas each, shall "henceforth be awarded on the day of the Anniversary Meeting of the "Royal Society, on each ensuing year, for the most important discoveries "in any one principal subject or branch of knowledge.

"Secondly, That the subject matter of inquiry shall be previously settled "and propounded by the Council of the Royal Society, three years pre- "ceding the day of such award.

"Thirdly, That Literary Men of all nations shall be invited to afford the "aid of their talents and research: and,

"Fourthly, That for the ensuing three successive years, the said Two "Medals shall be awarded to such important discoveries, or series of in- "vestigations, as shall be sufficiently established, or completed to the "satisfaction of the Council, within the last five years of the days of award, "for the years 1834 and 1835, including the present year, and for which "the Author shall not have previously received an honorary reward.

(Signed) "H. TAYLOR."

The Council propose to give one of the Royal Medals in the year 1836, to the most important unpublished paper in Astronomy, communicated to the Royal Society for

insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council also propose to give one of the Royal Medals in the year 1836 to the most important unpublished paper in Animal Physiology, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Royal Medals for the year 1833 were awarded to

SIR JOHN FREDERICK WILLIAM HERSCHEL, K.H. F.R.S.,

for his Paper on the Investigation of the Orbits of Revolving Double Stars; and to

PROFESSOR AUGUSTE PYRAME DE CANDOLLE, of Geneva, Foreign Member of the Royal Society,

for his Discoveries and Investigations in Vegetable Physiology.

Those for 1834 were awarded to

JOHN WILLIAM LUBBOCK, ESQ., V.P. & TREAS. R.S.,

for his Papers on the Tides published in the Philosophical Transactions; and to

CHARLES LYELL, ESQ.,

for his Work entitled "Principles of Geology."

The Council propose to give one of the Royal Medals in the year 1837 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in their Transactions, after the present date (November 27th, 1834,) and prior to the month of June in that year.

The Council also propose to give one of the Royal Medals in the year 1837 to the author of the best paper, to be entitled "Contributions towards a System of Geological Chronology founded on an examination of fossil remains, and their attendant phenomena," such paper to be communicated to the Royal Society after the present date (December 1st, 1834,) and prior to the month of June 1837.

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PHILOSOPHICAL TRANSACTIONS.

I. The Bakerian Lecture.—*On the Proofs of a gradual Rising of the Land in certain parts of Sweden.* *By* Charles Lyell, *Jun. Esq. F.R.S.*

Received October 4,—Read November 27, 1834.

It is now more than one hundred years since the Swedish naturalist Celsius expressed his opinion that the waters, not only of the Baltic, but of the whole Northern Ocean, were gradually sinking; and he represented their level as lowering at the rate of forty Swedish inches in a century*. He observed that several rocks which not long ago were sunken reefs and dangerous to navigators, had become in his time above water; that the sea was constantly leaving dry new tracts of land along its borders; that ancient ports had become inland towns; and that old fishermen and seafaring people could testify that at a variety of places, both on the shores of the Baltic and the ocean, considerable changes had taken place within the time of their memory, in the form of the coast and depth of the sea. Lastly he appealed to marks which had been cut in the rocks before his time expressly to indicate the former level, and the waters were observed to have fallen below these marks.

This notion of a change continually in progress in the relative level of land and sea was at first warmly controverted, and many facts were adduced to prove that there had not been a general fall of the waters even in the Baltic. It was supposed by many that there might have been some error in the observations, as the Baltic, though free from tides, is often raised for several days continuously two or three feet above its standard level by the melting of the snow, or by the prevalence of particular winds; and it was remarked that the altered form of the coast and the shallowing of the sea might be ascribed partly to new accessions of land at points where rivers entered, depositing sand and mud, and partly to the drifting of large blocks by ice, which are sometimes stranded and driven up on rocks and low islands so as to raise their height.

Playfair, in the year 1802, in his "Illustrations of the Huttonian Theory," de-

* I have used the Swedish measure throughout this paper, for the sake of uniformity, when alluding to the measurements made by Swedes. The Swedish foot, which is divided into twelve inches, agrees very nearly with our own, being less than ours by three eighths of an inch only.

clared that the supposed change of relative level of sea and land in Sweden, which appeared to him to be sufficiently ascertained, might be ascribed to the movement of the land rather than of the waters. He observed, "that in order to depress or elevate the absolute level of the sea, by a given quantity, in any one place, we must depress or elevate it by the same quantity over the whole surface of the earth; whereas no such necessity exists with respect to the elevation or depression of the land*." The hypothesis of the rising of the land, he adds, "agrees well with the Huttonian theory, which holds that our continents are subject to be acted upon by the expansive forces of the mineral regions; that by these forces they have been actually raised up, and are sustained by them in their present situation†."

In the year 1807 Von Buch, after returning from a tour in Scandinavia, announced his conviction "that the whole country from Frederickshall in Sweden to Åbo in Finland, and perhaps as far as St. Petersburgh, was slowly and insensibly rising;" a conclusion to which he appears to have been led principally by information obtained from the inhabitants, and in part by the occurrence of marine shells, of recent species, which he had found at several points on the coast of Norway above the level of the sea.

At several periods since this discussion began respecting the decline of the level of the Baltic Sea and German Ocean, marks have been cut on the rocks of exposed cliffs, both of islands and the main land, so as to indicate the then existing height of the waters, the year in which the marks were made being at the same time recorded. All these marks were examined in 1820–21 by the officers of the pilotage establishment of Sweden, and a report made by them to the Royal Academy at Stockholm, in which they declared, as the result of their measurement, that along the whole coast of the northern part of the Gulf of Bothnia the water is lower with respect to the land than formerly; but that the amount of variation, or change of level, has not been uniform. At the same time an account was given in, and published by the Academy, of new marks which were made in the same years, 1820–21, to record the level of the sea observed at the time of the survey.

Notwithstanding the numerous proofs recorded of the change of level, and the high authorities who had declared in its favour, I continued, in common with many others, to entertain some doubts respecting the reality of the phenomenon, partly because I suspected that it might be explained by reference to more ordinary causes, such as some of those above mentioned, and partly because it appeared to me improbable that such great effects of subterranean expansion should take place in countries which, like Sweden and Norway, have been remarkably free within the times of history from violent earthquakes. The slow, constant, and insensible elevation of a large tract of land, is a process so different from the sudden rising or falling known to have accompanied, in certain regions, the intermittent action of earthquakes and volcanos, that the fact appeared to require more than an ordinary weight of evidence for its confirmation. I am willing, however, to confess, after

* § 393. † § 398.

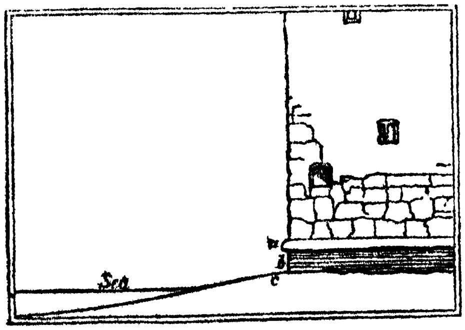
reviewing all the statements published previously to my late tour for and against the reality of the change of level in Sweden, that my scepticism appears to have been unwarrantable; but it will not be disputed that too many proofs cannot be accumulated to substantiate so remarkable a phenomenon.

I propose, therefore, to lay before the Royal Society the observations which I made during the summer of 1834, with a view of satisfying myself in regard to the data appealed to in support of the elevation of parts both of the eastern and western shores of Sweden. As much of the evidence could only have been derived from personal intercourse with the inhabitants, it may be proper to mention that I was accompanied throughout my excursion by a well-informed Swede, Mr. Johnson, who by his thorough knowledge of the English language was well qualified to assist me as interpreter.

On my way to Sweden I examined the eastern shores of the Danish islands of Möen and Seeland; but neither there, nor afterwards in Scania, could I discover any signs of a recent upward movement of the land, nor could I learn that the notion of such a change was entertained by the natives. Proceeding northwards along the coast of the Baltic, the first place which I visited where any elevation of land is supposed to be going on was Calmar. This port is situated in latitude 56° 41′. To the south of the town is the celebrated ancient castle in which was signed, in the year 1397, the famous treaty of union between Sweden, Denmark, and Norway. The castle is supposed to have remained in its present state from a still earlier period. There was a fortress on the site so long ago as the year 1030*. Two round-towers terminate the outworks of this fortress on the side of the sea; and when I observed that the base of one of these rested on the beach only two feet above the level of the water, and when I found that sea-weed had recently been washed up, so as to touch the lowest part of the building, I concluded, at first, that for the last four or five centuries there could have been no lowering of the Baltic at this place, for otherwise we should be compelled to suppose that part of the tower had been originally constructed under water. But on nearer inspection I was led to suspect that this had really been the case, and that the foundation was originally subaqueous. At the height of about two feet above the base of the tower (see sketch, fig. 1.), and four feet above the level of the sea, a projecting band of stone (a), one foot deep, encircles the tower like a hoop. This projecting band is of smooth stone, and the stones above it are large, and with an even, dressed surface. But below the hoop are many courses of thin slabs of a different stone (b), with layers of cement between. It oc-

Fig. 1.

Part of one of the round-towers of Calmar Castle.



a, Projecting band or hoop of stone; *b*, thin layers of slabs of stone and mortar, originally perhaps built under water; *c*, the beach covered by water when the sea is high.

* See Ankarsvard's Work on Calmar Castle.

curred to me that these rough slabs and cement may have been laid originally under water, and that the projecting rim of dressed stone may have formed the visible base of the building, which now rises to the height of about twenty-five feet above. This idea is rendered the more probable, as it is known that the castle had often defended itself from attacks on the side of the sea. I have since been informed by our eminent architect Mr. Wilkins, that it is highly probable, from the general analogy of buildings having a subaqueous foundation, that the courses of slaty stone were laid under water, and that the projecting fascia was alone intended to be seen above the level of the sea. Admitting this conjecture to be well founded, it would still prove that there has been a much slighter rise of the land since this building was erected, or during the last four centuries and upwards, than some writers have imagined, for it cannot have amounted to more than four feet in that time. Part of the moat on one side of the castle, which is believed to have been formerly filled with water from the sea, is now dry, and the bottom covered with green turf. It may have been in part silted up with sand and sediment, but a slight rise of the land would have contributed to its desiccation. A garden, composed of newly gained land in the harbour, between the castle and the town, in a place where there was sea half a century ago, clearly shows that the deposition of sedimentary matter may sometimes take place rapidly on this coast.

From Calmar I went to Stockholm, where I immediately found many striking geological proofs of a change in the relative level of land and sea, since the Baltic was inhabited by the same species of *Testacea* which it now supports.

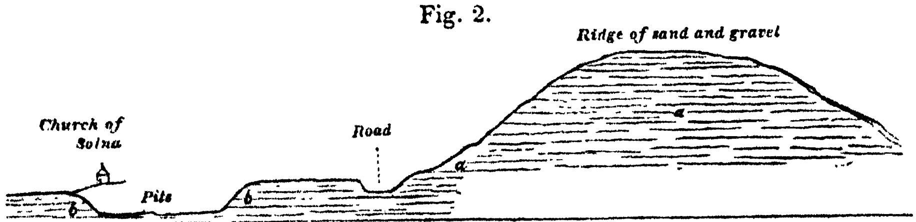
The country around Stockholm is in general low, seldom rising to more than 150 feet above the level of the sea, the fundamental rocks being gneiss and granite, which are often quite bare, presenting a surface for the most part smoothed and rounded, as if these rocks had formed for a long time the bottom of the sea, and had been worn and almost polished by the continual attrition of sand and pebbles. A mass of shingle and sand, here and there passing into loam, occasionally covers the rock; but it is rarely of great thickness, excepting along certain lines, where remarkable ridges of sand and gravel are seen, called in Sweden sand-oasar (åsar), the term 'oas' in Swedish corresponding to 'rigging' in Scotch, and for which we have no precise English synonym. These oasar are immense banks of sand, from fifty to several hundred yards broad, and from fifty to more than one hundred feet in height, which may often be traced in unbroken lines for a great many leagues through the country, but are breached occasionally by narrow transverse valleys. They usually run in a direction from north to south; generally terminate on both sides in a steep slope, and are sometimes so narrow at the top as to leave little more than room for a road. As they afford excellent materials for road-making, a great many of the highways in Sweden are carried either along the summit or base of these ridges, so that the traveller has many opportunities of observing their form and structure. In places where they are composed of large rounded boulders, of about the size of a man's

head, no stratification is observable; but where, as is more usual, they consist of gravel and fine sand, they are invariably stratified, in the same manner as sand and gravel in the beds of rivers. A great succession of thin layers repose one upon another, often at high inclinations. But this disposition can only be seen where there is a fresh section made in digging for gravel, the materials being so loose as to fall down and soon form a sloping talus.

I shall offer, in another place, some speculations on the probable origin of these ridges; and I have merely alluded to them now in order to explain the position of some fossil shells which I am about to describe. I had learnt from Professor NILSSON, of Lund, a gentleman well known to geologists by his valuable work on the fossils of Scania, that marine shells of species similar to those in the Baltic had been found near Stockholm; and soon after my arrival I was taken to the spot by Professor BERZELIUS. They occur at Solna, about a mile to the north-west of the city, at the foot of one of the great ridges of sand and gravel before mentioned; a ridge which, passing southward, traverses the city of Stockholm, and is said to have afforded fossil shells in the large pits at the Skantstull, in the southern suburbs.

The annexed section will show that there is little more than space for the road between the ridge and the gravel-pits at Solna.

Fig. 2.



a, a. Stratified sand and gravel. *b, b.* Same, with shells.

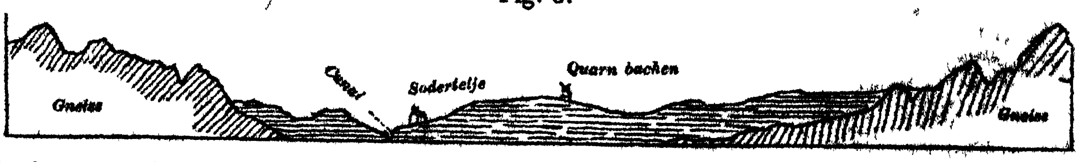
These pits lie between the church of Solna and the public cemetery of Stockholm. Both in the pits and in the adjoining ridge the gravel and sand is stratified, and in general no organic remains can be discovered in them; but in the pits, a little below the level of the road, there are some layers of loam mixed with vegetable matter, where shells occur in abundance. They consist principally of *Cardium edule* and *Tellina Baltica*, a great number of which have both valves united. Portions of the *Mytilus edulis* also occur; and there has evidently been a great accumulation of this shell in the stratum, but it is almost entirely decomposed, and is only recognized by the violet colour which it has imparted to the whole mass. The other shells which I found are, *Littorina crassior*, also the Common Periwinkle (*Littorina littorea*), and a small *Paludina* allied to, if not identical with, our English *Paludina ulva* (see Plate II. fig. 5.). The *Mytilus* and *Cardium* are all dwarfish in size, just as they are found in the brackish water of the neighbouring Gulf of Bothnia, and the whole assemblage of shells is such as characterizes the Baltic. The bed containing them has been ascertained by Colonel HÄLLSTROM to be thirty feet above the level of the Baltic;

so that they afford a clear indication of a change in the relative level of that sea to the amount of thirty feet since its waters were inhabited by the existing species of *Testacea*. On inquiring whether any other examples had been observed of similar deposits of shells, I was informed by Colonel Hällstrom that he had discovered them on the farm of Orby, near Bränkyrka, about three miles to the south of Stockholm. He obligingly accompanied me to the spot, where I found strata of marl and sand filling the bottom of a valley situated in a broken tract of ground where the fundamental rock is gneiss. This tract of land intervenes between Lake Maeler and the sea.

The shells are very numerous, and are for the most part imbedded in a peaty soil containing fragments of wood. The peat has perhaps been derived from sea-weed, large accumulations of which I saw recently heaped up in a bay of the Baltic near Sölvitsborg, intermixed with similar species of shells. The identity of the shells of Bränkyrka with those of the neighbouring sea was even more complete than at Solna; for in addition to the species before enumerated, I found the *Neritina fluviatilis*, a freshwater shell which lives in abundance in the brackish waters of the Baltic, and which I saw covering the rocks in the saltish water at Gräsö, near Oregrund. The Baltic variety is small, and usually black; but both in the recent and fossil individuals it sometimes exhibits its usual variety of colours. Some specimens also of a land shell (*Bulimus lubricus*) occurred with the marine at Bränkyrka.

The height of these shells has been determined by Colonel Hällstrom to be seventy Swedish feet above the Baltic; so that they indicate a fall of the waters, or rather a rise of the land, to that amount, since the neighbouring gulf was inhabited by this assemblage of *Testacea*. But the most remarkable spot where these Baltic shells occur in a fossil state is still further to the south, at Södertelje (see the Map, Plate I.), about sixteen miles south-west of Stockholm, where they are found elevated more than ninety feet above the sea. At Södertelje a canal was cut in 1819 across a barrier of sand, gravel, and clay, which separated Lake Maeler from a long narrow inlet or frith of the Baltic. The canal is, in fact, carried through the bottom of one of those valleys so common in this district, of which the sides consist of rocks of gneiss, and the bottom of the same covered by more recent deposits. The accompanying transverse section (fig. 3.) will explain this geological structure.

Fig. 3.



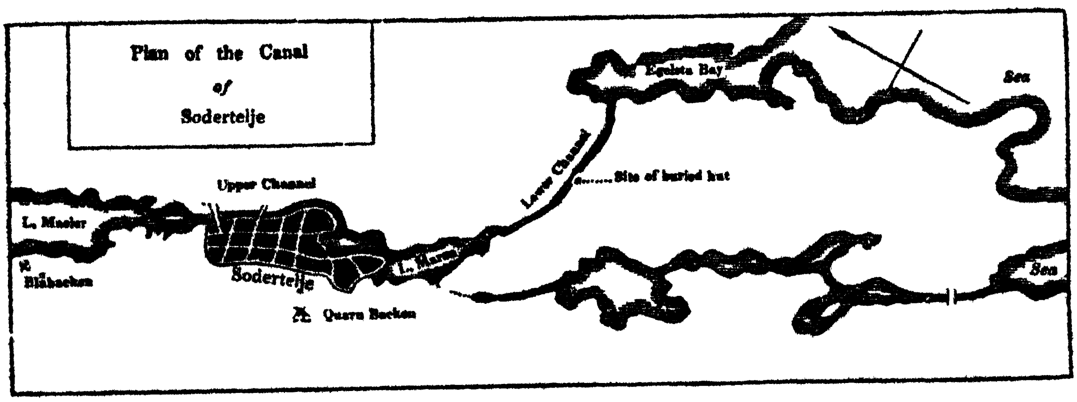
Section across the valley of Södertelje, showing the position of the new deposits in relation to the gneiss.

The boundary hills of bare rock rise to the height of two hundred feet, the newer formation being in some places about one hundred feet high, while on others, as on the site of the Lake Maren, there are hollows which sink beneath the level of the sea.


Christiania
Carlstadt
Orebro
Upsala
Sala
Westeras
Norrtelje
Linde
Koping
Askersund
Nykoping
Soderkoping
Linkoping
Grenna
Jonkoping
Eksjo
Gothenburg
Halmstadt
Laholm
Weksiö
Calmar
Carlskrona
Christianstadt
Lund
Malmo
Koenigsberg
Dantzig
Long. East of Green.

In these recent strata of loam, sand, and gravel, marine shells have been found at various altitudes, as may be seen by Colonel NORDSWALL's paper in the Transactions of the Royal Academy, where a ground plan is given of the canal and the surrounding district, of part of which I subjoin a reduced copy *. I found at the Quarnbacken (see diagram, fig. 4.), at the height of about ninety feet above the level of the sea, the same species of shells as those at Solna before mentioned, imbedded in a marly clay, which derives a violet colour from the decomposition of the *Mytilus edulis*. Again, the same assemblage of shells may be seen in the Blåbacken, or "blue hills," a neighbouring locality, where a bed of marl about three feet deep rests on the gneiss at the height of about one hundred feet above the sea. Here the violet colour of the decomposed *Mytilus edulis* is so remarkable as to have given a name to the hill. The shells, with the exception of the *Mytilus*, are in general very entire. The breadth of the Södertelje valley, between the opposite boundaries of gneiss, varies from about half to three quarters of a mile; and the newer shelly deposit, which sometimes constitutes a nearly level platform, at the height of sixty feet or more above the canal, has precisely the appearance of the Subapennine formations in Italy, or at the base of the Maritime Alps, where they are seen at inferior elevations, filling the bottom of valleys in the older rocks, or flanking hills of higher antiquity and of inclined stratification. It is only by aid of the shells so exactly corresponding to those of the Baltic that the geologist can at once decide on the comparatively modern origin of these Swedish strata.

Fig. 4.



The distance between the nearest points of Lake Maeler and the sea, now united by the Södertelje canal, is nearly a mile and a half English, the general line of the canal being from north-west to south-east, and the depth of the strata cut through varying from fifteen to more than sixty feet.

First a communication was made which united Lake Maeler with the small lake, or mere, called Maren (see plan); and this passage was called the upper channel. Here a horizontal bed of marl was passed through, of a violet colour, like that of the

* Kongl. Vetenskaps-Academiens Handlingar, 1832.

Blåbacken, and containing the *Cardium edule.* Besides the shells, several buried vessels were found in this channel, some of them apparently of high antiquity, there being no iron in them, and the planks being fixed together by wooden pegs. In another place, however, an anchor was dug up, as also, in one spot, some iron nails. In the lower channel, or that which united Lake Maeler with the bay of the sea called Egelsta Wiken, two similar beds of marine shells were found, one at the height of eighteen and the other of forty Swedish feet above the level of the sea.

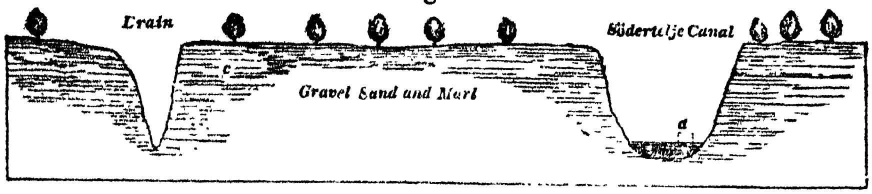
But a much more remarkable discovery was made in the lower channel. Here the excavation commenced in a hill, or platform, covered with a forest; and after digging down about fifty feet through stratified sand, gravel, and clay, they came upon what appears to have been a small wooden house, the site of which is marked on the plan *a*, fig. 4. The floor of this building was on a level with the sea. Colonel NORDEWALL has stated in his account, that the mass which covered the house was thirty-four feet thick: but he perhaps wrote ells (a Swedish ell is two feet); for Captain CRONSTRAND, an engineer who superintended the whole excavation, and who accompanied me to the spot, assured me that it was at the depth of about sixty-four feet. In other respects this engineer's account agrees with that of Colonel NORDEWALL; but he has enabled me to add some particulars, which I shall now mention.

The stratification of the mass over the house was very decided, but for the most part of that wavy and irregular kind which would result from a meeting of currents. It contained here and there very coarse gravel, and some boulders of considerable size. At the bottom of the whole, a mass of very fine sand was entered, in which the appearance of the four walls of a square building was discovered. Attention was not paid to this phenomenon soon enough to decide whether there were any remains of a roof. An attempt was made to dig round the walls, and leave them standing; but the wood was perfectly decomposed, and crumbled down like dust when all support was removed. But when they reached the level of the sea they found the timber of the walls preserved. At the bottom, on what may have constituted the floor of the hut, an irregular ring of stones was found, having the appearance of a rude fireplace, and within these was a heap of charcoal and charred wood. On the outside of the ring was a heap of unburnt fir-wood, broken up as for fuel, the dried needles of the fir and the bark of the branches being still preserved. The building was about eight feet square, and was supposed to have been merely a fishing-hut, occasionally resorted to at the fishing-season. Captain CRONSTRAND says that the building was enveloped with sand as fine as if blown by the wind.

I visited the nearest spot at which shells were found, in a deep drain not far from the former site of the fossil house, (see plan, fig. 5.) and am satisfied, from their position and from the occurrence of shells at different spots and heights in excavating the "upper channel," that the strata which covered the house, like all the rest cut through by the Södertelje canal, were marine. It appears evident, therefore, that this building must have been submerged beneath the waters of the Baltic to the

depth of sixty-four feet; and before it was raised again to its present position, which is about even with the level of the sea, it had become covered with strata more than sixty feet thick.

Fig. 5.



a. Site of the buried hut. *b*. Water in the canal. *c*. Bed of violet-coloured marl with *Cardium edule*.

If the buried vessels alone had been found, we should merely have been called upon to suppose that they had sunk to the bottom of a fiord, which was afterwards silted up and then upraised; but the situation of this house seems to require far greater changes of level. Had nothing been observed but the wooden walls, we might have imagined that the hut was carried away during an inundation, for I was told of a house that was floated off entire during a flood, in the north-east of Sweden, in consequence of the artificial drainage of a lake. But the fireplace and charred wood on the floor seem entirely opposed to such an hypothesis. To imagine a subsidence of the land to the amount of more than sixty feet, and a subsequent elevation, or in other words a series of movements analogous to those by which the phenomena of the Temple of Serapis have been explained, appears necessary; yet this is undoubtedly to assume far greater revolutions in the level of the land, since fishing-huts were first erected in Sweden, than history or tradition would have led us to anticipate. As to the fine sand in which the house was enveloped, it may be compared to the sand which is known to collect rapidly and form a mound over wrecked vessels which have sunk and presented an obstacle to a marine current charged with sediment.

I ought to state that I was unable to examine the remains of the house, since it was entirely cut away, having stood, as will be seen by the section (fig. 5.), in the exact line of the canal, the surface of the waters of which, like the foundation of the house, were situated at about the mean level of the sea; for Lake Maeler and the Baltic are so nearly on a level, that when the Baltic rises two or three feet above its mean height, the same lock at Södertelje which usually serves to convey vessels from the Baltic up into Lake Maeler, is used to convey them up in a contrary direction from the lake into the sea. But although I could not see the relic of the fishing-hut itself, I may observe that I had the advantage of conversing with the two eminent engineers who were witnesses to the fact, and who, being greatly astonished at the discovery, took careful notes of the phenomena at the time. They at first conceived that the building might have been part of some well, although this seemed highly improbable, not only from the size of the wooden structure, but from the occurrence of springs at the surface in the immediate neighbourhood. It was only when the fire-

place was found that they could form no other opinion than that it had been a human habitation. In order to explain the position of beds of shells at various heights in the strata intersected by the canal, an hypothesis was suggested by Colonel NORDEWALL, in his published report, that Lake Maeler may once have been shut out from the sea by a high barrier. Sand, gravel, and shells may then have been deposited at its bottom, which on the subsequent removal of the barrier were left at their present height above the lake. But if the shells had been submitted to a conchologist, they would have been at once recognized as consisting for the most part of marine species, such as do not exist in the present waters of Lake Maeler, but are characteristic of the Baltic. Whatever doubts, therefore, may hang over the causes which brought the hut into the extraordinary position in which it was discovered, it is impossible to reflect on this and the other facts brought to light during the excavation of the Södertelje canal, without being convinced that very important movements have taken place in the land and the bed of the sea since the Baltic was inhabited by the existing *Testacea*, and even since the sea was navigated by vessels, and this country inhabited by man.

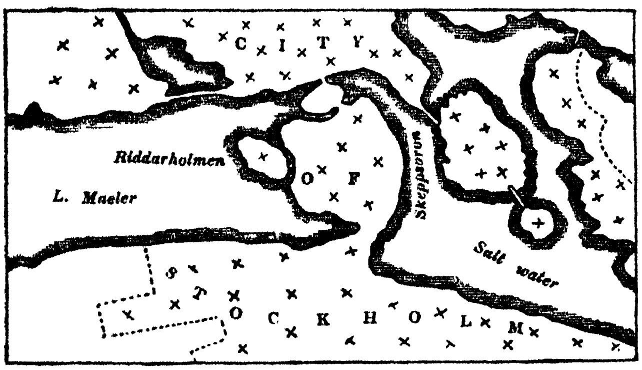
In regard to the shells, I may observe that the *Mya arenaria* is the only one found by me in great abundance in any part of the Baltic which I did not see among the fossils of any of the localities already mentioned, or those afterwards to be alluded to, further to the north. But this shell does not, I believe, extend so far north in the Gulf of Bothnia as Södertelje; I could not find it even at Calmar, and further south, at Sölvitzborg, it was rare, and of very small size. The analogy, in fact, of the fossil shells to those now living in the Bothnian Gulf is most complete: the shells are the same species, partly freshwater and partly marine, the species taken collectively being few in number, and the marine attaining a smaller average size than in the ocean, where the water is more salt. The *Tellina Baltica* is everywhere in great abundance. Hence we may conclude, that since the time when an inland sea of brackish water, like the Baltic, existed in the North of Europe, considerable fluctuations in the position of land and sea have taken place; a conclusion to which I shall revert in the sequel.

The elevated position of the marine shells around Södertelje prepares us to expect similar deposits scattered far and wide over the valleys bordering the various branches of Lake Maeler. Accordingly, in examining the country about forty-five miles north-west from Södertelje, between the towns Torshälla and Arboga, I was fortunate enough to meet with abundance of *Tellina Baltica* (see the variety represented in Plate II. figs. 3, 4.) in an unctuous clay, of a deep blue colour when wet, which filled the bottom of a valley near Lake Maeler, in a district of gneiss covered with huge erratic blocks. This locality, which is by far the most distant from the Baltic of all the places where similar beds with marine shells had previously been observed, lies between the village of Smedby and Kongsör, about seventy miles from Stockholm, and more than eighty from the general coast line. The clay is exposed to the depth of fifteen feet, being cut through by a streamlet, which is crossed by a small bridge

on the high road. The deposit is elevated only a few yards above Lake Maeler, and is therefore about the same above the Baltic; but the formation extends to greater heights in this and adjoining low lands, as do associated beds of gravel and sand, in which I could not detect any fossils.

After viewing these geological phenomena, I was well inclined to receive favourably any probable evidence brought forward to prove that the land has been rising in recent times in the neighbourhood of Stockholm; but I must confess that, on close investigation, I was disappointed in finding that several of the proofs relied on by some writers were very equivocal. Among other facts, it has been noticed that the level of Lake Maeler has been lowered in very modern times; and it is clear that the waters of this lake would appear to fall, together with the sea, if there be a general rise of the land, since Lake Maeler joins an arm, or fiord, of the Gulf of Bothnia at Stockholm, the salt and fresh water meeting in the middle of the city. The lake is generally three feet higher than the sea; but the line of separation is not constant, and when the Baltic rises very high, its waters flow for some miles into the lake. In that part of the town called the Riddarholmen, immediately above where the waters of the lake meet the sea, (see Map, fig. 6.,) some of the buildings have of late years become insecure, because the level of Lake Maeler has fallen, so that the piles on which the buildings rest are not constantly under water as of old. The tops of these piles being now every year alternately wet and dry, they are continually rotting away. This fact is unquestionable; and I saw the houses, which, in consequence of this failure of support, are much rent, and out of the perpendicular.

Fig. 6.

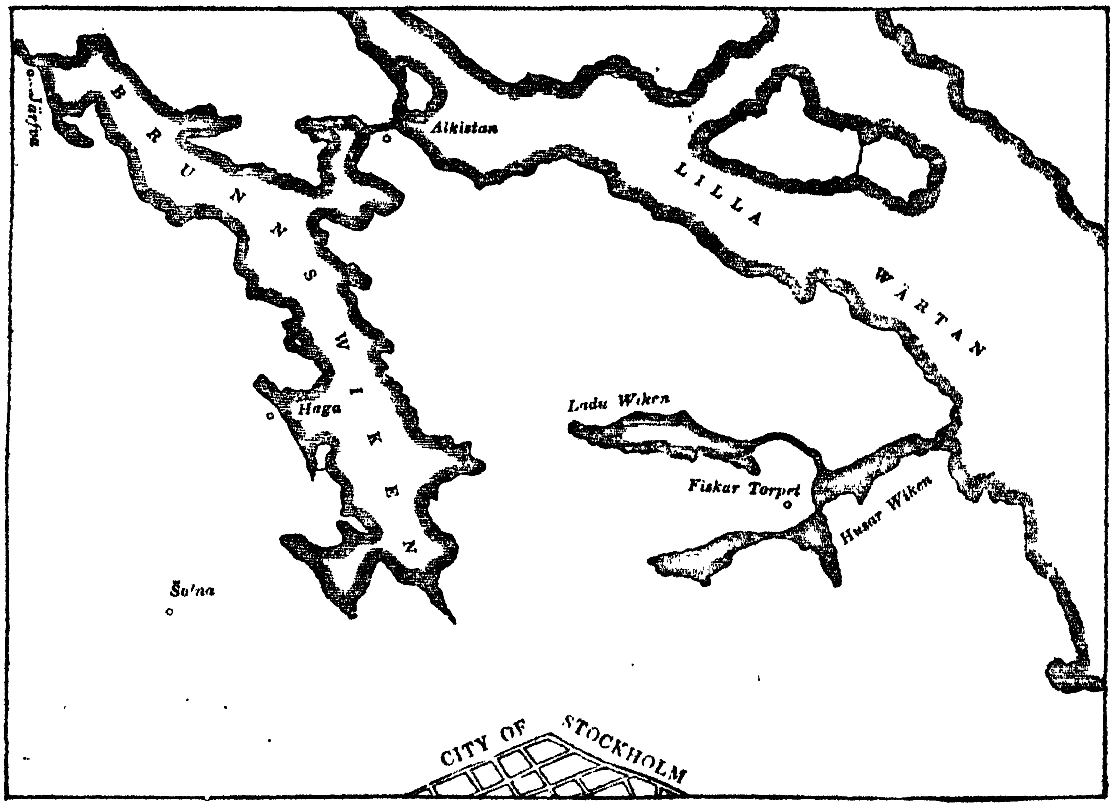


But during the time that this change has occurred, no corresponding fall has been observed in the neighbouring quay, or Skeppsbron, which is filled with brackish water, and which ought to have been equally affected on the supposition of a general rise of the land; and we naturally, therefore, inquire whether some particular circumstances have not of late years given a freer outlet to the waters of Lake Maeler, so as to cause them to sink. Now several Swedish engineers remarked to me, that the decay of the piles had taken place since the removal of the two old bridges in Stockholm, which being supported on a great number of wooden piles, obstructed the free discharge of the lake, the waters of which now pour in a rapid and unbroken current through the large arches of the new bridge; and secondly, they observed that the canal of Södertelje has formed, since the year 1819, an entirely new line of communication, by which the waters of Lake Maeler have of late years flowed out into the sea. Can any one doubt for a moment, that if the old bridge should be restored

and the Telje canal again closed up, the waters of the lake would immediately stand at a higher level*?

There are some marks in the suburbs of Stockholm which serve, I think, to set narrow limits to the extreme amount of elevation which can by possibility have taken place during the last three or four centuries. To one of these, the Fiskartorp of Charles XI., I shall particularly allude, (see Map, fig. 7.,) because an attempt has been made to draw from it the opposite inference of a rapid elevation of the land.

Fig. 7.



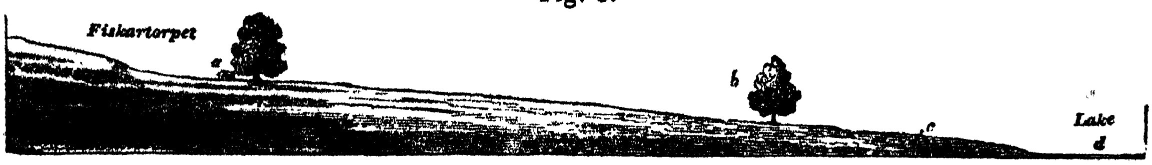
Map of the northern environs of Stockholm, showing the site of the Fiskartorp.

This fishing lodge is situated on a promontory surrounded on three sides by lakes (see Map, fig. 7.). The lodge is 131 yards distant from the nearest water, and twenty-three feet above its level. By the side of it is a large oak, and a second one of considerable age between it and the lake, only forty-six yards from the margin of the water, and having its base only ten feet above the level of the lake, which at the time that I visited it stood at least one foot below its mean height. (See Section, fig. 8.) Mr. Strom, Keeper of the Royal Woods and Forests, assured me that the age of this oak cannot be less than four centuries. There are already some signs of decay at its top, and its diameter at the height of five feet above the ground is four feet four

* Professor Johnston, in his paper in the Edinburgh New Philosophical Journal, No. 29, July 1833, has by mistake represented the houses where the piles are giving way as situated on the side of the Skeppsbron instead of the Riddarholmen.

inches. As Mr. STROM is perfectly acquainted with the average rate of growth of the oak in different kinds of soil in this country, and has cut down some in the neigh-

Fig. 8.



a. The fishing-house. *b*. The lower oak. *c*. Ancient site of the small cabin for fishing-tackle. *d*. The Husar Wiken.

bouring grounds which could be shown by their rings of annual increase to be more than six hundred years old, I consider his opinion as worthy of full confidence. This gentleman showed me an ancient plan in which the Fiskartorp and both the oaks were laid down; as also a small cabin, which, in the time of CHARLES XI., who died in 1697, was placed between the lower oak and the lake. It was not a boat-house, but had been merely used for preserving the oars and fishing-tackle. Being in a state of great decay in 1824, it was removed by Mr. STROM. Now it is improbable from what is known of the habits of the oak in this country, that the lower oak grew close to the water's edge originally; and if its base be now only eight feet above the mean level of the lake, it is clear that the rise in each century must have been very slight, although it may undoubtedly have amounted to ten inches in a hundred years, which would accord with the estimate of the best-informed scientific men in Sweden, in regard to the gradual rate of the rise of land at Stockholm. Professor JOHNSTON appears to have confounded the cabin, which has been removed, with the Fiskartorp, which is still standing, the latter having been frequently repaired, as a memorial of CHARLES XI.; for Mr. JOHNSTON states, that "the fishing-hut formerly stood close by the deep water, though no longer near any spot where the favourite amusement of the monarch can be enjoyed*."

Even the lower cabin did not stand near deep water so lately as a century and a half ago, but appears by the ancient plans to have been nearly as remote as now from the shallow Husar Wiken. I fully agree, however, with Professor JOHNSTON, that it appears clear from ancient documents and tradition, that the three lakes Husar, Ladu, and Uggel, which together formed, in the time of CHARLES XI., what was called the Gulf of Fiskartorp, have since grown much shallower, and have been in part converted into land; a change which may perhaps have been due, in part at least, to a slight general upheaving of the whole country. But although I do not dissent from Mr. JOHNSTON's general proposition, I ought to mention here that I consider another of his proofs derived from the neighbourhood of Stockholm as altogether untenable. Speaking of the Bruns Wiken, a beautiful lake in the northern suburbs of the city which skirts the woods and pleasure-grounds of the palace of Haga, (see Map, fig. 7.,)

* Edinburgh Philosophical Journal, No. 29, p. 39.

he says, "the position of this lake shows that it has formerly communicated with the sea, though now it is considerably above it and entirely inland. As the sea retired, this sheet of water would also have been drained off, had it not been dammed up at the only outlet (at Alkistan) to preserve the beauty of the promenade, one of the finest in the neighbourhood of the city. At present it is dammed up to the height of four or five feet, and the character of all the land around shows that in ancient times it has been very much higher and more extensive."

Now a reader would infer from this description, that but for an artificial dam this lake would have been laid dry; but the fact is that it fills a deep hollow in the granitic rocks of this district; and the only effect of the small dam is that the mean height of the water is somewhat more uniform throughout the year. The outlet alluded to is at Alkistan (see Map, fig. 7.), where a slight wooden dam has been erected, so small, that every year in the spring the water flows over it; so that the annual extreme height of the water is still the same as it would be if the dam were removed. When I visited the spot in June, the water was two feet lower than the top of the dam, and scarcely more than a foot above the bottom. The tract of land which separates the lake from the sea is about a hundred paces broad, and is composed of granite, over which the stream flows which issues from the lake.

I shall now pass to the country around Upsala, about forty miles north-north-west of that around Stockholm last described (see general Map, Plate I.). In its geological structure it resembles that of Stockholm, the fundamental rocks being here also gneiss and granite, partially covered with newer deposits and with erratic blocks; but near Upsala there is a much larger quantity of clay in the overlying formation. A section of this clay is well seen at Ulfva on the banks of Fyrisä, a spot which I visited with Mr. Marklin of Upsala. The thickness of clay here exposed in a vertical section is between thirty and forty feet, and the river is probably as much more above the level of the sea. This stiff blue clay reminded me much of the Subapennine clay of Italy. In some parts it contains no shells; but in others the *Tellina Baltica* entire, with both its valves and the epidermis, is very abundant. It is precisely the same variety of this shell as I found before near Torshälla (see p. 10, and Plate II. figs. 3, 4.). The *Mytilus edulis* also occurs, often much flattened, and occasionally covered with the small white flustra now so commonly attached to it in the Baltic. In some of the associated strata there is much vegetable matter, exactly resembling sea-weed. I could find none of the littoral shells which I before mentioned as associated with the *Mytilus* and *Tellina* near Stockholm.

One of those ridges of sand and gravel which I have before described as being frequent in Sweden, passes through the suburbs of Upsala, running in the usual direction nearly north and south. Its summit, according to the barometrical measurement of Professor Wahlenberg, rises more than a hundred feet above the river which flows at its base. Its structure is laid open in large pits, one of them about seventy feet deep; and these sections show that the mass consists for the most part of a con-

tinued series of thin layers of sand, loam, and gravel, in part horizontal, but in some places, and for a limited space, inclined at an angle of more than fifty degrees, with numerous small vertical rents occasionally traversing the beds. Whether these have been occasioned by subterranean movements, or during the drying and settling of the mass when it was first raised above the waters, is a point on which I can offer no conjecture. The inclination of the strata, resembling that in gravel-beds, I attribute chiefly to original inequalities in the mode of its deposition. Here, as in other places, I could find no fossils in the beds of pure sand and gravel, nor did I meet with any in the blue clay which seems to crop out from beneath the sand at the bottom of the hill. But fortunately, near the castle at Upsala a thin bed of violet-coloured marl, full of shells, has been cut through at the bottom of a gravel-pit near the top of the ridge. This marl, which forms a horizontal layer only three inches thick, is within about twelve feet of the summit of the ridge, and about eighty above the sea. It contains the *Mytilus edulis, Cardium edule, Tellina Baltica, Littorina littorea, Paludina ulva?* Both above and below this marl are strata of gravel, and some of the overlying beds contain round boulders a foot or more in diameter.

This is the only place in Sweden where I met with any fossils in the midst of one of the sand-oasar, or ridges of sand and gravel. The fact of finding the recent shells of the Baltic in such a position appears to me of the highest interest, especially because on the summit of this, as of other ridges, I found large erratic blocks resting immediately on the uppermost layers of gravel or fine sand. In that part of the ridge south of the town called Pålacksbacken, these blocks are abundant, and are on the very summit, appearing to be all superficial, for I could find none *in situ* in the deep gravel-pits which intersect the ridge. I examined these blocks in company with Professor WAHLENBERG, and found them to consist of angular masses of gneiss and granite, the larger ones rarely exceeding nine feet in length; but we measured one which was no less than sixteen feet long, thirteen high, and eight broad. It follows, therefore, that by whatever cause these enormous fragments of granite rocks have been conveyed to their present sites, some of them at least have been transported thither since the Baltic was separated from the ocean and inhabited by the existing species of *Testacea.*

I may observe also, that the occurrence of layers of marl containing littoral shells, as above described, in the midst of a stratified ridge of sand and gravel, is opposed to the theory of those geologists who refer the formation of such ridges to a violent flood or debacle rushing from the north. The perfect preservation of the shells at Upsala, and the repeated succession of thin alternating layers of gravel, sand, and loam, which are seen almost everywhere, imply a gradual, and at times a very tranquil, deposition of transported matter. If I am asked for a more probable hypothesis in the room of that to which I object, I may state that these ridges appear to me to be ancient banks of sand and shingle, which have been thrown down at the bottom of the Gulf of Bothnia, in lines parallel to the ancient coast during the

successive rise of the land; or in other words, during the gradual conversion of part of the gulf into land. I conceive that they may have been formed in those tracts where a marine current, flowing as now, during the spring when the ice and snow melt, from north to south, came in contact with flooded rivers rushing from the continent, or from the west, charged with gravel, sand, and mud. According to this view, these large Swedish ridges may be compared to smaller banks known to have been formed within the last five or six centuries on the eastern coast of England, at points where a prevailing marine current from the north meets rivers descending from the interior, or from the east. In such situations the river, instead of entering the sea in a straight line, is deflected at a right angle, and runs from north to south between the land and the new-formed sand-bank. The deep narrow breaches which occasionally occur in many of these ridges in Sweden, precisely resemble those which a flooded river or an inundation from the sea sometimes makes through our smaller banks above alluded to. If this explanation be admitted, I conceive that the steep escarpments often presented on both sides of the oasar or ridges of sand, may be almost entirely due to their original form, and not to subsequent denudation. As to the manner in which the erratic blocks have been lodged on the highest parts of these sand-banks, I fully adopt the opinion of those who believe them to have been carried by ice, respecting the agency of which I shall have more to say in another place.

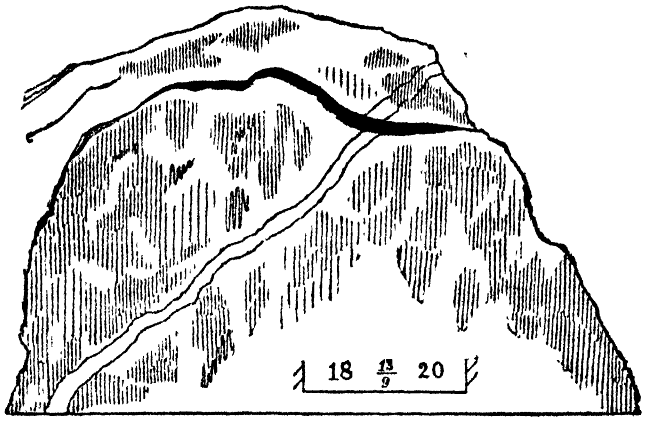
The low meadows near the town of Upsala are not many feet above the level of Lake Maeler, the most northern arm of which reaches near to that place, which is distant about fifty miles from Södertelje, before alluded to, at the south-eastern extremity of the same lake. If the opinion, therefore, of the rise of the land be well founded, the whole of Lake Maeler, and the low lands adjoining, must have been covered with salt water at no very remote period in history. Professor Wahlenberg pointed out to me a meadow to the south of Upsala in which the *Glaux maritima* and the *Triglochin maritimus* now flourish, plants which inhabit salt marshes bordering the sea. These same species have, it is true, been found in the interior of Germany and France near saline springs; but in the country of Upsala there are no salt springs; and this botanical phenomenon seems to confirm the opinion that the salt waters have only receded in very modern times from these lands, and that the rains have not yet had time to dissolve and wash away all the salt which may have been originally precipitated when this tract was laid dry.

Oregrund.

The next region which I examined was the coast near Oregrund, a port about forty miles north-east of Upsala. During the survey of 1820, before alluded to, a mark was made near this place on the rocky cliffs of Gräsö, a long narrow island which lies opposite to Oregrund. On my visit to this island I was accompanied by Lieut. Olof Flumen, a gentleman of the pilotage establishment, who cut the mark in 1820. It is much to be regretted that neither he nor any other observer, as far as I could learn,

had visited these spots since the marks were made. No place could have been better chosen for the purpose: the letters and lines, which are still as fresh as if newly made, have been cut upon the vertical face of a cliff of gneiss, which is free from lichens, and which plunges to the depth of about three fathoms perpendicularly beneath the water. I subjoin a sketch (fig. 9.) which I made of the rock and mark as they appeared on the 1st of July 1834. A vein of granite, composed of felspar and quartz, traverses the gneiss in an oblique direction above the mark. The rock is stated by BRUNCRONA to be in latitude 60° 18′ N. It is situated at the south of Strandtorpet and north of Käringsundet. The length of the horizontal line is twenty inches and a half; the figures express that the mark was cut on the 13th day of the ninth month (September) in the year 1820, and the runic letters at the beginning and end of the line are the initials of OLOF FLUMEN.

Fig. 9.



Mark at Gräsö near Oregrund.

At the above date the horizontal line was exactly at the level of the sea on a calm day, when the water was supposed to be at its standard level. When I visited the place on the 1st of July 1834, the line was five inches and a half above the surface of the water; and Lieutenant FLUMEN and the seamen thought that a slight wind which was then blowing from the north-north-west, directly down the sound between Oregrund and Gräsö, caused the water to be an inch or two higher than it would have been had the sea been as perfectly calm as on the day preceding my visit. I found the pilots, both here and at other places on this coast, to be of opinion, that notwithstanding the fluctuations of level caused by the wind, a person well accustomed to this sea can decide whether, on a particular day, the water is an inch or two above or below its standard level. There had been several calm days without wind before I arrived at Oregrund, and I was assured that the sea was in a state of rest similar to that of the day which had been chosen fourteen years before for making the mark. Before we came to the spot, both Lieutenant FLUMEN and the boatmen expressed their persuasion that I should find the sea below the mark, because they declared that either the waters of the gulf were always sinking, or the land on this coast was gradually rising. To confirm this opinion the sailors pointed out several rocks which they well remembered to have been barely covered with water in their younger days, or about forty years ago, but which now rise between one and two feet above the water. Among others they took me to a small insulated rock in the sea, opposite Domaskärsund, which they recollected to have once been nearly two feet lower, at which time the neighbouring channel, which I saw nearly dry, had allowed a loaded

boat to pass. So strong is the conviction of the fishermen here, and of the seafaring inhabitants generally, that a gradual change of level, to the amount of three feet or more in a century, is taking place, that they seem to feel no interest whatever in the confirmation of the fact afforded by artificial marks, for they observed to me that they can point out innumerable natural marks in support of the change; and they mentioned this as if it rendered any additional evidence quite superfluous.

The sea deepens rapidly near the coast at Oregrund, and there is twenty-eight fathoms water in the bay. Along the shore is a broad band of bare gneiss traversed by granite veins, which ramify in every direction, and consist chiefly of felspar in large crystals. In many places this sloping band of bare rock, having a smooth surface, extends up for a hundred paces from the sea, covered only with a scanty coating of lichens. The gneiss, where it approaches within eighteen paces of the sea, is so smooth and polished that it is difficult to walk upon it. The surface swells into those rounded flattened forms which are so common in the forests in the interior of Sweden, where grass is frequently unable to establish itself on so hard a foundation. Not even lichens can grow in some parts where veins and beds of quartz appear; but trees take root in the clefts of the granite and gneiss, rising amidst vast erratic blocks, resembling those which, in equal numbers and of equal dimensions, crowd the greater part of the shores and islands of the Bothnian gulf.

From Oregrund I went on to Gefle, about forty miles to the north-west. In a low part of the intervening country, near the village of Skjerplinge, I came to a large tract of stiff blue clay, like that near Upsala, covered with sand six or eight feet deep. In the clay I found the *Mytilus edulis* and the *Tellina Baltica*. I was informed that marine shells are met with abundantly at a much higher level in a hill of sand near Skjerplinge, where also, according to tradition, a large iron ring, such as ships are attached to, was formerly found fixed in the soil.

My attention was repeatedly called to low pastures from one to three miles inland, where the old inhabitants or their fathers remembered that boats and ships had sailed. The traveller would not have suspected such recent conversions of sea into *terra firma*; but there are few regions where a valley newly gained from the sea may so rapidly assume an air of considerable antiquity. Every small island and rock off this coast is covered with wood, and it only requires that the intervening channels and fiords should dry up and become overspread with green turf for the country to wear at once an inland aspect, with open glades and plains surrounded by well-wooded heights.

Among other stories of wrecked vessels found in the interior, I was told at Gefle that a vessel and an anchor had been found in a hill of sand and gravel at Uggleby, sixteen miles from the sea, in the parish of that name. Colonel Hällstrom tells me that similar traditions are common in Finland, and that a wreck is said to have been found there at Laihela, two miles from the sea.

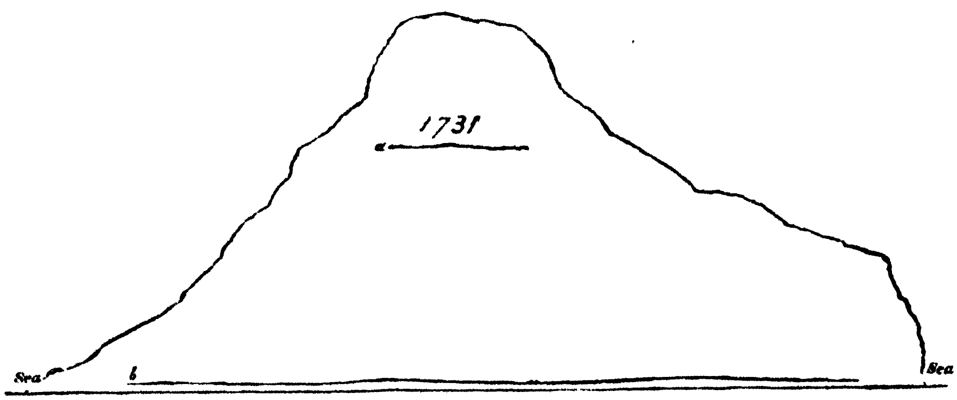
On both sides of the river at Gefle I found land gained from the sea, within the

memory of persons now living; and its gradual extension here, and in other places to the north and south, is attributed by the natives to a slow but constant change in the relative level of land and sea. In this place the deposition of fluviatile sediment must cooperate with other causes; but the shallowing of the water and its conversion into land are too universal to be explained by sedimentary accumulations alone. Preparations are making to remove the harbour farther from the town, in consequence, as I was assured, of the continued fall of the water rendering it every year more difficult for ships to reach the ancient wharfs.

I visited two marks near Gefle, one of them cut in 1731 in the island of Löfgrund, twelve miles north-east of that port, and another made in 1820, about six miles farther north. The first of these marks (that of Löfgrund *) was carved by one Rudberg in 1731, on a fixed rock of mica-schist, in the middle of a small sheltered bay on the east side of the island. The mica-schist is very hard and full of garnets, the highest part of the rock being only four feet above the water, and its length and breadth about fourteen feet. There is a depth of water of about seven feet and a half on the side where the mark is made. The annexed sketch (fig. 10.) will give some idea of the outline of that side of the rock and of the mark.

Fig. 10.

Rock in the Harbour of Löfgrund.



a. The upper mark. *b*. The lower mark.

The horizontal line, which is somewhat irregularly cut, is known to have been originally made at the mean water-level. When I measured it on the 3rd of July 1834, this line was two feet six inches and a half above the mean level of the water; but as the wind was blowing from the east-north-east, the chief pilot of Gefle, who accompanied me, declared that I ought to add at least four inches more in order to express the full difference of the ancient as compared to the present level of the sea. It will appear that I had afterwards good reason to believe that this estimate was not exaggerated. Even when this allowance is made, the fall, in the space of somewhat more than a century, is not quite equal to three feet. There is a lower horizontal mark two feet five inches long, irregular and without any date, which, when I ex-

* Sometimes called Löfgrundet, the final *et* being the definite article in Swedish.

amined it, was washed and almost covered by the ripple on the surface of the water. It is not enumerated by BRUNCRONA as among those which were cut in 1820; but my boatmen and the fishermen on the island said it was cut in 1820. Although occasionally covered by the small waves, it was one inch and a half above the mean level of the water, and would probably have been four inches or more above it on a calm day.

It has been observed that lichens grow nearly to the water's edge on the rocks skirting the Gulf of Bothnia, and certainly the lower border of this line of vegetation often appears very distinct when viewed at a short distance; the rock below, where it is alternately wet and dry, remaining of its natural colour, which is usually very much contrasted with that of the surface, where it is coated with lichens. Now it has been proposed to measure and note the distance of this line of vegetation above the sea, and then to determine, after a certain lapse of years, the rate of elevation of the land, by observing how much lower the lichens have descended. With a view of furnishing data to future observers for such comparisons, I endeavoured, at Löfgrundet and other places, to ascertain the height of this line of vegetation, but without success, for it always appeared to me undefinable. Not only is it very uneven, but sometimes, after passing over a space of bare rock, we come down again to some straggling lichens growing luxuriantly nearly to the water's edge.

VON BUCH mentions in his Travels* that he found a large quantity of fine-grained red sandstone, used as a building-stone, at Gefle, containing small nodules of asphaltum. He was told that these stones were found nowhere *in situ*, but were thrown up by the sea upon the skär, or that line of rocks and islands which bounds the coast off Gefle. I found the shore of the isle of Löfgrund strewed over with these schistose red-sandstone blocks. They have the form of large flat slabs, with angular edges, as if they had been just taken from a quarry. They were exposed to a hot sun, and the black pitchy matter was oozing out abundantly from numerous pores. The planes of stratification presented those undulations called ripple-marks. On my inquiring from whence they came, I was assured by the fishermen that a fresh supply of such masses was brought to the coast from time to time by the sea. I remarked that their size was such that the waves could not have power to move them, that there were no rocks like them in the neighbourhood, and that they were not rounded by attrition as if rolled at the bottom of the sea. One of the fishermen replied that the ice might have brought them, and he undertook to show me much larger blocks which had been stranded recently on different parts of the skär. I accordingly went to a small island called Hvitgrund in order to see proofs of this fact, and there I observed blocks of red granite, five or six feet in diameter, perfectly free from lichens, amidst other blocks of various sizes which were coloured grey, white, and black, by a coating of these plants. The sailors named other spots where I might see much larger blocks, perfectly bare, or only beginning to be covered, amidst

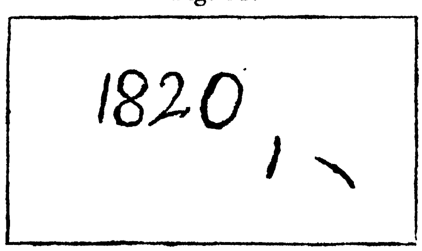
* Vol. ii. chap. v. French edition, p. 303.

thousands which, having probably lain for a great many years at the same height above the mean level of the sea, had entirely changed their colour. They declared that they well knew the exact date of the arrival of some of these blocks, which they observed would in time become as well coloured (or as thickly clad with lichens) as those older ones among which they had been thrown. On my demanding whether any of my informants had seen great stones floated by ice, they admitted that they had not; but the chief pilot stated that the drift ice on this coast is often *packed* so as to be eighteen feet thick, different sheets from five to six feet in thickness being driven one over the other; and when this happens, fragments of rocks might be frozen in and floated off on a rise of the water or a change of wind. The more usual mode, however, of explaining the manner in which ice operates is somewhat different. When the sea freezes in winter to the depth of about five or six feet, detached masses of rock lying on shoals are necessarily frozen in. Afterwards, when the water rises on the approach of summer, the ice, being buoyed up, lifts with it these stones, and they may then be transported by floating ice-islands to a great distance.

The next mark which I examined was that of St. Olof's Stone in Edskö (or Edsjo) Sund*, in the parish of Hille. There was no one at Gefle who was present in 1820 when the mark was cut, and unfortunately it is imperfectly and even incorrectly described in Bruncrona's Report. St. Olof's Stone is an immense erratic block, about thirty-six feet high above the water, forty long, and thirty broad, with precipitous and in some places overhanging sides. It consists of micaceous schist with garnets. It is situated in lat. 60° 52′ N. The mark is cut on the precipitous south-east side, at the base of which there is about a fathom's depth of water.

Bruncrona, in his Report, states that the mark consists of a horizontal line, upon which the date of the year 1820 is carved. Directions were probably given by him to this effect, but they have only been in part executed, for there is neither a horizontal nor vertical mark, but only two irregular lines to the right of the figures, as shown in the annexed sketch. It is also stated in the Report, that the water stood 1·92 foot under the lowest edge or base of the ciphers. Now unfortunately, the base of the letters do not form a perfectly horizontal line, the bottom of the last cipher being three quarters of an inch below the bottom of the figure 8. On the evening of July 3rd, I found the water-level to be exactly two feet below the base of the cipher, or the 0. The wind was blowing from the east-south-east, so that the water in the Sound, according to the pilot's opinion, was four or five inches above its level of equilibrium.

Fig. 11.



As this was the third time I had been told that the sea was several inches above its standard height, I determined to pass the night in Edskö, in hopes that the wind might fall, and that I might have an opportunity of repeating my observation during

* Colonel Bruncrona has called this Assiasund, but it is not known by this name at Gefle.

a perfect calm. At a very early hour the next morning the wind shifted to the north-north-west, and fell almost entirely, so that when I revisited St. Olof's Stone the surface of the water was perfectly smooth. I then found the level of the sea, as the pilot had expected, 3½ inches lower than on the preceding evening. This circumstance gave me much confidence in the opinion which he had previously expressed, that the water at Löfgrund was three or four inches above its standard level at the time of my observation.

The result, then, of my second visit was, that on a moderately calm day, with a slight wind blowing north-north-west, I found the level of the water, on July 4, 1834, two feet three inches and a half below the bottom of the 0, at the end of the figures 1820, or 3·58 inches lower than the water in the year 1820, supposing the measurement to have been then taken from the base of the last cipher. If it was taken from the base of the figure 8, then the difference between the water-level at the two periods compared would be three quarters of an inch greater.

It is much to be regretted that in the printed account of the cutting of this and other marks in the year 1820—21, no exact mention is made of the state of the sea and direction of the wind. I was merely assured generally that calm days were chosen, and circumstances avoided which are known to cause the Gulf to deviate from its standard level. This precaution I know to have been carefully attended to at Oregrund.

Mr. Von Hoff, in his important work entitled "The History of Natural Changes on the Earth's Surface proved by Tradition," has objected to the marks cut on the rocks of this coast that they were made on loose blocks, which may have been heaved up from their position by the sea and ice*. But the greater number of the marks have been set on fixed rocks; and even where this is not the case, the proof derived from such enormous masses as St. Olof's Stone is quite unexceptionable. I ought, however, to add, that Mr. Von Hoff has, in the third volume of his work just published, withdrawn his opposition to the validity of the evidence in favour of the rise of land now going on in the Baltic †.

Before I pass from Gefle to another part of Sweden, I may state that Colonel Hällstrom, to whom we are indebted for an interesting article on the marks made to determine the rate of change of level in the Bothnian gulf‡, informed me that the inhabitants of the opposite coast of Finland are as fully persuaded as those between Gefle and Torneo that either the waters are falling in their country or the land rising. The same gentleman observed, that notwithstanding the fluctuations of level in the Baltic at certain seasons, he never happened to examine any of the ancient marks, either on the Swedish or Finland side of the gulf, without finding the water below the marks. He also gave me some marl of a violet colour, which he had lately brought from Nådendal, near Åbo, in Finland, found at the height of sixty feet above the level

* Geschichte der Veränderungen, Part I. p. 425.

† Ibid. vol. iii. p. 316.

‡ Kongl. Vetenskaps-Academiens Handlingar, Stockholm, 1823, p. 30.

of the sea near the coast. It is composed principally, like that before mentioned near Stockholm and Upsala, of the decomposition of the *Mytilus edulis*, but also contains perfect specimens of the *Tellina Baltica*, *Littorina littorea*, *L. rudis*, and *Paludina ulva*.

The castle of Åbo on the Finland coast has been cited by several writers* as proving that the ground on which it stands has not been elevated, that building being many centuries old and yet close to the water's edge. But Colonel Hällstrom assured me that the base of the walls is ten feet above the water; so that the castle may be four centuries old, and yet there may have been a gradual rise of the land at that point to the amount of more than two feet in a century.

Not being able to visit Sundsvall, I applied by letter to Mr. James Dickson, resident at that port, who at my request put a series of questions, which I had drawn up, to the most experienced pilots and fishermen on their return in November last from their fishing-stations in the Gulf of Bothnia. In their answers they stated:

1st, That they could not conceive the possibility of the land rising, but were of opinion that the sea had been sinking gradually in the Gulf of Bothnia, the fall during the last thirty years amounting to two feet, or thereabouts:

2ndly, They had never seen any of the marks cut in the rocks in 1820; but from other appearances they inferred that the fall of the waters in the last fourteen years, in the neighbourhood both of Sundsvall and Hernösand, was from six to eight inches:

3rdly, They had found it necessary in their own time, in consequence of the retiring and shallowing of the waters, to remove their stations or fishing-posts nearer to the sea:

4thly, They could point out examples of large blocks of rock which had been moved and even conveyed from one place to another by ice, both on the shores of the islands of the Gulf of Bothnia and on those of the main land.

I shall now pass over from the shores of the Baltic to the opposite coast of Sweden between Uddevalla and Gothenburg, a district from 250 to 300 miles south-west of that before described, and about three degrees of latitude farther south. The deposits containing recent shells at Uddevalla, raised in some spots to the height of more than two hundred English feet above the sea, have long been celebrated; as also the discovery, made by M. Alexandre Brongniart, of barnacles attached to elevated rocks of gneiss on the spots where they must have grown. I was desirous of seeing this phenomenon, as it appeared to me that it might throw some light on the time which has elapsed since the shelly beds were raised from the sea; for if the *Balani* had been exposed in the open air ever since the emergence of the rocks to which they were fixed, it could hardly be supposed that the time had been indefinitely great, since in that case the shells must have been decomposed. The fact recorded by M. Brongniart was, I believe, observed at Capellbacken, immediately south of Uddevalla, where there is a narrow valley in the gneiss, the bottom of which is filled up with a great deposit of shells, sand, and clay, which rise, according to Hisinger, at their greatest eleva-

* See Von Hoff, Part I. p. 438.

tion 206 English feet above the sea*. I searched in vain for the *Balani* round the boundary of gneiss at its contact with the beds of shells, as also on some insulated rocks of gneiss which had been newly laid bare by the workmen, the shelly matter being removed as materials for the repair of the roads. I presume, however, that it was in just such a situation as that last mentioned that M. BRONGNIART found the adhering barnacles; for under similar circumstances I afterwards found them in another place, called Kured, about two miles north of Uddevalla †. Here a mass of white shells has been laid open to the depth of forty feet, in a quarry resembling singularly, when seen at a distance, one of our chalk-pits. Although now two miles from the nearest sea, and a hundred feet or more above it, they evidently fill what has once been a narrow channel, or fiord, bounded by rocks of gneiss. The deposit now forms a flat inland meadow, the fertility of which is contrasted with the steep and barren rocks which rise above it on all sides. It consists here almost exclusively of broken and entire shells, which lie in thin strata. They have been used largely both for making lime and for road materials; and the removal of part of them has exposed a ledge and precipice of gneiss, which they must previously have covered to some depth. Adhering to the face of this precipice, I found the circular supports of many large *Balani*. Some of these supports (see Plate II. figs. 38, 39.) were three quarters of an inch in diameter; and being white, they spotted the rock, so as to present at a distance exactly the appearance of lichens. I also found in horizontal clefts between the rocks pendent barnacles, fixed to the roof so firmly that I was able to break off pieces of the hard gneiss on which the shells still remained attached. In some places small zoophytes (*Cellepora*? LAM.) were adhering to the rock or to the *Balani*; and I also found some of the Cellepores with the support of the *Balani* partially covering them. These corals and adhering shells, therefore, must have grown upon the gneiss before the accumulation of drift shells had filled up this valley, once a submarine hollow. I had always imagined that the shelly formations near Uddevalla resembled ancient beaches of the ocean which had been upraised, but they are in fact stratified formations of sand, clay, and gravel, and in several places almost entirely of shells, which have filled up at some former period the deep bays and fiords of a sea like that now bounding this coast. The quantity and variety of the shells at Capellbacken, Kured, and Bräcke reminded me of the deposits of Grignon and Damerie in the Paris basin; but it is curious to reflect, that although the shells are almost equally well preserved in both these regions, they are specifically so distinct, that in the one it is scarcely possible to find a recent species, while in the other nearly all, perhaps every one of the species, belong to the German Ocean. The list of the shells which I collected here in one day will be found at the end of this paper; and although it will probably give but an imperfect idea of the

* Anteckningar, &c., v. p. 81.

† M. BRONGNIART says that he found the barnacles "un peu au dessus de l'amas coquillier," (Tableau des Terr. p. 89); but this may refer to what then remained of the shelly mass.

entire number which might be found, it will serve to show that a considerable variety exists here.

The difference of this assemblage of shells from the fossils which I had before examined near the shores of the Baltic was very striking. A considerable proportion of the whole mass, especially at Kured, was made up of the loose valves of a large barnacle (*Balanus tulipa*,—see Appendix), to which I imagine the large supports belong which covered the surface of the gneiss at Kured. These supports exhibit a number of concentric rings of growth, often very regular (see Plate II. figs. 38, 39.). When the animal died, the shell seems to have been easily broken off from the rock, and we must suppose successive crops of them to have been supplied for ages before such enormous heaps of stratified shells were amassed. The *Balanus sulcatus* is also very common, of a large size, remaining entire, with its support. Some of these I saw fixed to the rock as before mentioned; but generally they are found adhering to valves of the *Mytilus edulis*, or large valves of the *Pecten islandicus*, of which last the colour is preserved. Not one of these *Balani*, nor any species of that genus, inhabits the Baltic. The shell next perhaps in abundance to the large Balanus is *Saxicava rugosa*, of which the valves are often of extraordinary thickness, and must have belonged to very aged individuals. The two valves are sometimes united; but I never found them lodged in any cavity either of a rock or zoophyte: perhaps they may have inhabited the roots of large sea-weeds. (See remarks on this shell in the Appendix.) The thick shells of *Mya truncata* are also in great quantity; and the *Mytilus edulis* four or five times larger than in the Baltic, and retaining much of its colour. A *Fusus* also (*Murex Rumphius*, MONT.) occurs in profusion.

I found at Uddevalla many bivalve shells, in which small holes had been drilled by predaceous Trachelipodes, whereas among the fossils near Stockholm and Upsala I could never meet with a single bivalve so perforated; and there are, I believe, no zoophagous *Mollusca* now living in the Baltic.

From Uddevalla I went to the small island of Gulholmen (see Map), in the parish of Morlanda, part of the coast not far from Uddevalla, where CELSIUS declared, at the beginning of the last century, that the sea was sinking. On my way I crossed Orust, an island about fourteen miles in diameter, consisting chiefly of micaceous schist, forming low hills a few hundred feet high, resting upon which, at different elevations, are beds of sand, gravel, and clay, sometimes entirely destitute of shells, but often inclosing many recent shells, for the most part the same species as at Uddevalla, but with the addition of the *Ostrea edulis* and *Cerithium reticulatum*. I met with some of these fossils between Hogan and Morlanda in a blue clay, which seemed to lie at a higher elevation than any of the shells near Uddevalla. The features of the scenery in the interior of Orust are precisely such as we might suppose the present coast to exhibit if it should be lifted up with its small islands, rocks, and friths, and if the intervening level flats, where sand, mud, and shells are known to be now accumulating, should be laid dry. An account was given me of the finding of an anchor near Morlanda,

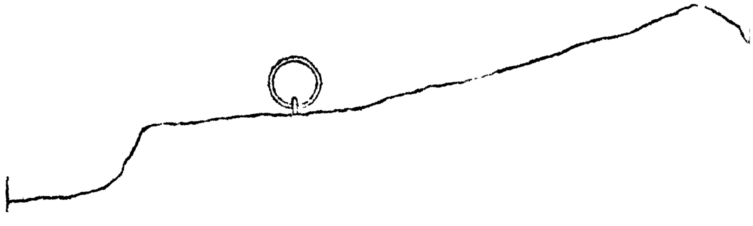
in a valley, the lower part of which had gained considerably in extent, within the memory of persons now living, by the retreat of the waters. In descending to Ellelös on the eastern coast, opposite the island of Gulholmen, I observed shelly deposits about fifteen feet above the level of the sea, in which were many specimens of the *Ostrea edulis, Saxicava rugosa, Cerithium reticulatum*, and other shells, some of which I had seen at Uddevalla, and others cast up on the shores in Orust.

In regard to the island of Gulholmen, Celsius tells us that in his time forty pilots, none of whom were under sixty years of age, having been assembled there, had unanimously declared to one Mr. Kalm that there was only fifteen feet depth of water in places where in their youth there had been eighteen feet. He also mentions that one of the pilots pointed out a small rock near Gulholmen, then rising two feet above the water, which, when he was a child, was not visible *.

The present inhabitants, as far as I conversed with them, are entirely ignorant of any such statements having been recorded a century ago; but on my demanding whether the water stood now at the same level as in their younger days, they unanimously declared that it did not. Mr. Bruncrona, in his memoir before cited, mentions that on an insulated rock called Gulleskär, near the harbour of Gulholmen, there was an iron ring to which ships were moored, and that this ring, when measured in 1820, was eight feet above the level of the water. Unfortunately, no particulars are given; and as both the chief pilot of 1820 and another who assisted him in the measurement were dead at the time of my visit, I could not ascertain with certainty from what point of the ring they had begun their measurement, nor the means they had taken to secure accuracy. Having obtained the assistance of Johan Wunsch, now chief pilot, I found the point where the ring is fixed into the rock to be only seven feet five inches above the level of the sea, which was then declared to be at its usual level, a very slight wind only blowing from the north-north-west, and there being never any tides in the sea here. The iron ring, which has remained for more than half a century in its present place, is fifteen inches in diameter, and the top of it stands more than eighteen inches above the level of the rock when it is erect, in which position I found it, thus (see fig. 12), having been so placed for the sake

Fig. 12.

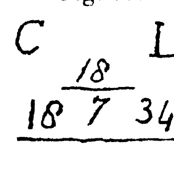
Summit of the Gulleskär, with the Ring.



* Celsius, Observations on the Diminution of the Waters of the Baltic and German Ocean.—Trans. Roy. Acad. of Sweden.

of drying the fresh paint, with which it had been just covered; but the islanders suppose the measure to have been taken from the bottom, or point where the staple enters the rock, which seems most probable. Curiosity led a great many of the inhabitants to accompany me; and when I declared that the height of the ring was seven inches less above the water than that recorded by BRUNCRONA, many of the older men with one accord pronounced this to be impossible, and said that the former observation must have been incorrect, for that the sea must, on the contrary, have fallen since 1820. Some of them affirmed that the pilot who received orders in 1820 to make the measurement was ignorant in what manner to proceed, the place of the ring not being perpendicularly over the water, and he having no instrument for levelling, so as to ascertain that the line which he first carried out from the ring was strictly horizontal. Whether there was any foundation for this charge I cannot pretend to decide; but I mention it as proving that the islanders believe that there is a change of level going on. It may be useful to those who may make future measurements to state what length of line it required to reach from the iron staple of the ring to the nearest point of the rock to which the sea comes up, this point being now exactly in the direction north-west and by north of the ring. I stretched the rope from one angle to another of the rock, not applying it to the surface of the intervening hollows, and found its length to be fifteen feet five inches and a half. As the Gulleskär, however, is by no means well chosen for the facility of observations, I had a new mark cut on the face of a vertical cliff on the south side of the harbour, about a hundred yards from the post-house. I subjoin a copy of the mark, the lower part of which was cut in my presence, and which the chief pilot promised to see completed. The horizontal line was cut six inches above the water-level, and the vertical line at the right end of it, six inches in length, was terminated at the bottom by a short cross line, which the surface of the water just covered. The vertical depth of water below the mark was four feet two inches and a half. I may suggest, that whenever horizontal lines or any marks are made, like that of St. Olof's Stone before mentioned, *not* at the level of the sea, but at a certain height above it, on a vertical face of rock, there should always be a perpendicular line cut down to the then existing level of the water, to facilitate subsequent observations and prevent mistakes. Marks cut at given heights above the standard level are perhaps the best, as they are not concealed by a temporary rise of the water.

Fig. 13.



Before leaving Gulholmen I visited the Skefverskär, an isolated rock which, according to the testimony of several old people, was always covered, except at very low water, about forty years ago. In their younger days, before the year 1799, when the present church of Gulholmen was built, they went to church at Morlanda, and passed near this rock, the exposure of the summit of which was a well-known sign to them of a particular state of the weather. This rock is now always seen except when the sea is very high. I found the highest point of it to be sixteen inches above the level

of the water; and its extreme length from east to west, including a detached point at one end, measured fifty-two feet four inches and a half.

From Gulholmen I went to Marstrand, an island about twenty miles to the south, in order to observe another of the marks enumerated by BRUNCRONA. I first re crossed the ferry at Svansund to the main land, and then passed to that of Tjufkil, which leads to Koon. On the shore at Tjufkil I found a bed of oysters and other shells, five or six feet thick, with pebbles intermixed, rising to the height of sixteen feet or more above the water. The oysters, which were in great number, all belonged to the *Ostrea edulis*, which is taken on this coast; and the other shells were the same as at Uddevalla and Ellelös, with the addition of *Anomia striata*. This shelly deposit has been overwhelmed by a great fall of rock from the steep heights of gneiss behind, some of the fragments which cover the shells being about nine feet square.

Not far from the harbour at Marstrand is an artificial channel, which, in the year 1770, was cut through an isthmus which formerly connected two parts of Koon island. The excavation was made through a mass of clay and sand with shells, similar to that of Tjufkil, already mentioned; so that there can be no doubt that there must originally have been a natural passage in this place. One Captain CONSTANT, who superintended the digging of the channel in 1770, caused a mark, of which the following is a sketch, to be hewn on the face of a vertical rock of micaceous schist on the shore of Koon, nearly opposite Marstrand.

Fig. 14.
Mark at Koon Island, near Marstrand.

An horizontal line, ten inches long, is seen twenty-one inches below the bottom of the last cipher. This line I found to be just ten inches above the level of the

water. My observation was made on the 19th of July 1834, sixty-four years after the mark was cut. Now my boatmen stated that the horizontal line was originally intended to express the lowest level to which the sea fell at the time of digging the Koon canal; and this information was confirmed by Mr. O. J. Westbeck, who resides in the immediate neighbourhood. On my applying to this gentleman to learn whether the water at the time of my observation might be considered as unusually low, he said that as the wind was easterly, the sea was certainly below its mean level, but it had by no means reached its extreme point of depression, for there still was water in the Koon canal, immediately opposite his villa; whereas, after the prevalence of a strong easterly wind for two days, the sea falls so low that certain parts of this canal are dried up. He suggested, therefore, that by measuring the depth of water in those parts of the canal which dry up, and adding that depth to the ten inches which I had already obtained below the mark only half an hour before, I should ascertain the point of extreme low water as compared to that of 1770. We accordingly found that the water in the places alluded to was fourteen inches deep; so that the lowest water now is two feet below the maximum of depression sixty-four years ago. Mr. Westbeck said that he had always heard from his father that the mark, which was cut the year he was born, was intended to express the lowest level of the sea during the digging of the canal in 1770.

I have already stated that there is no tide on the coast here, a circumstance which seems very extraordinary; but all the pilots and seamen agree in asserting the fact. A strong wind off the shore causes the water to fall two or three feet, and to rise as much if it be in the opposite direction. Notwithstanding these occasional oscillations, the inhabitants pretend to determine whether the sea is two or three inches above or below its standard level. I was shown here, as at other places, rocks which forty or fifty years ago could rarely be seen, but are now permanently above water. I was also told of numerous rocky channels where boats could once pass, but which had now grown too shallow, and of meadows which were yielding from time to time a larger quantity of hay, in consequence of their increased extension on the side towards the sea.

I know not how much further to the south the same signs of a rise of the land have been observed, but it is certain that the narrow frith in which the port of Gothenburg is situated has been gradually filling up, in such a manner as would happen if the same cause of change was cooperating there with the deposition of river-sediment. It is well known that in the sixteenth century the ancient port was placed twenty miles further up, and called Lödese; and this was afterwards removed further down, and called New Lödese, to distinguish it from what remained of the more ancient harbour. But now the newer of these places is called Gammle Staden, or the old town, and is a mile or more above Gothenburg.

On the banks of the river at Gothenburg I found a deposit of blue clay, filled with a great variety of recent marine shells. Among others, *Lutraria compressa*; *Mactra*

subtruncata, very abundant; *Tellina solidula*; *Donax trunculus*? DILLWYN; *Cyprina Islandica, Venus gallina, Cardium edule, Littorina littorea, Turritella terebra, Rostellaria pes pelicani,* and *Buccinum reticulatum.* This part of the estuary is now always filled with fresh water, except on rare occasions, and for a short time, when a strong wind drives the sea up the river, and causes the water to rise six feet, in which case it becomes brackish. At different heights above the sea, in the valley of the Götha Elf, between Gothenburg and Trolhättan, marine shells have been found similar to those of Uddevalla.

Some persons who have been long resident in Gothenburg pointed out to me, as a proof that the water was falling there, that the rocks several feet above the highest water-mark were bare and uncoloured, by which they meant that no lichens grew upon them.

A similar remark had been made to me at Tjufkil, Svansund, and other places on this coast. It seems probable that some species of lichen may require a much longer time to establish themselves on newly exposed rocks than others; and I could observe distinctly, near Gothenburg, that some kinds approached nearer the water's edge than others, and that the variety of species became greater and the colour different on ascending to greater heights. It would therefore be an interesting point for a geologist sufficiently skilled in botany to determine whether the extent of the lichens and mosses downwards towards the water on this coast, where the rocks are supposed to be always rising, presents different phenomena from the line of vegetation on other coasts, where the relative level of the land and sea is known to have remained stationary.

On many parts of the eastern coast, above described, the sea freezes in severe winters in the Skär; that is to say, among the rocks and islets which skirt the main land, and where there is almost always still water. As I have before mentioned the accounts which I received of the transporting power of ice in the Gulf of Bothnia, it may be well to state some facts bearing on the same subject which I learnt at Gothenburg. In the harbour of that port there are a great number of strong wooden piles, called dolphins, three or four feet in circumference, the lower parts of which are sunk to a considerable depth in the mud, and firmly fixed in it, so that vessels may be moored to their tops. As these dolphins are annually frozen in, it is found necessary to break the ice round them; but sometimes this has been neglected, and Mr. HARRISON, the English Vice-Consul, informed me, that on such occasions he has known a great number of the piles drawn up together out of the mud six feet perpendicular, a rise of the river having caused the ice to float up to that amount.

Mr. WESTBECK of Marstrand, to whom I have already alluded, mentioned to me, that having been formerly employed in the Swedish Diving Company for thirty years, he had opportunities of witnessing the extraordinary power of ice to lift up from the bottom of the sea and remove to a distance very heavy masses. In two instances the ice collected round sunken vessels which were under his charge, and having frozen

round them, floated them off with their cargo and ballast from shallow into deep water.

I shall now state some general conclusions to which I have been led by the observations above described. It is evident from the position of the fossil shells of recent species on the coast of the Baltic between Gefle and Södertelje, and on the shores of the ocean between Uddevalla and Gothenburg, that the tract of land (see Map, Plate I.) which once separated the two seas in this region was much narrower at a comparately modern period. Shells like those of Uddevalla have not only been found a few miles due east of that place, but as far inland as Trolhättan in digging the canal there*; and still further in the interior, about fifty miles from the coast at Tusenddalersbacken, and other places near Lake Rogvarpen in Dalsland, on the west side of Lake Wener (see Map, Plate I.). Of these fossils an account will be seen in the works of Mr. Hisinger, to whom we are indebted for a valuable geological map of the whole of the south of Sweden. They are found in Dalsland about as far above the sea as near Uddevalla, or about two hundred feet high; so that when they were deposited, we must suppose the whole of that extensive Lake Wener, the surface of which lies at an inferior level, to have formed part of the ocean. On the other hand, when the marine shells of the environs of Upsala, Stockholm, and Torshälla lived in the Baltic, we must suppose the whole of Lake Maeler to have been a bay of that sea. Now the distance between the nearest points of Lakes Wener and Maeler is only about seventy English miles, whereas there is more than three times that distance between Stockholm and Uddevalla, the nearest points at which the two seas now approach each other in the same direction. It is very desirable that Swedish geologists should pursue this subject still further, and ascertain precisely how far the shells of the two seas can be traced inland in opposite directions.

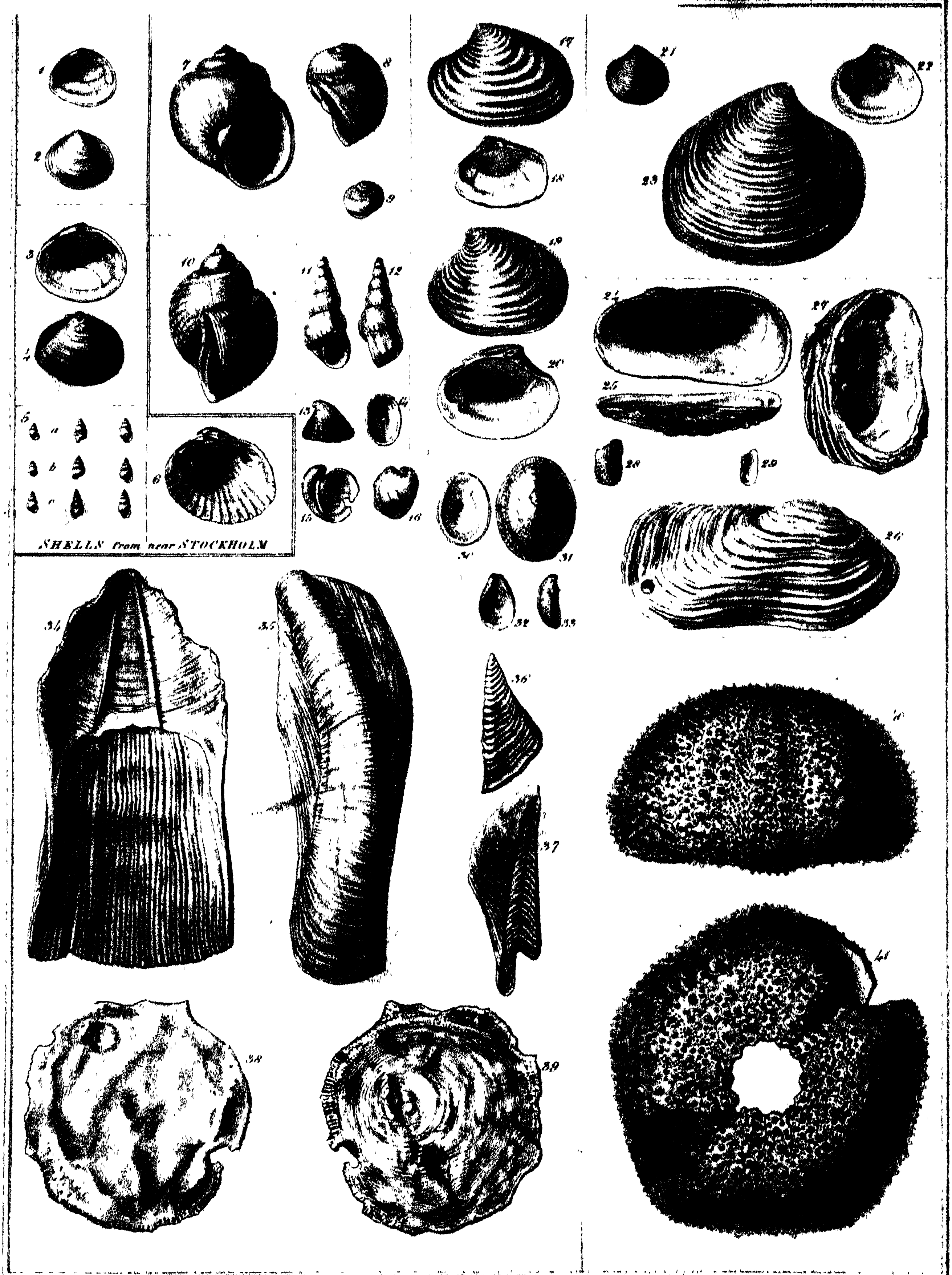
In crossing from Stockholm to Södertelje, Arboga, Orebro, Mariestadt, and Wenersborg to Uddevalla, I passed the summit level of the country, about half-way between the Baltic and the ocean, near Bodarne, where the hills, as Von Buch remarks, do not probably exceed five or six hundred feet in height. I found erratic blocks scattered widely over the whole of this country, but they were much larger and more numerous on the eastern than on the western watershed. There were also deposits of stratified sand and gravel on the heights, but I was never able to discover any shells in them, nor in the blue clay in the lower grounds bordering the lakes, except very rarely, and these were of freshwater species; as, for example, at the place before mentioned near Lake Maeler not far from Torshälla, between Smedby and Kongsör. It will naturally be asked, whether the appearance of the interior is generally such as would agree with the hypothesis of a gradual rise, according to which we must suppose that every tract has in its turn been first a shoal in the sea, and then for a time a shore. It appeared to me, on comparing both the eastern and western coasts and their islands with the interior, that the geological appearances and physical

* See Hisinger's Anteckningar, vol. iv. p. 42.

features of those parts of the country which I examined answered well to all the conditions of such a theory. In passing from Gefle to Fahlun, and from thence to Sala, I found the number of erratic blocks very great, as now on the islands and shores of the Bothnian Gulf; whereas on the opposite or western coast they are smaller in size and quantity, both in the interior of the country around Uddevalla and Gothenburg, and at the sea-side in the contiguous Skär. I saw some considerable boulders overlying the deposits of recent shells at Capellbacken near Uddevalla, a phenomenon analogous to that described near Upsala, where these huge erratic blocks repose upon the sand-hills, characterized by fossil shells of Baltic species. The transportation, therefore, of these rocky fragments into their present position continued after the period when the modern shelly formations of both coasts were accumulated; and it may be inferred from several facts mentioned in this memoir, that the drifting of such blocks may now be going on by means of ice every year. I am at a loss to conceive from what data some geologists have inferred the simultaneous dispersion of the erratic blocks of the North of Europe; but it would carry me into too wide a digression should I endeavour to controvert that theory. I can, however, confirm the statement of Professor HAUSMANN, that, in the ridges of sand and gravel, the largest blocks occur in the highest parts of each ridge; a fact which seems to me to point to the mode in which they may have been drifted into their present position. For if these ridges were originally sand-banks in the sea, as the marine shells found in some of them incline me to believe, the summits of such banks would have arrested the progress of ice-islands which might transport fragments of rock in the manner before suggested.

In regard to the proposition, that the land in certain parts of Sweden is gradually rising, I have no hesitation in assenting to it after my visit to the districts above alluded to. Independently of the geological proofs derived from strata containing recent shells, the evidence in favour of an upward movement consists of two kinds: first, the testimony of the inhabitants; and secondly, the altered level indicated by artificial marks cut in the rocks. More than one generation has passed away since CELSIUS recorded the stories of pilots, fishermen, and the inhabitants of the two opposite coasts at Gefle and Gulholmen respecting the increased extension of land and apparent sinking of the sea. It was at the same places that I heard precisely similar accounts from persons now living; so identical, indeed, that if related, they would appear mere repetitions of the words of CELSIUS, with scarcely any change except in the names of the witnesses. But I am aware, from what I myself experienced when reading formerly on this subject, that it is not easy to convey to the minds of those who do not visit the country, the impression made by the testimony now under consideration, deriving as it does almost all its weight from an accumulation of minute particulars, each, separately considered, of but small importance.

From what I saw at Calmar and Stockholm as compared with Oregrund and Gefle, I have no doubt that the rate of elevation is very different in different places; and in



FOSSIL SHELLS from UDDEVALLA in SWEDEN.

J.D.C.S.

the south of Scania I could not ascertain, either from the testimony of the inhabitants or from any appearances on the coast, that the slightest change of relative level can be detected. The difference of about three feet in a century, indicated by the mark at Löfgrundet, and of about two feet in sixty-four years, by that of Marstrand, are in such complete accordance with the results of the surveys of Bruncrona, Hällstrom, and others, as to lead me to place entire reliance on the conclusions to which they have arrived from a larger number of data, and respecting a territory of much greater extent. The slight amount of difference between the level of the sea and the marks of 1820 which I observed at Oregrund and Gefle, although corroborating the same result, are undoubtedly in themselves of small value; and a difference of level amounting only to about four or six inches may be easily attributed to accident or the particular state of the weather at the time of my visit. Subsequent observers might find the same marks submerged beneath the waters; but I nevertheless believe, that if the summer season and a calm day be selected, so that the circumstances shall correspond with those under which the marks were originally cut, there will be found to have been a real depression of level, to the amount of several inches, in the course of the last fourteen years.

Be this as it may, I may be allowed to congratulate the scientific world that this wonderful phenomenon is every day exciting increased attention among the philosophers of Sweden, and especially of Professor Berzelius, who, in his reports to the Academy of Sciences at Stockholm, has already recorded many valuable observations on the levels of the water of Lake Maeler at different seasons, and who is understood to be now exerting himself to secure more frequent observations in future of the marks in the Bothnian Gulf. It is only by multiplying such measurements, and repeating them within short intervals of time, that we shall be able to determine whether the movement of the land be oscillatory or always in one direction, and whether it be intermittent or constant.

Appendix.

List of Fossil Shells from the Country near Stockholm.

Names.	Observations.
1. *Tellina Baltica.* Var. *a.* Pl. II. figs. 1. & 2.	The variety of this shell, found fossil in sand and marl at Solna, Bränkyrka, and Södertelje, where it is associated with littoral shells, is smaller, thinner, and deprived of epidermis, resembling those which I collected in the sand on the shores of the Gulf of Bothnia and at Solvitzborg.

Names.	Observations.
Tellina Baltica. Var. *b.* Pl. II. figs. 3. & 4.	This variety of *T. baltica* was found in stiff blue clay between Smedby and Kongsör (see page 10), as also at Ulfva, near Upsala (see page 14). It is larger, thicker, and covered with a strong green epidermis; but there is a passage between it and the preceding variety.
2. *Cardium edule.* Var. Pl. II. fig. 6.	This *Cardium* is generally of a small size in the brackish waters of the Baltic, and often more elongated transversely than individuals of the same species in the ocean. This transverse form is seen in the fossils found at Solna and other places near Stockholm mentioned in the memoir; and Mr. Gray tells me that the same variety has been observed elsewhere in brackish waters. But individuals of the more ordinary form, though of a dwarfish size, are also found living in the Baltic, and fossil in the localities above mentioned.
3. *Mytilus edulis.*	The variety of this shell, which occurs fossil at Solna, Bränkyrka, Södertelje, Ulfva, &c., is small, about half an inch long, like that now inhabiting the brackish waters of the Baltic. It is almost always found in a state of decomposition, and converted into a violet-coloured marl.
4. *Littorina littorea.* (*Turbo littoreus*, Linn.)	Found fossil at Solna, Bränkyrka, Södertelje, Skerplinge, and other places bordering the Baltic. I found varieties of different ages, but never any which approached the larger size which the same species often attains on the borders of the ocean.
5. *Littorina rudis.* (*Turbo rudis.*)	A young specimen of this occurred fossil with the former at Bränkyrka; also in the violet-coloured marl from Nädendal in Finland, given me by Colonel Hallstrom (see page 22).
6. *Littorina crassior.* (*Turbo crassior*).	I found specimens of this at Solna.
7. *Paludina ulva*? Pl. II. fig. 5. *a. b. c.*	A great number of small univalves, of which I have given figures, are found fossil with littoral shells at Solna, Bränkyrka, and Sodertelje, resembling those which occur generally in the sands of the shores of the Baltic, as well as on those of the ocean between Uddevalla and Gothenburg. The three principal varieties which are figured are

Names.	Observations.
	selected from the different localities of fossils before mentioned near Stockholm. In var. *a* there are five volutions, which are of a squarish form; in var. *b* five, which are rounded; and in var. *c* six, which are rounded. On comparing a great number of individuals, there appeared to be so many passages from one form to another as to render it difficult, if not impossible, to establish distinct species.
8. *Rissoa parva.* (*Turbo parvus*, Mont.)	I found at Bränkyrka a few individuals which Mr. Gray referred to this species.
9. *Neritina fluviatilis.*	A small black variety of this species was met with at Bränkyrka, which I also saw recent in abundance on the shores of Möen, in Denmark. Dr. Beck, of Copenhagen, regards it as a distinct species. It is smaller than the same shell living in fresh water. I found some varieties both fossil at Bränkyrka and recent at Gräsö, near Gefle, which had the ordinary colours of the *N. fluviatilis.*
10. *Bulimus lubricus.*	Fossil at Bränkyrka. (See page 6.)

List of Fossil Shells from Uddevalla, on the West Coast of Sweden.

Names.	Observations.
1. *Pholas crispata.*	I met with one valve only of this species, at Capellbacken, near Uddevalla.
2. *Mya truncata.*	Found in very great abundance around Uddevalla.
3. *Anatina myalis*, Lam. (*Mya pubescens*, Turt. *Ligula pubescens*, Mont.)	I met with one very perfect specimen, with its ligament, fossil near Uddevalla.
4. *Saxicava rugosa.* (*Mytilus rugosus*, Mont.) Pl. II. figs. 24—29.	The small individuals, figs. 28, 29., would be called by some conchologists *Hiatella arctica*; but many naturalists are now of opinion that the shells called *Saxicava* or *Hiatella rugosa* (*Mytilus rugosus*, Linn.), and the *Hiatella arctica*, are not specifically distinct; and the fossils which I collected in great abundance at Uddevalla confirm me in this opinion. This shell is more abundant perhaps than any other, and some individuals are of great thickness, and must evidently have been very aged (see fig. 27.). I never found any of them lodged in cavities in

Names.	Observations.
	rocks, and I presume that they must have lived in the roots of fuci, in which situation they are sometimes met with on our coast.
5. *Tellina triangularis.*	Common at Capellbacken.
6. *T. Baltica.*	I found one individual, which seems not distinguishable, in size or shape, either from the fossil or recent *T. baltica* of the neighbourhood of Stockholm.
7. *Astarte.* Figs. 17, 18.	Shell rather convex, transversely elliptical, thin; its surface strongly furrowed; furrows rounded, about sixteen. Lunette deep, elliptical. Lateral tooth slender, elongated, more transverse than the recent *Astarte Garensis*, and with somewhat fewer furrows, but perhaps a variety of the same?
8. *Astarte.* Figs. 19, 20.	Shell convex, transversely elongated, but less so than the former; both the anterior and posterior margins more rounded than in the preceding; rather thin; its surface strongly furrowed; furrows deep, rounded, about sixteen. Lunette deep, lanceolate, elongated. The lateral tooth slender. Perhaps, like the former, a variety of *A. Garensis*, to which it approaches much nearer.
9. *Astarte.* Figs. 21, 22, 23.	Shell compressed, suborbicular, slightly truncated on the posterior margin; thin; its surface rugose, marked with many transverse furrows when young. Lunette deep, lanceolate, short, pointed. Lateral tooth small, short. Fulcrum long.
10. *Cardium edule.*	
11. *Mytilus edulis.*	In great abundance, and preserving a portion of its colour; about two inches in length.
12. *Modiola barbata.*	From Kured.
13. *Pecten Islandicus.*	In great abundance, often preserving its colour, and covered with *Balani.*
14. *Terebratula.* Pl. II. figs. 32, 33.	A single perforated valve is all that I found of this genus.
15. *Patella*, allied to *testudinaria*, CHEMN. (*P. Clelandi*, Sow.) Pl. II. figs. 30, 31.	This *Patella* is referable to the genus *Lottia*, GRAY, (Philosophical Transactions, 1834.).

Names.	Observations.
16. *Patella Noachina*, CHEMN. (*Puncturella*, LOWE.) Pl. II. figs. 13, 14.	Mr. G. SOWERBY informs me that this species has been lately found fossil with other recent shells, at a slight elevation above the level of the sea near Glasgow.
17. *Margarita striata*, LOWE. (*Trochus*, LAM.)	
18. *Littorina littorea*. (*Turbo littoreus*, LINN.)	Some young individuals at Uddevalla retain their colour in great perfection.
19. *Littorina*? Plate II. fig. 10.	The shell here figured has lost its outer coat, and may perhaps belong to the genus *Littorina*.
20. *Turritella*? Plate II. figs. 11, 12.	This shell is very like a worn *Scalaria*, but perhaps belongs to the genus *Turritella*.
21. *Natica*, allied to *N. clausa*. Pl. II. figs. 7, 8, 9.	This shell is common at Uddevalla, especially at Kured, and differs decidedly from the *N. glaucina*, having a less flattened spire, and being more ventricose, I presume that it is the *N. glaucina* of Mr. HISINGER's list of Uddevalla shells.
22. *Velutina*, GRAY. Pl. II. figs. 15, 16.	Probably *Helix lævigata*, MONT. An imperfect specimen.
23. *Fusus*. (*Murex rumphius*, MONT.)	Very common.
24. *Fusus corneus*.	
25. *Buccinum undatum*.	Abundant.
26. *Balanus sulcatus*.	Very abundant, and of large size, and occurs both attached to other shells and fixed to the rocks of gneiss. (see p. 25.)
27. *Balanus tulipa*. (*Lepas tulipa*, MULLER, CHEMNITZ, viii. t. 92. f. 832.) Pl. II. figs. 34, 35, 36, 37, 38, 39.	Mr. GRAY informs me that this shell is not noticed by LAMARCK, and that it differs from other *Balani* in the substance of the shells being solid, and the base being only longitudinally grooved on the inner side; also in the side edges of the valves being entire and not crenulated. By the aid of these characters Mr. GRAY has formed of this and a few other species which are in the collection of the British Museum, a particular section, to which he has given the name of *Chirona*. This I presume is the species called *B. Uddevallensis* in some of the Swedish lists of Uddevalla fossils. It is of great size, frequently three or four inches long. The supports, figs. 38 and 39, were found adhering in great numbers to the face of the

Names.	Observations.
	rocks of gneiss, and they appeared to me, from their large size, to belong to this species.
28. *Echinus.* (*Echinometra.*) Pl. II. figs. 40, 41.	Fragments of this *Echinus* were found at Capellbacken near Uddevalla, and they have been put together as represented Plate II. figs. 40, 41.

This collection of Uddevalla fossils must be very incomplete, as they are only such as I could obtain by diligent search, and with assistance, in one day. I did not meet with *Pileopsis Ungarica*, but Mr. HISINGER showed me specimens of that shell which he obtained there.

Description of the PLATES.

PLATE I.

Map of part of Sweden, to indicate the principal localities referred to in the preceding paper.

PLATE II.

Figs. 1, 2. *Tellina Baltica*, var. *a*, from Solna, Bränkyrka, and Södertelje. (See Appendix, p. 33.)

3, 4. The same, var. *b*, from Ulfva. (Appen. p. 34.)

Fig. 5. *Paludina ulva*?, three varieties, from Solna, Bränkyrka, and Södertelje. (Appen. p. 34.)

6. Transverse variety of *Cardium edule*, from Solna. (Appen. p. 34.)

Figs. 7, 8, 9. *Natica*, allied to *N. clausa*, from Kured. (Appen. p. 37.)

Fig. 10. *Littorina*? of which the outer coat is lost. (Appen. p. 37.)

Figs. 11, 12. *Turritella*? (Appen. p. 37.)

13, 14. *Patella*, LAM. (Appen. p. 37.)

15, 16. *Velutina*. (Appen. p. 37.)

17, 18. *Astarte*. (Appen. p. 36.)

19, 20. *Astarte*. (Appen. p. 36.)

21, 22, 23. *Astarte*. (Appen. p. 36.)

24, 25, 26, 27, 28, 29. *Saxicava rugosa*, from Uddevalla. (Appen. p. 35.)

30, 31. *Patella*. (Appen. p. 36.)

32, 33. *Terebratula*. (Appen. p. 36.)

34, 35. Large valves of *Balanus tulipa*, from Uddevalla. (Appen. p. 37.)

36, 37. Opercular pieces of the same. (Appen. p. 37.)

38, 39. Supports of the same. ? (Appen. p. 37.)

40, 41. *Echinus* (*Echinometra*), from Capellbacken. (Appen. p. 38.)

II. *Note on the Electrical Relations of certain Metals and Metalliferous Minerals. By* R. W. Fox. *Communicated by* Davies Gilbert, *Esq. F.R.S.*

Received and read January 15th, 1835.

I HAVE ascertained that the crystallized grey oxide of manganese holds a much higher place in the electro-negative scale than any other body with which I have compared it, when immersed in various acids, and alkaline solutions; and the other metals and minerals which I have examined, appear to rank after it in the following order:

Manganese.
Rhodium.
Loadstone.
Platina.
Arsenical pyrites.
Plumbago.
Iron pyrites.
Arsenical cobalt.
Copper pyrites.
Purple copper.
Galena.
Standard gold.
Copper nickel.
Vitreous copper.
Silver.
Copper.
Pan brass.
Sheet iron.

These five hold nearly the same place, varying in their mutual relations according to the time of their remaining immersed, and the nature of the liquid.

The same may in some degree be said of the three other bodies included in the larger bracket.

I have also compared the action of different metalliferous combinations in various diluted acids, &c. on the needle of the galvanometer, and some of the results are given in the following Table, in which cases sea-water, and also muriatic acid diluted with thirty-two parts of water, were employed. The figures show the angles of deflection observed when the needle became stationary, which may serve to give some idea of the relative effect of the combinations in question on the needle; but I find that the results are often considerably modified by the bodies being exposed for a longer or shorter time to the action of the acids, &c.; indeed this is so remarkable in the case of copper with zinc, that the needle often moves back much more than ten degrees from its maximum angle of deflection in one or two minutes after immersion: whereas in the case of iron with zinc, for example, the immediate retrograde motion

of the needle is very inconsiderable, and it is still less, if anything, when some of the ores are substituted for one or both these metals. May not these phenomena depend on the relative degrees of tenacity with which the electric elements are retained by different bodies, it being apparently greatest in the case of compound bodies?

	Zinc.		Copper.		Iron.		Lead.	
	Sea Water.	Diluted Muriatic Acid.	Sea Water.	Diluted Muriatic Acid.	Sea Water.	Diluted Muriatic Acid.	Sea Water.	Diluted Muriatic Acid.
Manganese* (crystallized)	56°	60°	35°	45°	52°	54°	50°	56°
Loadstone..............	41	58	21	29	33	48	32	47
Platina	21	46	1	5	21	23	15	21
Plumbago..............	52	56	23	31	45	45	42	45
Iron pyrites	34	38	7	8	20	29	20	19
Copper pyrites..........	49	57	36	31	43	44	45	43
Purple copper ore	44	45	14	10	33	40	32	38
Galena.................	47	50	19	27	36	41	37	37
Gold...................	26	38	11	14	14	30	25	24
Vitreous copper ore......	42	51	16	24	17	25	24	32
Silver	56	59	22	21	47	45	44	42
Sheet† copper	55	58	....	....	19	37	40	38
Pan brass..............	34	43	—	—	11	17	11	30
Sheet iron	36	46	—	—	....	....	—	—

If we regard the electrical relations of different metalliferous minerals in a geological point of view, it is curious to observe how nearly many of those which are usually associated in the same veins agree in this respect, their reciprocal voltaic action being generally very small. Were it otherwise, it may be assumed that the evidences of decomposition *in situ* would be much more decided and general than they now are. There is, however, a sufficiently strong action in some cases to account for the electro-magnetic phenomena which have been observed in copper and lead veins: thus, when copper pyrites and vitreous copper form a voltaic combination in water taken from a mine, or *even in spring water*, they are capable of producing considerable deflections of the needle. It is not, therefore, surprising, that when two parallel veins, or two portions of the same vein separated by imperfect conductors, are connected with the galvanometer, the action on the needle should be very decided. The degree of influence on the needle does not seem to depend, in the case of metalliferous minerals, upon extensive voltaic surfaces; for only *one or two inches of surface* may produce nearly the maximum effect in deflecting it, if the wire used in the galvanometer be small. Hence, the considerable deflection, which has been sometimes observed when two masses of ore were connected by the wires, proves that their reciprocal action, taken in the aggregate, must be very great; and it appears to be highly probable that the metalliferous veins, and perhaps even the rocks themselves, impregnated as they are with different mineral waters, and thereby rendered imperfect conductors, if not exciters of electricity, may have an important influence in the economy of nature.

* The contact of the wire with the manganese and other *minerals* was produced by pressure only, and the deflections would doubtless have been greater if the contact had been more perfect.

† I have ascertained that the electro-magnetic action of mineral veins was the same whether copper or zinc conductors were employed for making the contact with the ores.

III. *Experimental Researches in Electricity.—Ninth Series.* *By* Michael Faraday, *D.C.L. F.R.S. Fullerian Prof. Chem. Royal Institution, Corr. Memb. Royal and Imp. Acadd. of Sciences, Paris, Petersburgh, Florence, Copenhagen, Berlin, &c. &c.*

Received December 18, 1834,—Read January 29, 1835.

§. 15. *On the influence by induction of an Electric Current on itself:—and on the inductive action of Electric Currents generally.*

1048. THE following investigations relate to a very remarkable inductive action of electric currents, or of the different parts of the same current, and indicate an immediate connexion between such inductive action and the direct transmission of electricity through conducting bodies, or even that exhibited in the form of a spark.

1049. The inquiry arose out of a fact communicated to me by Mr. Jenkin, which is as follows. If an ordinary wire of short length be used as the medium of communication between the two plates of an electromotor consisting of a single pair of metals, no management will enable the experimenter to obtain an electric shock from this wire; but if the wire which surrounds an electro-magnet be used, a shock is felt each time the contact with the electromotor is broken, provided the ends of the wire be grasped one in each hand.

1050. Another effect is observed at the same time, which has long been known to philosophers, namely, that a bright electric spark occurs at the place of disjunction.

1051. A brief account of these results, with some of a corresponding character which I had observed in using long wires, was published in the Philosophical Magazine for 1834*; and I added to them some observations on their nature. Further investigations led me to perceive the inaccuracy of my first notions, and ended in identifying these effects with the phenomena of induction which I had been fortunate enough to develop in the First Series of these Experimental Researches†. Notwithstanding this identity, the extension and the peculiarity of the views respecting electric currents which the results supply, lead me to believe that they will be found worthy of the attention of the Royal Society.

1052. The *electromotor* used consisted of a cylinder of zinc introduced between the two parts of a double cylinder of copper, and preserved from metallic contact in the usual way by corks. The zinc cylinder was eight inches high and four inches in diameter. Both it and the copper cylinder were supplied with stiff wires, surmounted by cups containing mercury; and it was at these cups that the contacts of wires, he-

* Vol. v. p. 349.

† Philosophical Transactions, 1832, p. 126.

lices, or electro-magnets, used to complete the circuit, were made or broken. These cups I will call G and E throughout the rest of this paper (1079.).

1053. Certain *helices* were constructed, some of which it will be necessary to describe. A pasteboard tube had four copper wires, one twenty-fourth of an inch in thickness, wound round it, each forming a helix in the same direction from end to end: the convolutions of each wire were separated by string, and the superposed helices prevented from touching by intervening calico. The lengths of the wires forming the helices were 48, 49·5, 48, and 45 feet. The first and third wires were united together so as to form one consistent helix of 96 feet in length; and the second and fourth wires were similarly united to form a second helix, closely interwoven with the first, and 94·5 feet in length. These helices may be distinguished by the numbers i and ii. They were carefully examined by a powerful current of electricity and a galvanometer, and found to have no communication with each other.

1054. Another helix was constructed upon a similar pasteboard tube, two lengths of the same copper wire being used, each forty-six feet long. These were united into one consistent helix of ninety-two feet, which therefore was nearly equal in value to either of the former helices, but was not in close inductive association with them. It may be distinguished by the number iii.

1055. A fourth helix was constructed of very thick copper wire, being one fifth of an inch in diameter; the length of wire used was seventy-nine feet, independently of the straight terminal portions.

1056. The principal *electro-magnet* employed consisted of a cylindrical bar of soft iron twenty-five inches long, and one inch and three quarters in diameter, bent into a ring, so that the ends nearly touched, and surrounded by three coils of thick copper wire, the similar ends of which were fastened together; then each of these terminations was soldered to a copper rod, serving as a conducting continuation of the wire. Hence any electric current sent through the rods was divided in the helices surrounding the ring, into three parts, all of which, however, moved in the same direction. The three wires may therefore be considered as representing one wire, of thrice the thickness of the wire really used.

1057. Other electro-magnets could be made at pleasure by introducing a soft iron rod into any of the helices described (1053. &c.).

1058. The *galvanometer* which I had occasion to use was rough in its construction, having but one magnetic needle, and not at all delicate in its indications.

1059. The effects to be considered *depend on the conductor* employed to complete the communication between the zinc and copper plates of the electromotor; and I shall have to consider this conductor under four different forms: as the helix of an electro-magnet (1056.); as an ordinary helix (1053. &c.); as a *long* extended wire, having its course such that the parts can exert no mutual influence; and as a *short* wire. In all cases the conductor was of copper.

1060. The effects are best shown by the *electro-magnet* (1056.). When it was

used to complete the communication at the electromotor, there was no sensible spark on *making* contact, but on *breaking* contact there was a very large and bright spark, with considerable combustion of the mercury. Then, again, with respect to the shock: if the hands were moistened in salt and water, and good contact between them and the wires retained, no shock could be felt upon *making* contact at the electromotor, but a powerful one on *breaking* contact.

1061. When the helix i or iii (1053. &c.) was used as the connecting conductor, there was also a good spark on breaking contact, but none (sensibly) on making contact. On trying to obtain the shock from these helices, I could not succeed at first. By joining the similar ends of i and ii so as to make the two helices equivalent to one helix, having wire of double thickness, I could just obtain the sensation. Using the helix of thick wire (1055.) the shock was distinctly obtained. On placing the tongue between two plates of silver connected by wires with the parts which the hands had heretofore touched (1064.), there was a powerful shock on *breaking* contact, but none on *making* contact.

1062. The power of producing these phenomena exists therefore in the simple helix, as in the electro-magnet, although by no means in the same high degree.

1063. On putting a bar of soft iron into the helix, it became an electro-magnet (1057.), and its power was instantly and greatly raised. On putting a bar of copper into the helix, no change was produced, the action being that of the helix alone. The two helices i and ii, made into one helix of twofold length of wire, produced a greater effect than either i or ii alone.

1064. On descending from the helix to the mere long wire, the following effects were obtained. A copper wire, 0·18 of an inch in diameter, and 132 feet in length, was laid out upon the floor of the laboratory, and used as the connecting conductor (1059.); it gave no sensible spark on making contact, but produced a bright one on breaking contact, yet not so bright as that from the helix (1061.). On endeavouring to obtain the electric shock at the moment contact was broken, I could not succeed so as to make it pass through the hands; but by using two silver plates fastened by small wires to the extremity of the principal wire used, and introducing the tongue between those plates, I succeeded in obtaining powerful shocks upon the parts of the mouth, and could easily convulse a flounder, an eel, or a frog. None of these effects could be obtained directly from the electromotor, i. e. when the tongue, frog, or fish was in a similar, and therefore comparative manner, interposed in the course of the communication between the zinc and copper plates, separated everywhere else by the acid used to excite the combination. The bright spark and the shock, produced only on breaking contact, are therefore effects of the same kind as those produced in a higher degree by the helix, and in a still higher degree by the electro-magnet.

1065. In order to compare an extended wire with a helix, the helix i, containing ninety-six feet, and ninety-six feet of the same-sized wire lying on the floor of the laboratory, were used alternately as conductors: the former gave a much brighter

spark at the moment of disjunction than the latter. Again, twenty-eight feet of copper wire were made up into a helix, and being used gave a good spark on disjunction with the electromotor; being then suddenly pulled out and again employed, it gave a much smaller spark than before, although nothing but its spiral arrangement had been changed.

1066. As the superiority of a helix over a wire is important to the philosophy of the effect, I took particular pains to ascertain the fact with certainty. A wire of copper sixty-seven feet long was bent in the middle so as to form a double termination which could be communicated with the electromotor; one of the halves of this wire was made into a helix and the other remained in its extended condition. When these were used alternately as the connecting wire, the helix gave by much the strongest spark. It even gave a stronger spark than when it and the extended wire were used conjointly as a double conductor.

1067. When a *short wire* is used, *all* these effects disappear. If it be only two or three inches long, a spark can scarcely be perceived on breaking the junction. If it be ten or twelve inches long and moderately thick, a small spark may be more easily obtained. As the length is increased, the spark becomes proportionately brighter, until from extreme length the resistance offered by the metal as a conductor begins to interfere with the principal result.

1068. The effect of elongation was well shown thus: 114 feet of copper wire, one eighteenth of an inch in diameter, were extended on the floor and used as a conductor; it remained cold, but gave a bright spark on breaking contact. Being crossed so that the two terminations were in contact near the extremities, it was again used as a conductor, only twelve inches now being included in the circuit: the wire became very hot from the greater quantity of electricity passing through it, and yet the spark on breaking contact was scarcely visible. The experiment was repeated with a wire one ninth of an inch in diameter and thirty-six feet long with the same results.

1069. That the effects, and also the action, in all these forms of the experiment are identical; is evident from the manner in which the former can be gradually raised from that produced by the shortest wire to that of the most powerful electro-magnet: and this capability of examining what will happen by the most powerful apparatus, and then experimenting for the same results, or reasoning from them, with the weaker arrangements, is of great advantage in making out the true principles of the phenomena.

1070. The action is evidently dependent upon the wire which serves as a conductor; for it varies as that wire varies in its length or arrangement. The shortest wire may be considered as exhibiting the full effect of spark or shock which the electromotor can produce by its own direct power; all the additional force which the arrangements described can excite being due to some affection of the current, either permanent or momentary, in the wire itself. That it is a *momentary* effect, produced only at the instant of breaking contact, will be fully proved (1089. 1100.).

1071. No change takes place in the quantity or intensity of the current during the

time the latter is *continued*, from the moment after contact is made up to that previous to disunion, except what depends upon the increased obstruction offered to the passage of the electricity by a long wire as compared to a short wire. To ascertain this point with regard to *quantity*, the helix i (1053.) and the galvanometer (1058.) were both made parts of the metallic circuit used to connect the plates of a small electromotor, and the deflection at the galvanometer was observed; then a soft iron core was put into the helix, and as soon as the momentary effect was over, and the needle had become stationary, it was again observed, and found to stand exactly at the same division as before. Thus the quantity passing through the wire when the current was continued was the same either with or without the soft iron, although the peculiar effects occurring at the moment of disjunction were very different in degree under such variation of circumstances.

1072. That the quality of *intensity* belonging to the constant current did not vary with the circumstances favouring the peculiar results under consideration, so as to yield an explanation of those results, was ascertained in the following manner. The current excited by an electromotor was passed through short wires, and its intensity tried by subjecting different substances to its electrolyzing power (912. 966. &c.); it was then passed through the wires of the powerful electro-magnet (1056.), and again examined with respect to its intensity by the same means and found unchanged. Again, the constancy of the *quantity* passed in the above experiment (1071.) adds further proof that the intensity could not have varied; for had it been increased upon the introduction of the soft iron, there is every reason to believe that the quantity passed in a given time would also have increased.

1073. The fact is, that under many variations of the experiments, the permanent current *loses* in force as the effects upon breaking contact become *exalted*. This is abundantly evident in the comparative experiments with long and short wires (1068); and is still more strikingly shown by the following variation. Solder an inch or two in length of fine platina wire (about one hundredth of an inch in diameter) on to one end of the long communicating wire, and also a similar length of the same platina wire on to one end of the short communication; then, in comparing the effects of these two communications, make and break contact between the platina terminations and the mercury of the cup G or E (1079.). When the short wire is used, the platina will be *ignited by the constant current*, because of the quantity of electricity, but the spark on breaking contact will be hardly visible; on using the longer communicating wire, which by obstructing will diminish the current, the platina will remain cold whilst the current passes, but give a bright spark at the moment it ceases: thus the strange result is obtained of a diminished spark and shock from the strong current, and increased effects from the weak one. Hence the spark and shock at the moment of disjunction, although resulting from great intensity and quantity of the current *at that moment*, are no direct indicators or measurers of the intensity or quantity of the constant current previously passing, and by which they are ultimately produced.

1074. It is highly important in using the spark as an indication, by its relative brightness, of these effects, to bear in mind certain circumstances connected with its production and appearance. An ordinary electric spark is understood to be the bright appearance of electricity passing suddenly through an interval of air, or other badly conducting matter. A voltaic spark is sometimes of the same nature, but, generally, is due to the ignition and even combustion of a minute portion of a good conductor; and that is especially the case when the electromotor consists of but one or few pairs of plates. This can be very well observed if either or both of the metallic surfaces intended to touch be solid and pointed. The moment they come in contact the current passes; it heats, ignites, and even burns the touching points, and the appearance is as if the spark passed on making contact, whereas it is only a case of ignition by the current, contact being previously made, and is perfectly analogous to the ignition of a fine platina wire connecting the extremities of a voltaic battery.

1075. When mercury constitutes one or both of the surfaces used, the brightness of the spark is greatly increased. But as this effect is due to the action on, and probable combustion of, the metal, such sparks must only be compared with other sparks also taken from mercurial surfaces, and not with such as may be taken, for instance, between surfaces of platina or gold, for then the appearances are far less bright, though the same quantity of electricity be passed. It is not at all unlikely that the commonly occurring circumstance of combustion may affect even the duration of the light; and that sparks taken between mercury, copper, or other combustible bodies, will continue for a period sensibly longer than those passing between platina or gold.

1076. When the end of a short clean copper wire, attached to one plate of an electromotor, is brought down carefully upon a surface of mercury connected with the other plate, a spark, almost continuous, can be obtained. This I refer to a succession of effects of the following nature: first contact,—then ignition of the touching points,—recession of the mercury from the mechanical results of the heat produced at the place of contact, and the electro-magnetic condition of the parts at the moment*, —breaking of the contact and the production of the peculiar intense effect dependent thereon,—renewal of the contact by the returning surface of the undulating mercury, —and then a repetition of the same series of effects, and that with such rapidity as to present the appearance of a continued discharge. If a long wire or an electro-magnet be used as the connecting conductor instead of a short wire, a similar appearance may be produced by tapping the vessel containing the mercury and making it vibrate; but the sparks do not usually follow each other so rapidly as to produce an apparently continuous spark, because of the time required when the long wire or electro-magnet is used both for the full development of the current (1101. 1106.) and for its complete cessation.

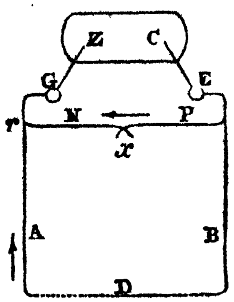
1077. Returning to the phenomena in question, the first thought that arises in the mind is, that the electricity circulates with something like *momentum or inertia* in

* Quarterly Journal of Science, vol. xii. p. 420.

the wire, and that thus a long wire produces effects at the instant the current is stopped, which a short wire cannot produce. Such an explanation is, however, at once set aside by the fact, that the same length of wire produces the effects in very different degrees, according as it is simply extended, or made into a helix, or forms the circuit of an electro-magnet (1069.). The experiments to be adduced (1089.) will still more strikingly show that the idea of momentum cannot apply.

1078. The bright spark at the electromotor, and the shock in the arms, appeared evidently to be due to *one* current in the long wire, divided into two parts by the double channel afforded through the body and through the electromotor; for that the spark was evolved at the place of disjunction with the electromotor, not by any direct action of the latter, but by a force immediately exerted in the wire of communication, seemed to be without doubt (1070.). It followed, therefore, that by using a better conductor in place of the human body, the *whole* of this extra current might be made to pass at that place; and thus be separated from that which the electromotor could produce by its immediate action, and its *direction* be examined apart from any interference of the original and originating current. This was found to be true; for on connecting the ends of the principal wire together by a cross wire two or three feet in length, applied just where the hands had felt the shock, the whole of the extra current passed by the new channel, and then no better spark than one producible by a short wire was obtained on disjunction at the electromotor.

1079. The *current* thus separated was examined by galvanometers and decomposing apparatus introduced into the course of this wire. I will always speak of it as the current in the cross wire or wires, so that no mistake, as to its place or origin, may occur. In the wood-cut, Z and C represent the zinc and copper plates of the electromotor; G and E the cups of mercury where contact is made or broken (1052.); A and B the terminations of D the long wire, the helix, or the electro-magnet, used to complete the circuit; N and P are the cross wires, which can either be brought into contact at x, or else have a galvanometer (1058.) or an electrolyzing apparatus (312. 316.) interposed there. The production of the *shock* from the current in the cross wire, whether D was a long extended wire, or a helix, or an electro-magnet, has been already described (1064. 1061. 1060.).



1080. The *spark* of the cross-wire current could be produced at x in the following manner: D was made an electro-magnet; the metallic extremities at x were held close together, or rubbed lightly against each other, whilst contact was broken at G or E. When the communication was perfect at x, little or no spark appeared at G or E. When the condition of vicinity at x was favourable for the result required, a bright spark would pass there at the moment of disjunction, *none* occurring at G and E: this spark was the luminous passage of the extra current through the cross-wires. When there was no contact or passage of current at x, then the spark ap-

peared at G or E, the extra current forcing its way through the electromotor itself. The same results were obtained by the use of the helix or the extended wire at D in place of the electro-magnet.

1081. On introducing a fine platina wire at *x*, and employing the electro-magnet at D, no visible effects occurred as long as contact was continued; but on breaking contact at G or E, the fine wire was instantly ignited and fused. A longer or thicker wire could be so adjusted at *x* as to show ignition, without fusion, every time the contact was broken at G or E.

1082. It is rather difficult to obtain this effect with helices or wires, and for very simple reasons: with the helices i, ii, or iii, there was such retardation of the electric current, from the length of wire used, that a full inch of platina wire one fiftieth of an inch in diameter could be retained ignited at the cross wires during the *continuance of contact*, by the portion of electricity passing through it. Hence it was impossible to distinguish the particular effects at the moments of making or breaking contact from this constant effect.

1083. On using the thick wire helix (1055.), the same results ensued. Proceeding, however, upon the known fact that electric currents of great quantity but low intensity, though able to ignite thick wires, cannot produce that effect upon thin ones, I used a very fine platina wire at *x*, reducing its diameter until a spark appeared at G or E, when contact was broken there. A quarter of an inch of such wire might be introduced at *x* without being ignited by the *continuance* of contact at G or E; but when contact was broken at either place, this wire became red hot; proving, by this method, the production of the induced current at that moment.

1084. *Chemical decomposition* was next effected by the cross-wire current, an electro-magnet being used at D, and a decomposing apparatus, with solution of iodide of potassium in paper (1079.), employed at *x*. The conducting power of the connecting system A B D was sufficient to carry all the primary current, and consequently no chemical action took place at *x* during the *continuance* of contact at G and E; but when contact was broken, there was instantly decomposition at *x*. The iodine appeared against the wire N, and not against the wire P; thus demonstrating that the current through the cross-wires, when contact was broken, was in the *reverse direction* to that marked by the arrow, or that which the electromotor would have sent through it.

1085. In this experiment a bright spark occurs at the place of disjunction, indicating that only a small part of the extra current passed the apparatus at *x*, because of the small conducting power of the latter.

1086. I found it difficult to obtain the chemical effects with the simple helices and wires, in consequence of the diminished inductive power of these arrangements, and because of the passage of a strong constant current at *x* whenever a very active electromotor was used (1082.).

1087. The most instructive set of results was obtained, however, when the *galvano-*

meter was introduced at x. Using an electro-magnet at D, and continuing contact, a current was then indicated by the deflection, proceeding from P to N, in the direction of the arrow; the cross wire serving to carry one part of the electricity excited by the electromotor, and the arrangement A B D, as indicated by the arrows, the other and far greater part. The magnetic needle was then forced back, by pins applied upon opposite sides of its two extremities, to its natural position when uninfluenced by a current; after which, contact being *broken* at G or E, it was deflected strongly in the opposite direction; thus showing, in accordance with the chemical effects (1084.), that the extra current followed a course in the cross wires *contrary* to that indicated by the arrow, i. e. the one produced by the direct action of the electromotor *.

1088. With the helix only, these effects could scarcely be observed, in consequence of the smaller inductive force of this arrangement, the opposed action from induction in the galvanometer wire itself, the mechanical condition and tension of the needle from the effect of blocking (1087.) whilst the current due to continuance of contact was passing round it, and other causes. With the extended wire all these circumstances had still greater influence, and therefore allowed less chance of success.

1089. These experiments, establishing as they did, by the quantity, intensity, and even direction, a distinction between the primary or generating current and the extra current, led me to conclude that the latter was identical with the induced current described (6. 26.) in the first series of these Researches; and this opinion I was soon able to bring to proof, and at the same times obtained not the partial (1078.) but entire separation of one current from the other.

1090. The double helix (1053.) was arranged so that i should form the connecting wire between the plates of the electromotor, ii being out of the current, and its ends unconnected. In this condition i acted very well, and gave a good spark at the time and place of disjunction. The opposite ends of ii were then connected together so as to form an endless wire, i remaining unchanged: but now no spark, or one scarcely sensible, could be obtained from the latter at the place of disjunction. Then, again, the ends of ii were held so nearly together that any current running round that helix should be rendered visible as a spark; and in this manner a spark was obtained from ii when the junction of i with the electromotor was broken, in place of appearing at the disjoined extremity of i itself.

1091. By introducing a galvanometer or a decomposing apparatus into the circuit formed by the helix ii, I could easily obtain the deflections and decomposition occasioned by the induced current due to the breaking contact at helix i, or even to that occasioned by making contact of that helix with the electromotor; the results in both cases indicating the contrary directions of the two induced currents thus produced (26.).

* It was ascertained experimentally, that if a strong current was passed through the galvanometer only, and the needle restrained in one direction as above in its natural position, when the current was stopped, no vioration of the needle in the opposite direction took place.

1092. All these effects, except those of decomposition, were reproduced by two extended long wires, not having the form of helices, but placed close to each other; and thus it was proved that the *extra current* could be removed from the wire carrying the original current to a neighbouring wire, and was at the same time identified, in direction and every other respect, with the currents producible by induction (1089.). The case, therefore, of the bright spark and shock on disjunction may now be stated thus: If a current be established in a wire, and another wire, forming a complete circuit, be placed parallel to the first, at the moment the current in the first is stopped it induces a current in the *same* direction in the second, the first exhibiting then but a feeble spark; but if the second wire be away, disjunction of the first wire induces a current in itself in the same direction, producing a strong spark. The strong spark in the single long wire or helix, at the moment of disjunction, is therefore the equivalent of the current which would be produced in a neighbouring wire if such second current were permitted.

1093. Viewing the phenomena as the results of the induction of electrical currents, many of the principles of action, in the former experiments, become far more evident and precise. Thus the different effects of short wires, long wires, helices, and electro-magnets (1069.) may be comprehended. If the inductive action of a wire a foot long upon a collateral wire also a foot in length, be observed, it will be found very small; but if the same current be sent through a wire fifty feet long, it will induce in a neighbouring wire of fifty feet a far more powerful current at the moment of making or breaking contact, each successive foot of wire adding to the sum of action; and by parity of reasoning, a similar effect should take place when the conducting wire is also that in which the induced current is formed: hence the reason why a long wire gives a brighter spark on breaking contact than a short one (1068.), although it carries much less electricity.

1094. If the long wire be made into a helix, it will then be still more effective in producing sparks and shocks on breaking contact; for by the mutual inductive action of the convolutions each aids its neighbour, and will be aided in turn, and the sum of effect will be very greatly increased.

1095. If an electro-magnet be employed, the effect will be still more highly exalted; because the iron, magnetized by the power of the continuing current, will lose its magnetism at the moment the current ceases to pass, and in so doing will tend to produce an electric current in the wire around it (37. 38.), in conformity with that which the cessation of current in the helix itself also tends to produce.

1096. By applying the laws of the induction of electric currents formerly developed (6. &c.), various new conditions of the experiments could be devised, which by their results should serve as tests of the accuracy of the view just given. Thus, if a long wire be doubled, so that the current in the two halves shall have opposite actions, it ought not to give a sensible spark at the moment of disjunction: and this proved to be the case, for a wire forty feet long, covered with silk, being doubled and tied closely together to within four inches of the extremities, when used in that state,

gave scarcely a perceptible spark; but being opened out and the parts separated, it gave a very good one. The two helices i and ii being joined at their similar ends, and then used at their other extremities to connect the plates of the electromotor, thus constituted one long helix, of which one half was opposed in direction to the other half: under these circumstances it gave scarcely a sensible spark, even when the soft iron core was within, although containing nearly two hundred feet of wire. When it was made into one consistent helix of the same length of wire it gave a very bright spark.

1097. Similar proofs can be drawn from the mutual inductive action of two separate currents (1110.); and it is important for the general principles that the consistent action of two such currents should be established. Thus, two currents going in the same direction should, if simultaneously stopped, aid each other by their relative influence; or if proceeding in contrary directions, should oppose each other under similar circumstances. I endeavoured at first to obtain two currents from two different electromotors, and passing them through the helices i and ii, tried to effect the disjunctions mechanically at the same moment. But in this I could not succeed; one was always separated before the other, and in that case produced little or no spark, its inductive power being employed in throwing a current round the remaining complete circuit (1090.): the current which was stopped last always gave a bright spark. If it were ever to become needful to ascertain whether two junctions were accurately broken at the same moment, these sparks would afford a test for the purpose, having an infinite degree of perfection.

1098. I was able to prove the points by other expedients. Two short thick wires were selected to serve as terminations, by which contact could be made or broken with the electromotor. The compound helix, consisting of i and ii (1053.), was adjusted so that the extremities of the two helices could be placed in communication with the two terminal wires, in such a manner that the current moving through the thick wires should be divided into two equal portions in the two helices, these portions travelling, according to the mode of connexion, either in the same direction or in contrary directions at pleasure. In this manner two streams could be obtained, both of which could be stopped simultaneously, because the disjunction could be broken at G or F by removing a single wire. When the helices were in contrary directions, there was scarcely a sensible spark at the place of disjunction; but when they were in accordance there was a very bright one.

1099. The helix i was now used constantly, being sometimes associated, as above, with helix ii in an according direction, and sometimes with helix iii, which was placed at a little distance. The association i and ii, which presented two currents able to affect each other by induction, because of their vicinity, gave a brighter spark than the association i and iii, where the two streams could not exert their mutual influence; but the difference was not so great as I expected.

1100. Thus all the phenomena tend to prove that the effects are due to an induc-

tive action, occurring at the moment when the principal current is stopped. I at one time thought they were due to an action continued during the *continuance* of the current, and expected that a steel magnet would have an influence according to its position in the helix, comparable to that of a soft iron bar, in assisting the effect. This, however, is not the case; for hard steel, or a magnet in the helix, is not so effectual as soft iron; nor does it make any difference how the magnet is placed in the helix, and for very simple reasons, namely, that the effect does not depend upon a permanent state of the core, but a change of state, and that the magnet or hard steel cannot sink through such a difference of state as soft iron, at the moment contact ceases, and therefore cannot produce an equal effect in generating a current of electricity by induction (34. 37.).

1101. As an electric current acts by induction with equal energy at the moment of its commencement as at the moment of its cessation (10. 26.), but in a contrary direction, the reference of the effects under examination to an inductive action, would lead to the conclusion that corresponding effects of an opposite nature must occur in a long wire, a helix, or an electro-magnet, every time that *contact is made* with the electromotor. These effects will tend to establish a resistance for the first moment in the long conductor, producing a result equivalent to the reverse of a shock or a spark. Now it is very difficult to devise means fit for the recognition of such negative results; but as it is probable that some positive effect is produced at the time, if we knew what to expect, I think the few facts bearing upon this subject with which I am acquainted are worth recording.

1102. The electro-magnet was arranged with an electrolyzing apparatus at *x*, as before described (1084.), except that the intensity of the chemical action at the electromotor was increased until the electric current was just able to produce the feeblest signs of decomposition whilst contact was continued at G and E (1079.); (the iodine of course appearing against the end of the cross wire P;) the wire N was also separated from A at *r*, so that contact there could be made or broken at pleasure. Under these circumstances the following set of actions was repeated several times: contact was broken at *r*, then broken at G, next made at *r*, and lastly renewed at G; thus any current from N to P due to *breaking* of contact was avoided, but any additional force to the current from P to N due to *making* contact could be observed. In this way it was found, that a much greater decomposing effect (causing the evolution of iodine against P) could be obtained by a few completions of contact than by the current which could pass in a much longer time if the contact was *continued*. This I attribute to the act of induction in the wire A B D at the moment of contact rendering that wire a worse conductor, or rather retarding the passage of the electricity through it for the instant, and so throwing a greater quantity of the electricity which the electromotor could produce, through the cross wire passage N P. The instant the induction ceased, A B D resumed its full power of carrying a constant current of

electricity, and could have it highly increased, as we know by the former experiments (1060.) by the opposite inductive action brought into activity at the moment contact at Z or C was *broken.*

1103. A galvanometer was then introduced at *x*, and the deflection of the needle noted whilst contact was continued at G and E: the needle was then blocked as before in one direction (1087.), so that it should not return when the current ceased, but remain in the position in which the current could retain it. Contact at G or E was broken, producing of course no visible effect; it was then renewed, and the needle was instantly deflected, passing from the blocking-pins to a position still further from its natural place than that which the constant current could give, and thus showing by the temporary excess of current in this cross communication, the temporary retardation in the circuit A B D.

1104. On adjusting a platina wire at *x* (1081.) so that it should not be ignited by the current passing through it whilst contact at G and E was *continued,* and yet become red hot by a current somewhat more powerful, I was readily able to produce its ignition upon *making contact,* and again upon *breaking contact.* Thus the momentary retardation in A B D on making contact was again shown by this result, as well also as the opposite result upon breaking contact. The two ignitions of the wire at *x* were of course produced by electric currents moving in opposite directions.

1105. Using the *helix* only, I could not obtain distinct deflections at *x*, due to the extra effect on making contact, for the reasons already mentioned (1088.). By using a very fine platina wire there (1083.), I did succeed in obtaining the igniting effect for making contact in the same manner, though by no means to the same degree, as with the electro-magnet (1104.).

1106. We may also consider and estimate the effect on *making contact,* by transferring the force of induction from the wire carrying the original current to a lateral wire, as in the cases described (1090.); and we then are sure, both by the chemical and galvanometrical results (1091.), that the forces upon making and breaking contact, like action and reaction, are equal in their strength but contrary in their direction. If, therefore, the effect on making contact resolves itself into a mere retardation of the current at the first moment of its existence, it must be, in its degree, equivalent to the high exaltation of that same current at the moment contact is broken.

1107. Thus the case, under the circumstances, is, that the intensity and quantity of electricity moving in a current are smaller when the current commences or is increased, and greater when it diminishes or ceases, than they would be if the inductive action occurring at these moments did not take place; or than they are in the original current wire if the inductive action be transferred from that wire to a collateral one (1090.).

1108. From the facility of transference to neighbouring wires, and from the effects generally, the inductive forces appear to be lateral, *i. e.* exerted in a direction perpendicular to the direction of the originating and produced currents: and they also

appear to be accurately represented by the magnetic curves, and closely related to, if not identical with, magnetic forces.

1109. There can be no doubt that the current in one part of a wire can act by induction upon other parts of the *same* wire which are lateral to the first, *i. e.* in the same section, or in the parts which are more or less oblique to it (1112.), just as it can act in producing a current in a neighbouring wire. It is this which gives the appearance of the current acting upon itself: but all the experiments and all analogy tend to show that the elements (if I may so say) of the currents do not act upon themselves, and so cause the effect in question, but produce it by exciting currents in conducting matter which is lateral to them.

1110. It is possible that some of the expressions I have used may seem to imply, that the inductive action is essentially the action of one current upon another, or of one element of a current upon another element of the same current. To avoid any such conclusion I must explain more distinctly my meaning. If an endless wire be taken, we have the means of generating a current in it which shall run round the circuit without adding any electricity to what was previously in the wire. As far as we can judge, the electricity which appears as a current is the same as that which before was quiescent in the wire; and though we cannot as yet point out the essential condition of difference of the electricity at such times, we can easily recognise the two states. Now when a current acts by induction upon conducting matter lateral to it, it probably acts upon the electricity in that conducting matter whether it be in the form of a *current* or *quiescent*, in the one case increasing or diminishing the current according to its direction, in the other producing a current, and the *amount* of the inductive action is probably the same in both cases. Hence, to say that the action of induction depended upon the mutual relation of two or more currents, would, according to the restricted sense in which the term current is understood at present (283. 517. 667.), be an error.

1111. Several of the effects, as, for instances, those with helices (1066.), with according or counter currents (1097. 1098.), and those on the production of lateral currents (1090.), appeared to indicate that a current could produce an effect of induction in a neighbouring wire more readily than in its own carrying wire, in which case it might be expected that some variation of result would be produced if a bundle of wires were used as a conductor instead of a single wire. In consequence the following experiments were made. A copper wire one twenty-third of an inch in diameter was cut into lengths of five feet each, and six of these being laid side by side in one bundle, had their opposite extremities soldered to two terminal pieces of copper. This arrangement could be used as a discharging wire, but the general current could be divided into six parallel streams, which might be brought close together, or, by the separation of the wires, be taken more or less out of each other's influence. A somewhat brighter spark was, I think, obtained on breaking contact when the six wires were close together than when held asunder.

1112. Another bundle, containing twenty of these wires, was eighteen feet long: the terminal pieces were one fifth of an inch in diameter, and each six inches long. This was compared with nineteen feet in length of copper wire one fifth of an inch in diameter. The bundle gave a smaller spark on breaking contact than the latter, even when its strands were held together by string: when they were separated, it gave a still smaller spark. Upon the whole, however, the diminution of effect was not such as I expected; and I doubt whether the results can be considered as any proof of the truth of the supposition which gave rise to them.

1113. The inductive force by which two elements of one current (1109. 1110.) act upon each other, appears to diminish as the line joining them becomes oblique to the direction of the current, and to vanish entirely when it is parallel. I am led by some results to suspect that it then even passes into the repulsive force noticed by AMPÈRE*; which is the cause of the elevations in mercury described by Sir HUMPHRY DAVY†, and which again is probably directly connected with the quality of intensity.

1114. Notwithstanding that the effects appear only at the making and breaking of contact, (the current remaining unaffected, seemingly, in the interval,) I cannot resist the impression that there is some connected and correspondent effect produced by this lateral action of the elements of the electric stream during the time of its continuance (60. 242.). An action of this kind, in fact, is evident in the magnetic relations of the parts of the current. But admitting (as we may do for the moment) the magnetic forces to constitute the power which produces such striking and different results at the commencement and termination of a current, still there appears to be a link in the chain of effects, a wheel in the physical mechanism of the action, as yet unrecognised. If we endeavour to consider electricity and magnetism as the results of two forces of a physical agent, or a peculiar condition of matter, exerted in determinate directions perpendicular to each other, then, it appears to me, that we must consider these two states or forces as convertible into each other in a greater or smaller degree; i. e. that an element of an electric current has not a determinate electric force and a determinate magnetic force constantly existing in the same ratio, but that the two forces are, to a certain degree, convertible by a process or change of condition at present unknown to us. How else can a current of a given intensity and quantity be able, by its direct action, to sustain a state which, when allowed to react, (at the cessation of the original current,) shall produce a second current, having an intensity and quantity far greater than the generating one? This cannot result from a direct reaction of the electric force; and if it result from a change of electrical into magnetic force, and a reconversion back again, it will show that they differ in something more than mere direction, as regards *that agent* in the conducting wire which constitutes their immediate cause.

1115. With reference to the appearance, at different times, of the contrary effects

* Recueil d'Observations Electro-Dynamiques, p. 285.

† Philosophical Transactions, 1823, p. 155.

produced by the making and breaking contact, and their separation by an intermediate and indifferent state, this separation is probably more apparent than real. If the conduction of electricity be effected by vibrations, or by any other mode in which opposite forces are successively and rapidly excited and neutralized, then we might expect a peculiar and contrary development of force at the commencement and termination of the periods during which the conducting action should last (somewhat in analogy with the colours produced at the outside of an imperfectly developed solar spectrum): and the intermediate actions, although not sensible in the same way, may constitute the very essence of conductibility. It is by views and reasons such as these, which seem to me connected with the fundamental laws and facts of electrical science, that I have been induced to enter, more minutely than I otherwise should have done, into the experimental examination of the phenomena described in this paper.

1116. Before concluding, I may briefly remark, that on using a voltaic battery of fifty pairs of plates instead of a single pair (1052.), the effects were exactly of the same kind. The spark on making contact, for the reasons before given, was very small (1101. 1107.); that on breaking contact, very excellent and brilliant. The *continuous* discharge did not seem altered in character, whether a short wire or the powerful electro-magnet were used as a connecting discharger.

1117. The effects produced at the commencement and end of a current, (which are separated by an interval of time when that current is supplied from a voltaic apparatus,) must occur at the same moment when a common electric discharge is passed through a long wire. Whether, if happening accurately at the same moment, they would entirely neutralize each other, or whether they would not still give some definite peculiarity to the discharge, is a matter remaining to be examined; but it is very probable that the peculiar character and pungency of sparks drawn from a long wire depend in part upon the increased intensity given at the termination of the discharge by the inductive action then occurring.

1118. In the wire of the helix of magneto-electric machines, (as, for instance, in Mr. SAXTON's beautiful arrangement,) an important influence of these principles of action is evidently shown. From the construction of the apparatus the current is permitted to move in a complete metallic circuit of great length during the first instants of its formation: it gradually rises in strength, and is then suddenly stopped by the breaking of the metallic circuit; and thus great intensity is given *by induction* to the electricity, which at that moment passes (1064. 1060.). This intensity is not only shown by the brilliancy of the spark and the strength of the shock, but also by the necessity which has been experienced of well insulating the convolutions of the helix, in which the current is formed; and it gives to the current a force at these moments very far above that which the apparatus could produce if the principle which forms the subject of this paper were not called into play.

Royal Institution,
December 8th, 1834.

IV. *On the Determination of the Terms in the Disturbing Function of the fourth order as regards the Eccentricities and Inclinations which give rise to Secular Inequalities.* *By* J. W. LUBBOCK, *V.P. and Treas. R.S.*

Received October 16,—Read November 20, 1834.

HITHERTO in the theory of the secular inequalities the terms in the disturbing function of the fourth order as regards the inclinations have been neglected. As the magnitude of these terms depends, in great measure, upon certain numerical coefficients, it is impossible to form any precise notion *à priori* with respect to their amount, and as to the error which may arise from neglecting them. I have therefore thought it desirable to ascertain their analytical expressions; and the details of this calculation form the subject of this paper. Some of the secular inequalities which result from these terms are far within the limits of accuracy which LAPLACE appears to have contemplated in the third volume of the Mécanique Céleste.

The method which I have here adopted for developing the disturbing function rests upon principles which I have already explained *. Very little trouble is requisite to obtain certain analytical expressions for the terms upon which the secular inequalities depend, or for any others, in the development of the disturbing function; but it is not so easy to put these expressions in the simplest form of which they are susceptible; and this is a point to which I think hitherto sufficient attention has not been paid. It will be found that I have obtained, finally, expressions of very remarkable simplicity: to accomplish this, however, I have been obliged to go through tedious processes of reduction, the details of which are here subjoined, in order that my results may be verified or corrected without difficulty. In order to give an additional example of the great facility with which terms in the disturbing function are arrived at by my method, I have calculated one of those given by Professor AIRY, and which is required in the determination of his inequality of Venus; and I have arrived at the result which he has given. The same method, with certain modifications, is applicable to the development of the disturbing function in terms of the true longitudes. The terms in the disturbing function which give rise to the secular inequalities of the elliptic constants, when the terms of the order of the fourth powers of the eccentricities and inclinations are retained, and higher powers of those quantities are neglected, are as follows: and I propose, as they form, in fact, a system apart, to distinguish them by the indices given in the left-hand column.

* Philosophical Transactions, 1832, Part II.

o. 0
I. $\tau - \xi + \xi_{\prime}$
II. $2\tau - 2\xi + 2\xi_{\prime}$
III. $\tau + \xi + \xi_{\prime} - 2\eta$
IV. $\tau - \xi - \xi_{\prime} + 2\eta_{\prime}$
V. $\xi - \xi_{\prime} - \eta + \eta_{\prime}$
VI. $\xi + \xi_{\prime} - \eta - \eta_{\prime}$
VII. $2\tau + 2\xi_{\prime} - 2\eta$
VIII. $2\xi - 2\eta$
IX. $2\tau - 2\xi + 2\eta_{\prime}$
X. $2\xi_{\prime} - 2\eta_{\prime}$
XI. $\tau - \eta + \eta_{\prime}$
XII. $\tau - 2\xi + \eta + \eta_{\prime}$
XIII. $\tau + 2\xi_{\prime} - \eta - \eta_{\prime}$
XIV. $2\tau - \xi + \xi_{\prime} - \eta + \eta_{\prime}$
XV. $2\tau - 2\eta + 2\eta_{\prime}$.

After extensive reductions I find

$$R = -\frac{m_{\prime}}{2a_{\prime}} b_{1,0} - \frac{m_{\prime} a}{8 a_{\prime}^2} b_{3,1} \left\{ e^2 + e_{\prime}^2 - \gamma^2 - \gamma_{\prime}^2 \right\} + m_{\prime} \left\{ -\frac{3a^3}{32 a_{\prime}^4} b_{5,1} + \frac{3a^2}{128 a_{\prime}^3} b_{5,2} \right\} e^4$$

$$- \frac{9 m_{\prime} a^2}{32 a_{\prime}^3} b_{5,0}\, e^2 e_{\prime}^2 + m_{\prime} \left\{ -\frac{3a}{32 a_{\prime}^2} b_{5,1} + \frac{3a^2}{128 a_{\prime}^3} b_{5,2} \right\} e_{\prime}^4$$

$$+ m_{\prime} \left\{ -\frac{3a^2}{16 a_{\prime}^3} b_{5,1} + \frac{3a^2}{32 a_{\prime}^3} b_{5,2} \right\} \left\{ \gamma^4 + \gamma_{\prime}^4 \right\}$$

$$+ m_{\prime} \left\{ -\frac{9a^2}{32 a_{\prime}^3} b_{5,0} - \frac{3a^2}{16 a_{\prime}^3} b_{5,2} \right\} \gamma^2 \gamma_{\prime}^2 + \frac{9 m_{\prime} a^2}{32 a_{\prime}^3} b_{5,0} \left\{ e^2 + e_{\prime}^2 \right\} \left\{ \gamma^2 + \gamma_{\prime}^2 \right\}$$

[o.]

$$+ m_{\prime} \left\{ \frac{a}{4 a_{\prime}^2} b_{3,2} + \left\{ \frac{15 a^2}{32 a_{\prime}^3} b_{5,1} - \frac{3a}{16 a_{\prime}^2} b_{5,2} \right\} e^2 + \left\{ \frac{15 a^2}{32 a_{\prime}^3} b_{5,1} - \frac{3a^3}{16 a_{\prime}^4} b_{5,2} \right\} e_{\prime}^2 \right.$$

$$\left. + \left\{ -\frac{15 a^2}{32 a_{\prime}^3} b_{5,1} - \frac{3a^3}{32 a_{\prime}^4} b_{5,2} \right\} \left\{ \gamma^2 + \gamma_{\prime}^2 \right\} \right\} e\, e_{\prime} \cos(\tau - \xi + \xi_{\prime})$$

[I.]

$$- \frac{9 m_{\prime} a^2}{64 a_{\prime}^3} b_{5,2}\, e^2 e_{\prime}^2 \cos(2\tau - 2\xi + 2\xi_{\prime}) + \frac{9 m_{\prime} a^2}{32 a_{\prime}^3} b_{5,1}\, e\, e_{\prime} \gamma^2 \cos(\tau + \xi + \xi_{\prime} - 2\eta)$$

[II.] [III.]

$$+ \frac{9 m_{\prime} a^2}{32 a_{\prime}^3} b_{5,1}\, e\, e_{\prime} \gamma_{\prime}^2 \cos(\tau - \xi - \xi_{\prime} + 2\eta_{\prime}) + \frac{9 m_{\prime} a^2}{16 a_{\prime}^3} b_{5,1}\, e\, e_{\prime} \gamma \gamma_{\prime} \cos(\xi - \xi_{\prime} - \eta + \eta_{\prime})$$

[IV.] [V.]

$$- \frac{9 m_{\prime} a^2}{16 a_{\prime}^3} b_{5,1}\, e\, e_{\prime} \gamma \gamma_{\prime} \cos(\xi + \xi_{\prime} - \eta - \eta_{\prime})$$

[VI.]

$$+ m_{,} \left\{ - \frac{3\,a^3}{16\,a_{,}^4} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} b_{5,2} \right\} e_{,}^2 \gamma^2 \cos (2\,\tau + 2\,\xi_{,} - 2\,\eta)$$
[VII.]

$$+ m_{,} \left\{ - \frac{3\,a}{16\,a_{,}^2} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} b_{5,2} \right\} e^2 \gamma^2 \cos (2\,\xi - 2\,\eta)$$
[VIII.]

$$+ m_{,} \left\{ - \frac{3\,a}{16\,a_{,}^2} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} \right\} e^2 \gamma_{,}^2 \cos (2\,\tau - 2\,\xi + 2\,\eta_{,})$$
[IX.]

$$+ m_{,} \left\{ - \frac{3\,a^2}{16\,a_{,}^3} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} b_{5,2} \right\} e_{,}^2 \gamma_{,}^2 \cos (2\,\xi_{,} - 2\,\eta_{,})$$
[X.]

$$+ m_{,} \left\{ - \frac{a}{4\,a_{,}^2} b_{3,1} - \frac{9\,a^2}{16\,a_{,}^3} b_{5,0} (e^2 + e_{,}^2) + \frac{3\,a^2}{16\,a_{,}^3} b_{5,0} (\gamma^2 + \gamma_{,}^2) \right\} \gamma\,\gamma_{,} \cos (\tau - \eta + \eta_{,})$$
[XI.]

$$+ m_{,} \left\{ \frac{3\,a}{8\,a_{,}^2} b_{5,1} - \frac{3\,a^2}{32\,a_{,}^3} b_{5,2} \right\} e^2 \gamma\,\gamma_{,} \cos (\tau - 2\,\xi + \eta + \eta_{,})$$
[XII.]

$$+ m_{,} \left\{ \frac{3\,a^2}{8\,a_{,}^3} b_{5,0} - \frac{3\,a^2}{32\,a_{,}^3} b_{5,2} \right\} e_{,}^2 \gamma\,\gamma_{,} \cos (\tau + 2\,\xi_{,} - \eta - \eta_{,})$$
[XIII.]

$$+ m_{,} \left\{ \frac{9\,a^2}{32\,a_{,}^3} b_{5,1} + \frac{33\,a^2}{32\,a_{,}^3} b_{5,3} \right\} e\,e_{,}\,\gamma\,\gamma_{,} \cos (2\,\tau - \xi + \xi_{,} - \eta + \eta_{,})$$
[XIV.]

$$- m_{,} \frac{3\,a^2}{32\,a_{,}^3} b_{5,0}\, \gamma^2 \gamma_{,}^2 \cos (2\,\tau - 2\,\eta + 2\,\eta_{,})$$
[XV.]

The method which I propose to employ in order to arrive at the terms in the disturbing function, independent of the inclinations, is sufficiently explained in the Philosophical Transactions. The following are the equations employed:

$$\frac{d R}{d e} = \frac{a\,d R}{d e}\,\frac{d r}{r\,d e} + \frac{d R}{d \tau}\,\frac{d \lambda}{d e}$$

$$\frac{d r}{r\,d e} = \frac{e}{2}\left(1 + \frac{e^2}{4}\right) - \left(1 - \frac{9}{8} e^2\right) \cos \xi - \frac{3}{2} e \left(1 - \frac{11}{9} e^2\right) \cos 2\,\xi$$
[0] [2] [8]

$$- \frac{17}{8} e^2 \cos 3\,\xi - \frac{71}{24} e^3 \cos 4\,\xi$$
[20] [35]

$$\frac{d \lambda}{d e} = 2 \left(1 - \frac{3 e^2}{8}\right) \sin \xi + \frac{5}{2}\, e \left(1 - \frac{28}{15} e^2\right) \sin 2\,\xi + \frac{13}{4} e^2 \sin 3\,\xi + \frac{103}{24} e^2 \sin 4\,\xi.$$
[2] [8] [20] [35]

Calculation of the Term in the non-periodical Portion of R multiplied by e^4.

If R'_2 denote the term in the coefficient of $\cos\xi$ multiplied by e^3,
R_8 . . . . $\cos 2\xi$ . . . . . e^2,
R'_0 . . . . non-periodical portion of R multiplied by e^2,
R_2 . . . . coefficient of $\cos\xi$ multiplied by e,

$$3R'_2 = \frac{a\,\mathrm{d}R_2}{2\,\mathrm{d}a} - \frac{a\,\mathrm{d}R_8}{2\,\mathrm{d}a} - \frac{3\,a\,\mathrm{d}R_2}{4\,\mathrm{d}a} - \frac{a\,\mathrm{d}R'_0}{\mathrm{d}a} + \frac{9\,a\,\mathrm{d}R_0}{8\,\mathrm{d}a}$$

$$R_2 = -\frac{a^2}{2a_{\prime}^3}b_{3,0} + \frac{a}{2a_{\prime}^3}b_{3,1} \qquad R_8 = -\frac{a^2}{2a_{\prime}^3}b_{3,0} + \frac{3a}{8a_{\prime}^2}b_{3,1}$$

$$R'_0 = -\frac{a}{8a_{\prime}^2}b_{3,1} \qquad R_0 = -\frac{1}{a_{\prime}}b_{1,0}$$

$$\frac{R_2}{2} - \frac{R_8}{2} - \frac{3}{4}R_2 - R'_0 + \frac{9}{8}R_0 = \frac{3a^2}{8a_{\prime}^3}b_{3,0} - \frac{3a}{16a_{\prime}^2}b_{3,1} - \frac{9}{16a_{\prime}}b_{1,0}$$

$$3R'_2 = \frac{3a^2}{4a_{\prime}^3}b_{3,0} - \frac{3a}{16a_{\prime}^2}b_{3,1} + \frac{9a}{16a_{\prime}^2}\left(\frac{a}{a_{\prime}}b_{3,0} - b_{3,1}\right) - \frac{9a^3}{8a_{\prime}^4}\left(\frac{a}{a_{\prime}}b_{5,0} - b_{5,1}\right)$$
$$+\frac{9a^2}{16a_{\prime}^3}\left(\frac{a}{a_{\prime}}b_{5,1} - \frac{1}{2}b_{5,0} - \frac{1}{2}b_{5,2}\right)$$

$$R'_2 = \frac{7a^2}{16a_{\prime}^3}b_{3,0} - \frac{a}{4a_{\prime}^2}b_{3,1} - \frac{3a^4}{8a_{\prime}^5}b_{5,0} + \frac{9a^3}{16a_{\prime}^4}b_{5,1} - \frac{3a^2}{32a_{\prime}^3}b_{5,0} - \frac{3a^2}{32a_{\prime}^3}b_{5,2}$$
$$= \frac{7a^2}{16a_{\prime}^3}b_{3,0} - \frac{a}{4a_{\prime}^2}b_{3,1} - \frac{3a^2}{8a_{\prime}^3}b_{3,0} + \frac{3a^2}{16a_{\prime}^3}b_{5,0} - \frac{3a^3}{16a_{\prime}^4}b_{5,1} + \frac{a}{16a_{\prime}^2}b_{3,1}$$
$$= \frac{a^2}{16a_{\prime}^3}b_{3,0} - \frac{3a}{16a_{\prime}^2}b_{3,1} + \frac{3a^2}{16a_{\prime}^3}b_{5,0} - \frac{3a^3}{16a_{\prime}^4}b_{5,1}.$$

If R''_0 denote the term in the non-periodical portion of the disturbing function multiplied by e^4,

$$4R''_0 = \frac{a\,\mathrm{d}R'_0}{2\,\mathrm{d}a} + \frac{a\,\mathrm{d}R_0}{8\,\mathrm{d}a} - \frac{a\,\mathrm{d}R'_2}{2\,\mathrm{d}a} + \frac{9\,a\,\mathrm{d}R_2}{16\,\mathrm{d}a} - \frac{3\,a\,\mathrm{d}R_8}{4\,\mathrm{d}a}$$

$$4R''_0 = \frac{1}{16a_{\prime}}b_{1,0} - \frac{a}{16a_{\prime}^2}\left(\frac{a}{a_{\prime}}b_{3,0} - b_{3,1}\right) - \frac{1}{16a_{\prime}}b_{3,0} - \frac{a}{16a_{\prime}^2}b_{3,1} + \frac{9a^2}{32a_{\prime}^3}b_{5,0}$$
$$-\frac{3a}{16a_{\prime}^2}b_{5,1} + \frac{3a}{16a_{\prime}^2}\left(\frac{a}{a_{\prime}}b_{5,0} - b_{5,1}\right) + \frac{3a^2}{32a_{\prime}^3}\left(\frac{a}{a_{\prime}}b_{5,1} - \frac{1}{2}b_{5,0} - \frac{1}{2}b_{5,2}\right)$$
$$-\frac{15a^3}{32a_{\prime}^4}\left(\frac{a}{a_{\prime}}b_{7,0} - b_{7,1}\right) + \frac{15a^2}{32a_{\prime}^3}\left(\frac{a}{a_{\prime}}b_{7,1} - \frac{1}{2}b_{7,0} - \frac{1}{2}b_{7,2}\right)$$
$$= \frac{1}{16a_{\prime}}\left(\frac{a^2+a_{\prime}^2}{a_{\prime}^2}b_{3,0} - 2\frac{a}{a_{\prime}}b_{3,1}\right) - \frac{a}{16a_{\prime}^2}\left(\frac{a}{a_{\prime}}b_{3,0} - b_{3,1}\right) - \frac{1}{16a_{\prime}}b_{3,0} - \frac{a}{16a_{\prime}^2}b_{3,1}$$
$$+\frac{9a^2}{32a_{\prime}^3}b_{5,0} - \frac{3a}{16a_{\prime}^2}b_{5,1} + \frac{3a}{16a_{\prime}^2}\left(\frac{a}{a_{\prime}}b_{5,0} - b_{5,1}\right) + \frac{3a^2}{32a_{\prime}^3}\left(\frac{a}{a_{\prime}}b_{5,1} - \frac{1}{2}b_{5,0} - \frac{1}{2}b_{5,2}\right)$$

$$- \frac{15\,a^2}{32\,a_i^3}\left(\frac{a^2+a_i^2}{a_i^2} b_{7,0} - 2\,\frac{a}{a_i} b_{7,1}\right) + \frac{15\,a^2}{64\,a_i^3} b_{7,0} - \frac{15\,a^2}{64\,a_i^3} b_{7,2}$$

$$= - \frac{a}{8 a_i^3}\left(\frac{a^2+a_i^2}{a_i^2} b_{5,1} - \frac{a}{a_i} b_{5,0} - \frac{a}{a_i} b_{5,2}\right) + \frac{27\,a^2}{64\,a_i^3} b_{5,0} - \frac{3\,a}{8\,a_i^2} b_{5,1}$$

$$- \frac{3\,a^2}{64\,a_i^3} b_{5,2} - \frac{15\,a^2}{32\,a_i^3} b_{5,0} + \frac{3\,a}{32\,a_i^2} b_{5,1}$$

$$= \frac{5\,a^2}{64\,a_i^3} b_{5\,0} - \frac{13\,a}{32\,a_i^2} b_{5,1} - \frac{a^3}{32\,a_i^4} b_{5,1} + \frac{5\,a^2}{64\,a_i^3} b_{5,2}$$

$$= \frac{a}{32\,a_i^2} b_{5,1} + \frac{a^3}{32\,a_i^4} b_{5,1} + \frac{a^2}{64\,a_i^3} b_{5,2} - \frac{13\,a}{32\,a_i^2} b_{5,1} - \frac{a^3}{32\,a_i^4} b_{5,1} + \frac{5\,a^2}{64\,a_i^3} b_{5,2}$$

$$= - \frac{3\,a}{8\,a_i^2} b_{5,1} + \frac{3\,a^2}{32\,a_i^3} b_{5,2}.$$

Hence R contains the term

$$m_i \left\{ - \frac{3\,a}{32\,a_i^2} b_{5,1} + \frac{3\,a^2}{128\,a_i^3} b_{5,2} \right\} e^4.$$

Putting for $b_{5,1}$, $b_{5,2}$ their values in series, the first term is

$$- \frac{15\,a^2}{32\,a_i^3}.$$

This result agrees with what I have before arrived at in the Lunar theory. I have neglected no similar opportunity of verifying the terms in the disturbing function given in this paper; these opportunities are however but few, as the terms multiplied by γ_i^2 may be dispensed with in the lunar theory.

Calculation of the Term in the non-periodical Portion of the disturbing Function multiplied by $e^2\,e_i^2$.

If R'_2 now denote the term in the coefficient of $\cos \xi$ multiplied by $e\,e_i^2$,
R'_0 . . . . non-periodical portion . . . e_i^2,
R''_0 . . . . $e^2\,e_i^2$,

$$R'_0 = - \frac{a}{8\,a_i^2} b_{3,1} \qquad R'_2 = - \frac{a\,\mathrm{d}\,R'_0}{2\,\mathrm{d}\,a} - R'_0$$

$$R'_2 = - \frac{3\,a^2}{8\,a_i^3}\left(\frac{a}{a_i} b_{5,1} - \frac{1}{2} b_{5,3} - \frac{1}{2} b_{5,2}\right) + \frac{a}{8\,a_i^2} b_{3,1}$$

$$2\,R''_0 = \frac{a\,\mathrm{d}\,R'_0}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R'_2}{2\,\mathrm{d}\,a}$$

$$R'_0 - R'_2 = - \frac{a}{8\,a_i^2} b_{3,1} - \frac{a}{8\,a_i^2} b_{3,1} + \frac{3\,a^3}{8\,a_i^4} b_{5,1} - \frac{3\,a^2}{16\,a_i^3} b_{5,0} - \frac{3\,a^2}{16\,a_i^3} b_{5,2}$$

$$2\,R''_0 = - \frac{a}{8\,a_i^2} b_{3,1} + \frac{3\,a^2}{8\,a_i^3}\left(\frac{a}{a_i} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) + \frac{9\,a^3}{16\,a_i^4} b_{5,1}$$

$$- \frac{3\,a^2}{16\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{16\,a_i^3}\,b_{5,2} - \frac{15\,a^4}{16\,a_i^5}\left(\frac{a}{a_i}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)$$

$$+ \frac{15\,a^2}{32\,a_i^3}\left(\frac{a}{a_i}\,b_{7,0} - b_{7,1}\right) + \frac{15\,a^2}{32\,a_i^3}\left(\frac{a}{a_i}\,b_{7,2} - \frac{1}{2}\,b_{7,1} - \frac{1}{2}\,b_{7,3}\right)$$

$$= - \frac{a}{8\,a_i^2}\,b_{3,1} + \frac{15\,a^3}{16\,a_i^4}\,b_{5,1} - \frac{3\,a^2}{8\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{4\,a_i^3}\,b_{5,2}$$

$$- \frac{15\,a_i^3}{16\,a^4}\left(\frac{a^2 + a_i^2}{a_i^2}\,b_{7,1} - \frac{a}{a_i}\,b_{7,0} - \frac{a}{a_i}\,b_{7,2}\right) + \frac{15\,a^3}{64\,a_i^4}\,b_{7,1} - \frac{15\,a^3}{64\,a_i^4}\,b_{7,3}$$

$$= - \frac{3\,a^2}{16\,a_i^3}\,b_{5,0} + \frac{3\,a^2}{16\,a_i^3}\,b_{5,2} - \frac{3\,a^2}{8\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{16\,a_i^3}\,b_{5,2} = - \frac{9\,a^2}{16\,a_i^3}\,b_{5,0}.$$

Hence the disturbing function contains the term

$$- \frac{9\,m_i\,a^2}{32\,a_i^3}\,b_{5,0}\,e^2\,e_i^2\text{; in the lunar theory } - \frac{9\,m_i\,a^2}{16\,a_i^3}\,e^2\,e_i^2.$$

Calculation of the Term in the non-periodical Portion of R multiplied by e_i^4.

If R'_5 denote the term in the coefficient of $\cos \xi_i$ multiplied by e_{i3},

R_{17} . . . . $\cos 2\,\xi_i$ . . . . e_i^2,

R_5 . . . . $\cos \xi_i$ . . . . e_i,

R'_0 . . . . non-periodical portion of R multiplied by e_i^2,

R''_0 . . . . e_i^4,

$$3\,R'_5 = \frac{a_i\,\mathrm{d}\,R_5}{2\,\mathrm{d}\,a_i} - \frac{a_i\,\mathrm{d}\,R_{17}}{2\,\mathrm{d}\,a_i} - \frac{3\,a_i\,\mathrm{d}\,R_5}{4\,\mathrm{d}\,a_i} - \frac{a_i\,\mathrm{d}\,R'_0}{\mathrm{d}\,a_i} + \frac{9\,a\,\mathrm{d}\,R_0}{8\,\mathrm{d}\,a_i}$$

$$R_5 = - \frac{1}{2\,a_i}\,b_{3,0} + \frac{a}{2\,a_i^2}\,b_{3,1} \qquad R_{17} = - \frac{1}{2\,a_i}\,b_{3,0} + \frac{3\,a}{8\,a_i^2}\,b_{3,1} \qquad R'_0 = - \frac{a}{8\,a_i^2}\,b_{3,1}$$

$$R_0 = - \frac{1}{a_i}\,b_{1,0}\;\frac{R_5}{2} - \frac{R_{17}}{2} - \frac{3}{4}\,R_5 - R'_0 + \frac{9}{8}\,R_0 = - \frac{R_5}{4} - \frac{R_{17}}{2} - R'_0 + \frac{9}{8}\,R_0$$

$$= \frac{3}{8\,a_i}\,b_{3,0} - \frac{3\,a}{16\,a_i^2}\,b_{3,1} - \frac{9}{16}\,b_{1,0}$$

$$3\,R'_5 = - \frac{3}{8\,a_i}\,b_{3,0} + \frac{3\,a}{8\,a_i^2}\,b_{3,1} + \frac{9}{16\,a_i}\,b_{1,0} - \frac{9\,a}{16\,a_i^2}\left(\frac{a}{a_i}\,b_{3,0} - b_{3,1}\right)$$

$$+ \frac{9\,a}{8\,a_i^2}\left(\frac{a}{a_i}\,b_{5,0} - b_{5,1}\right) - \frac{9\,a^2}{16\,a_i^3}\left(\frac{a}{a_i}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$= - \frac{3}{8\,a_i}\,b_{3,0} - \frac{9\,a^2}{16\,a_i^3}\,b_{3,0} + \frac{15\,a}{16\,a_i^2}\,b_{3,1} + \frac{9}{16\,a_i}\left(\frac{a^2 + a_i^2}{a_i^2}\,b_{3,0} - \frac{2\,a}{a_i}\,b_{3,1}\right)$$

$$- \frac{9\,a}{16\,a_i^2}\left(\frac{a^2 + a_i^2}{a_i^2}\,b_{5,1} - \frac{a}{a_i}\,b_{5,0} - \frac{a}{a_i}\,b_{5,2}\right) + \frac{27\,a^2}{32\,a_i^3}\,b_{6,0} - \frac{9\,a}{16\,a_i^2}\,b_{5,1} - \frac{9\,a^2}{32\,a_i^3}\,b_{5,2}$$

$$R'_5 = \frac{1}{16\,a_i}\,b_{3,0} - \frac{3\,a}{16\,a_i^2}\,b_{3,1} + \frac{3\,a^2}{16\,a_i^3}\,b_{5,0} - \frac{3\,a}{16\,a_i^2}\,b_{5,1}$$

$$4\,R''_0 = \frac{a_i\,\mathrm{d}\,R'_0}{2\,\mathrm{d}\,a_i} + \frac{a_i\,\mathrm{d}\,R_0}{8\,\mathrm{d}\,a_i} - \frac{a_i\,\mathrm{d}\,R_5}{2\,\mathrm{d}\,a_i} + \frac{9\,a_i\,\mathrm{d}\,R_5}{16\,\mathrm{d}\,a} - \frac{3\,a_i\,\mathrm{d}\,R_{17}}{4\,\mathrm{d}\,a_i}$$

$$\frac{R'_0}{2} + \frac{R_0}{8} - \frac{R'_5}{2} + \frac{9}{6} R_5 - \frac{3}{4} R_{17}$$

$$= - \frac{a}{16 a_{\prime}^2} b_{3,1} - \frac{1}{16 a_{\prime}} b_{1,0} - \frac{1}{32 a_{\prime}} b_{3,0} + \frac{3 a}{32 a_{\prime}^2} b_{3,1} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,0}$$

$$+ \frac{3 a}{32 a_{\prime}^2} b_{5,1} - \frac{9}{32 a_{\prime}} b_{3,0} + \frac{9 a}{32 a_{\prime}^2} b_{3,1} + \frac{3}{8 a_{\prime}} b_{3,0} - \frac{9 a}{32 a_{\prime}^2} b_{3,1}$$

$$4 R''_0 = \frac{1}{16 a_{\prime}} b_{1,0} - \frac{a}{16 a_{\prime}^2} \left(\frac{a}{a_{\prime}} b_{3,0} - b_{3,1}\right) - \frac{1}{16 a_{\prime}} b_{3,0} - \frac{a}{16 a_{\prime}^2} b_{3,1} + \frac{9 a^2}{32 a_{\prime}^3} b_{5,0}$$

$$- \frac{3 a}{16 a_{\prime}^2} b_{5,1} + \frac{3 a}{16 a_{\prime}^2} \left(\frac{a}{a_{\prime}} b_{5,0} - b_{5,1}\right) + \frac{3 a^2}{32 a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$- \frac{15 a^3}{32 a_{\prime}^4} \left(\frac{a}{a_{\prime}} b_{7,0} - b_{7,1}\right) + \frac{15 a^2}{32 a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right)$$

$$= \frac{1}{16 a_{\prime}} \left\{ \frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{3,0} - 2 \frac{a}{a_{\prime}} b_{3,1} \right\} - \frac{a}{16 a_{\prime}^2} \left(\frac{a}{a_{\prime}} b_{3,0} - b_{3,1}\right) - \frac{1}{16 a_{\prime}} b_{3,0} - \frac{a}{16 a_{\prime}^2} b_{3,1}$$

$$+ \frac{9 a^2}{32 a_{\prime}^3} b_{5,0} - \frac{3 a}{16 a_{\prime}^2} b_{5,1} + \frac{3 a}{16 a_{\prime}^2} \left(\frac{a}{a_{\prime}} b_{5,0} - b_{5,1}\right) + \frac{3 a^2}{32 a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$- \frac{15 a^2}{32 a_{\prime}^3} \left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{7,0} - 2 \frac{a}{a_{\prime}} b_{7,1}\right) + \frac{15 a^2}{64 a_{\prime}^3} b_{7,0} - \frac{15 a^2}{64 a_{\prime}^3} b_{7,2}$$

$$= - \frac{a}{8 a_{\prime}^2} \left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{5,1} - \frac{a}{a_{\prime}} b_{5,0} - \frac{a}{a_{\prime}} b_{5,2}\right) + \frac{27 a^2}{64 a_{\prime}^3} b_{5,0} - \frac{3 a}{8 a_{\prime}^2} b_{5,1}$$

$$- \frac{3 a^2}{64 a_{\prime}^3} b_{5,2} - \frac{15 a^2}{32 a_{\prime}^3} b_{5,0} + \frac{3 a}{32 a_{\prime}^2} b_{5,1}$$

$$= \frac{5 a^2}{64 a_{\prime}^3} b_{5,0} - \frac{13 a}{32 a_{\prime}^2} b_{5,1} - \frac{a^3}{32 a_{\prime}^4} b_{5,1} + \frac{5 a^2}{64 a_{\prime}^3} b_{5,2}$$

$$= \frac{a}{32 a_{\prime}^2} b_{5,1} + \frac{a^3}{32 a_{\prime}^4} b_{5,1} + \frac{a^2}{64 a_{\prime}^3} b_{5,2} - \frac{13 a}{32 a_{\prime}^2} b_{5,1} - \frac{a^3}{32 a_{\prime}^4} b_{5,1} + \frac{5 a^2}{64 a_{\prime}^3} b_{5,2}$$

$$= - \frac{3 a}{8 a_{\prime}^2} b_{5,1} + \frac{3 a^2}{32 a_{\prime}^3} b_{5,2}.$$

Hence the disturbing function contains the term

$$m_{\prime} \left\{ - \frac{3 a}{32 a_{\prime}^2} b_{5,1} + \frac{3 a^2}{128 a_{\prime}^3} b_{5,2} \right\} e_{\prime}^4; \text{ in the lunar theory} - \frac{15 m_{\prime} a^2}{32 a_{\prime}^3} e_{\prime}^4.$$

In the preceding instance, and in the case of terms depending either upon e^4 or $e_{\prime}^4$ solely, the term in the disturbing function can only be obtained from $\frac{dR}{de}$ or $\frac{dR}{de_{\prime}}$; but in obtaining those which depend *both* upon e and $e_{\prime}$, they may be obtained indifferently either from the combinations which enter into the expression for $\frac{dR}{de}$ or from those which enter into the expression for $\frac{dR}{de_{\prime}}$.

Calculation of the Coefficient of $e^3 e_{,} \cos(\tau - \xi + \xi_{,})$ *in the Development of* R.

If R_9 denote that part of the coefficient of $\cos(\tau - 2\xi)$ which is multiplied by e^2

R_1 . . . . $\cos \tau$ . . . . e^0

R'_1 . . . . e^2

R_4 . . . . $\cos(\tau + \xi)$ . . . . e

R_3 . . . . $\cos(\tau + \xi)$ . . . . e

R'_3 . . . . e^3

$$3 R'_3 = \frac{a\,\mathrm{d} R_3}{2\,\mathrm{d} a} - \frac{a\,\mathrm{d} R_9}{2\,\mathrm{d} a} + R_9 + \frac{9}{16}\frac{a\,\mathrm{d} R_1}{\mathrm{d} a} + \frac{3}{8} R_1 - \frac{3}{4}\frac{a\,\mathrm{d} R_4}{\mathrm{d} a} - \frac{5}{4} R_4 - \frac{a\,\mathrm{d} R'_1}{2\,\mathrm{d} a} - R'_1$$

$$R_3 = -\frac{3a}{2a_{,}^2} + \frac{3a}{4a_{,}^2} b_{3,0} - \frac{a^2}{2a_{,}^2} b_{3,1} - \frac{a}{4a_{,}^2} b_{3,2} \qquad R_9 = \frac{a}{8a_{,}^2} - \frac{a}{16a_{,}^2} b_{3,0} - \frac{a}{16a_{,}^2} b_{3,2}$$

$$R_4 = \frac{a}{2a_{,}^2} - \frac{a}{4a_{,}^2} b_{3,0} - \frac{a}{2a_{,}^2} b_{3,1} + \frac{3a}{4a_{,}^2} b_{3,2} \qquad R'_1 = -\frac{a}{2a_{,}^2} + \frac{a}{4a_{,}^2} b_{3,0} - \frac{a}{2a_{,}^2} b_{3,2}$$

$$R_1 = \frac{a}{a_{,}^2} - \frac{b_{1,2}}{a_{,}}\frac{R_3}{2} - \frac{R_9}{2} + \frac{9}{16} R_1 - \frac{3}{4} R_4 - \frac{R'_1}{2} = -\frac{3a}{8a_{,}^2} + \frac{3a}{16a_{,}^2} b_{3,0} + \frac{a^2}{8a_{,}^3} b_{3,1} - \frac{a}{8a_{,}^2} b_{3,2}$$

$$R_9 + \frac{3}{8} R_1 - \frac{5}{4} R_4 - R'_1 = \frac{3a}{8a_{,}^2} - \frac{3a}{16a_{,}^2} b_{3,0} + \frac{5a^2}{8a_{,}^3} b_{3,1} - \frac{5a}{16a_{,}^2} b_{3,2}$$

$$\begin{aligned}
3 R_3 = & -\frac{3a}{8a_{,}^2} + \frac{3a}{16a_{,}^2} b_{3,0} + \frac{a^2}{4a_{,}^2} b_{3,1} - \frac{a}{8a_{,}^2} b_{3,2} - \frac{9a^2}{16a_{,}^3}\left(\frac{a}{a_{,}} b_{5,0} - b_{5,1}\right) \\
& - \frac{3}{8}\left\{\frac{a^3}{a_{,}^4}\left(\frac{a}{a_{,}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) - \frac{a^2}{a_{,}^3}\left(\frac{a}{a_{,}} b_{5,2} - \frac{1}{2} b_{5,1} - \frac{1}{2} b_{5,3}\right)\right\} \\
& + \frac{3a}{8a_{,}^2} - \frac{3a}{16a_{,}^2} b_{3,0} + \frac{5a^2}{8a_{,}^3} b_{3,1} - \frac{5a}{16a_{,}^2} b_{3,2} \\
= & \frac{7a^2}{8a_{,}^3} b_{3,1} - \frac{7a}{16a_{,}^2} b_{3,2} - \frac{9a^2}{16a_{,}^3}\left(\frac{a}{a_{,}} b_{5,0} - b_{5,1}\right) \\
& - \frac{3}{8}\left\{\frac{a^2}{a_{,}^3}\left(\frac{a^2 + a_{,}^2}{a_{,}^2} b_{5,1} - \frac{a}{a_{,}} b_{5,0} - \frac{a}{a_{,}} b_{5,2}\right) - \frac{a^2}{2a_{,}^3} b_{5,1} + \frac{a^2}{2a_{,}^3} b_{5,3} + \frac{a^3}{2a_{,}^4} b_{5,0} - \frac{a^3}{2a_{,}^4} b_{5,2}\right\} \\
& - \frac{3a^2}{8a_{,}^3} b_{3,1} - \frac{3a}{16a_{,}^2} b_{3,2} - \frac{9a^3}{16a_{,}^4} b_{5,0} + \frac{9a^2}{16a_{,}^3} b_{5,1}
\end{aligned}$$

$$R'_3 = -\frac{a}{16a_{,}^2} b_{3,2} + \frac{3a^2}{16a_{,}^3} b_{5,1} - \frac{3a^3}{16a_{,}^4} b_{5,2}.$$

If R'_{15} denote that part of the coefficient of $\cos(\tau - \xi + \xi_{,})$ which is multiplied by $e_{,} e^3$,

$$\begin{aligned}
R'_{15} = & -\frac{a_{,}\,\mathrm{d} R'_3}{2\,\mathrm{d} a_{,}} - R'_3 \\
= & -\frac{a}{16a_{,}^2} b_{3,2} + \frac{9a^2}{32a_{,}^3} b_{5,1} - \frac{3a^3}{8a_{,}^4} b_{5,2} + \frac{3a^2}{32a_{,}^3}\left(\frac{a}{a_{,}} b_{5,2} - \frac{1}{2} b_{5,1} - \frac{1}{2} b_{5,3}\right) \\
& - \frac{15}{32}\left\{\frac{a^3}{a_{,}^4}\left(\frac{a}{a_{,}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right) - \frac{a^4}{a_{,}^5}\left(\frac{a}{a_{,}} b_{7,2} - \frac{1}{2} b_{7,1} - \frac{1}{2} b_{7,3}\right)\right\} \\
& + \frac{a}{16a_{,}^2} b_{3,2} - \frac{3a^2}{16a_{,}^3} b_{5,1} + \frac{3a^3}{16a_{,}^4} b_{5,2}
\end{aligned}$$

$$= \frac{3a^2}{64a_i^3}b_{5,1} - \frac{3a^2}{32a_i^3}b_{5,2} - \frac{3a^2}{64a_i^3}b_{5,3} - \frac{15}{32}\left\{-\frac{a^3}{a_i^4}\left(\frac{a^2+a_i^2}{a_i^2}b_{7,2} - \frac{a}{a_i}b_{7,1} - \frac{a}{a_i}b_{7,3}\right)\right.$$

$$\left. - \frac{a^3}{2a_i^4}b_{7,0} + \frac{a^3}{2a_i^4}b_{7,2} + \frac{a^4}{2a_i^5}b_{7,1} - \frac{a^4}{2a_i^5}b_{7,3}\right\}$$

$$= \frac{3a^2}{64a_i^3}b_{5,1} - \frac{3a^2}{32a_i^3}b_{5,2} - \frac{3a^2}{64a_i^3}b_{5,3} + \frac{15a^3}{32a_i^4}b_{5,2} + \frac{3a^2}{32a_i^3}b_{5,1} - \frac{3a^3}{16a_i^4}b_{5,2}$$

$$= \frac{15a^2}{32a_i^3}b_{5,1} - \frac{3a}{16a_i^2}b_{5,2}.$$

Hence R contains the term

$$m_i\left\{\frac{15a^2}{32a_i^3}b_{5,1} - \frac{3a}{16a_i^2}b_{5,2}\right\}e_i e^3 \cos(\tau - \xi + \xi_i).$$

Calculation of the Coefficient of $e\,e_i^3 \cos(\tau - \xi + \xi_i)$.

If R_{19} denote that part of the coefficient of $\cos(\tau + 2\xi_i)$ which is multiplied by e_i^2,

R_1 . . . . $\cos\tau$ . . . . e_i^0,

R'_1 . . . . e_i^2,

R_6 . . . . $\cos(\tau - \xi_i)$ . . . . e_i,

R_7 . . . . $\cos(\tau + \xi_i)$ . . . . e_i,

R'_7 . . . . e_i^3,

$$3R'_7 = \frac{a\,\mathrm{d}R_7}{2\,\mathrm{d}a_i} - \frac{a_i\mathrm{d}R_{19}}{2\,\mathrm{d}a_i} + R_{19} + \frac{9}{16}\frac{a\,\mathrm{d}R_1}{\mathrm{d}a_i} + \frac{3}{8}R_1 - \frac{3}{4}\frac{a\,\mathrm{d}R_6}{\mathrm{d}a_i} - \frac{5}{4}R_6 - \frac{a_i\mathrm{d}R_1}{2\,\mathrm{d}a_i} - R'_1$$

$$R_7 = \frac{3a}{4a_i^2}b_{3,0} - \frac{1}{2a_i}b_{3,1} - \frac{a}{4a_i^2}b_{3,2} \qquad R_{19} = \frac{a}{8a_i^2} - \frac{a}{16a_i^2}b_{3,0} - \frac{a}{16a_i^2}b_{3,2}$$

$$R_6 = \frac{2a}{a_i^2} - \frac{a}{4a_i^2}b_{3,0} - \frac{1}{2a_i}b_{3,1} + \frac{3a}{4a_i^2}b_{3,2} \qquad R'_1 = -\frac{a}{2a_i^2} + \frac{a}{4a_i^2}b_{3,0} - \frac{a}{2a_i^2}b_{3,2}$$

$$R_1 = \frac{a}{a_i^2} - \frac{1}{a_i}b_{1,1}$$

$$\frac{R_7}{2} - \frac{R_{19}}{2} + \frac{9}{16}R_1 - \frac{3}{4}R_6 - \frac{R'}{2} = -\frac{3a}{4a_i^2} + \frac{3a}{16a_i^2}b_{3,0} + \frac{1}{8a_i}b_{3,1} - \frac{a}{8a_i^2}b_{3,2}$$

$$R_{19} + \frac{3}{8}R_1 - \frac{5}{4}R_6 - R'_1 = -\frac{3a}{2a_i^2} - \frac{3a}{16a_i^2}b_{3,0} + \frac{5}{8a_i}b_{3,1} - \frac{5a}{16a_i^2}b_{3,2}$$

$$3R'_7 = \frac{3a}{2a_i^2} - \frac{3a}{8a_i^2}b_{3,0} - \frac{1}{8a_i}b_{3,1} + \frac{a}{4a_i^2}b_{3,2} + \frac{9a^2}{16a_i^3}\left(\frac{a}{a_i}b_{5,0} - b_{5,1}\right)$$

$$+ \frac{3}{8}\left\{\frac{a}{a_i^2}\left(\frac{a}{a_i}b_{5,1} - \frac{1}{2}b_{5,0} - \frac{1}{2}b_{5,2}\right) - \frac{a^2}{a_i^3}\left(\frac{a}{a_i}b_{5,2} - \frac{1}{2}b_{5,1} - \frac{1}{2}b_{5,3}\right)\right\}$$

$$- \frac{3a}{2a_i^2} - \frac{3a}{16a_i^2}b_{3,0} + \frac{5}{8a_i}b_{3,1} - \frac{5a}{16a_i^2}b_{3,2}$$

$$= -\frac{9a}{16a_i^2}b_{3,0} + \frac{1}{2a_i}b_{3,1} - \frac{a}{16a_i^2}b_{3,2} + \frac{3}{8}\left\{\frac{a}{a_i^2}\left(\frac{a^2+a_i^2}{a_i^2}b_{5,2} - \frac{a}{a_i}b_{5,1} - \frac{a}{a_i}b_{5,3}\right)\right.$$

$$\left. + \frac{a^2}{2a_i^3}b_{5,1} - \frac{a^2}{2a_i^3}b_{5,3} - \frac{a}{2a_i^2}b_{5,0} + \frac{a}{2a_i^2}b_{5,2}\right\} + \frac{9a^3}{16a_i^4}b_{5,0} - \frac{9a^2}{16a_i^3}b_{5,1}$$

$$= -\frac{9a}{16a_{,}^2}b_{3,0} + \frac{1}{2a_{,}}b_{3,1} + \frac{a}{16a_{,}^2}b_{3,2} - \frac{3a}{8a_{,}^2}b_{3,2} + \frac{a}{4a_{,}^2}b_{3,2}$$

$$- \frac{1}{8a_{,}}b_{3,1} + \frac{9a^3}{16a_{,}^4}b_{5,0} - \frac{9a^2}{16a_{,}^3}b_{5,1}$$

$$= -\frac{3}{8a_{,}}b_{3,1} - \frac{3a^2}{8a_{,}^3}b_{3,1} + \frac{3a}{16a_{,}^2}b_{3,2} - \frac{3a}{16a_{,}^2}b_{3,2} + \frac{3}{8a_{,}}b_{3,1} + \frac{9a^3}{16a_{,}^4}b_{5,0} - \frac{9a^2}{16a_{,}^3}b_{5,1}$$

$$= -\frac{9a^2}{16a_{,}^3}b_{5,1} + \frac{9a^3}{16a_{,}^4}b_{5,2}.$$

$$R_7 = -\frac{3a^2}{16a_{,}^3}b_{5,1} + \frac{3a^3}{16a_{,}^4}b_{5,2}$$

If R'_{15} now denote that part of the coefficient of $\cos(\tau - \xi + \xi_{,})$ which is multiplied by $e\,e_{,}^3$,

$$R'_{15} = -\frac{a\,\mathrm{d}R_7}{2\,\mathrm{d}a} - R_7$$

$$= \frac{3a^2}{16a_{,}^3}b_{5,1} - \frac{9a^3}{32a_{,}^4}b_{5,2} + \frac{15a^3}{32a_{,}^4}b_{5,2} + \frac{3a_{,}^2}{32a_{,}^3}b_{5,1}$$

$$- \frac{3a^3}{16a_{,}^4}b_{5,2} + \frac{3a^2}{16a_{,}^3}b_{5,1} - \frac{3a^3}{16a_{,}^4}b_{5,2}$$

$$= \frac{15a^2}{32a_{,}^3}b_{5,1} - \frac{3a^3}{16a_{,}^4}b_{5,2}$$

Hence R contains the term

$$m_{,}\left\{\frac{15a^2}{32a_{,}^3}b_{5,1} - \frac{3a^3}{16a_{,}^4}b_{5,2}\right\}e\,e_{,}^3\cos(\tau - \xi + \xi_{,}).$$

I have found that the disturbing function contains the term

$$-\frac{9m_{,}a^2}{64a_{,}^3}b_{5,2}\,e^2\,e_{,}^2\cos(2\tau - 2\xi + 2\xi_{,}).$$

As I have given elsewhere the details of the calculation of this term, it is unnecessary to repeat them here.

In order to obtain the terms depending partly upon γ^2, $\gamma_{,}^2$, &c., from the same equation, the following transformations are necessary:

$$\cos\lambda' = \cos(\lambda' - \nu + \nu) = \cos(\lambda' - \nu)\cos\nu - \sin(\lambda' - \nu)\sin\nu$$

$$\tan(\lambda' - \nu) = \cos\iota\tan(\lambda - \epsilon)$$

$$\cos(\lambda' - \nu) = \frac{1}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}} \qquad \sin(\lambda' - \nu) = \frac{\cos\iota\tan(\lambda - \epsilon)}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}}$$

$$\cos\lambda' = \frac{\cos\nu - \cos\iota\tan(\lambda - \epsilon)\sin\nu}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}} = \frac{\cos^2\frac{\iota}{2}\cos(\lambda - \epsilon + \nu) + \sin^2\frac{\iota}{2}\cos(\lambda - \epsilon - \nu)}{\sqrt{1 - \sin^2\iota\sin^2(\lambda - \epsilon)}}$$

$$\sin\lambda' = \frac{\cos\iota\tan(\lambda - \epsilon)\cos\nu - \sin\nu}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}} = \frac{\cos^2\frac{\iota}{2}\sin(\lambda - \epsilon + \nu) - \sin^2\frac{\iota}{2}\sin(\lambda - \epsilon - \nu)}{\sqrt{1 - \sin^2\iota\sin^2(\lambda - \epsilon)}}$$

$$r' = r\sqrt{1 - \sin^2\iota\sin^2(\lambda - \epsilon)}$$

$$
\begin{aligned}
r' r'_{,} \{\cos(\lambda' - \lambda'_{,}) + s\, s_{,}\} = r\, r_{,} \Big\{ & \cos^2 \tfrac{\iota}{2} \cos^2 \tfrac{\iota_{,}}{2} \cos(\lambda - \lambda_{,} - \beta + \beta_{,} + \nu - \nu_{,}) \\
& + \sin^2 \tfrac{\iota}{2} \cos \tfrac{\iota_{,}}{2} \cos(\lambda + \lambda_{,} - \beta - \beta_{,} - \nu + \nu_{,}) \\
& + \cos^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\lambda + \lambda_{,} - \beta + \nu - \nu_{,}) \\
& + \sin^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\lambda - \lambda_{,} - \beta + \beta_{,} - \nu + \nu_{,}) \\
& + \frac{\tan\iota \tan\iota_{,}}{2} \cos(\lambda - \lambda_{,} - \beta + \beta_{,}) \\
& - \frac{\tan\iota \tan\iota_{,}}{2} \cos(\lambda + \lambda_{,} - \beta + \beta_{,}) \Big\}
\end{aligned}
$$

If $\tan\iota = \gamma$, $\quad \cos^2 \frac{\iota}{2} = 1 - \frac{\gamma^2}{4} + \frac{3}{16}\gamma^4 \quad \sin^2 \frac{\iota}{2} = \frac{\gamma^2}{4} - \frac{3}{16}\gamma^4$.

If $n t - n_{,} t + \varepsilon - \varepsilon_{,} - \beta + \beta_{,} + \nu - \nu_{,}$ be called τ, and if $n t + \varepsilon - \beta = \eta \qquad n_{,} t + \varepsilon_{,} - \beta_{,} = \eta_{,}$ since when the eccentricities are neglected $\lambda = n t + \varepsilon$, $\quad \lambda_{,} = n_{,} t + \varepsilon_{,}$, $\quad r = a$, $\quad r_{,} = a_{,}$

$$
\begin{aligned}
r' r'_{,} \{\cos(\lambda' - \lambda'_{,}) + s s_{,}\} = a\, a_{,} \Big\{ & \cos^2 \tfrac{\iota}{2} \cos^2 \tfrac{\iota_{,}}{2} \cos\tau + \sin^2 \tfrac{\iota}{2} \cos^2 \tfrac{\iota_{,}}{2} \cos(\tau - 2\eta) \\
& + \cos^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\tau + 2\eta_{,}) + \sin^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\tau - 2\eta + 2\eta_{,}) \\
& + \frac{\gamma\gamma_{,}}{2} \cos(\eta - \eta_{,}) - \frac{\gamma\gamma_{,}}{2} \cos(\eta + \eta_{,}) \\
= a\, a_{,} \Big\{ & \left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4} + \frac{3}{16}\gamma^4 + \frac{\gamma^2\gamma_{,}^2}{16} + \frac{3}{16}\gamma_{,}^4\right) \cos\tau \\
& + \frac{\gamma^2}{4}\left(1 - \frac{\gamma_{,}^2}{4}\right) \cos(\tau - 2\eta) + \frac{\gamma_{,}^2}{4}\left(1 - \frac{\gamma^2}{4}\right) \cos(\tau + 2\eta_{,}) \\
& + \frac{\gamma^2\gamma_{,}^2}{16} \cos(\tau - 2\eta + 2\eta_{,}) + \frac{\gamma\gamma_{,}}{2}\left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4}\right) \cos(\eta - \eta_{,}) \\
& - \frac{\gamma\gamma_{,}}{2}\left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4}\right) \cos(\eta + \eta_{,}) \Big\}.
\end{aligned}
$$

In order to have the terms required depending upon the squares of the inclina tions, it is sufficient to take

$$
\begin{aligned}
R = & -\frac{m_{,}}{a_{,}} \left\{ \frac{1}{2} b_{1,0} + b_{1,1} \cos\tau + b_{1,2} \cos 2\tau + \&c. \right\} \\
& + \frac{m_{,} a}{2 a_{,}^2} \left\{ \frac{1}{2} b_{3,0} + b_{3,1} \cos\tau + b_{3,2} \cos 2\tau + \&c. \right\} \\
& \left\{ \left(\frac{\gamma^2 + \gamma_{,}^2}{2}\right) \cos\tau - \frac{\gamma^2}{2} \cos(\tau - 2\eta) - \frac{\gamma_{,}^2}{2} \cos(\tau + 2\eta_{,}) \right. \\
& \left. - \gamma\gamma_{,} \cos(\eta - \eta_{,}) + \gamma\gamma_{,} \cos(\eta + \eta_{,}) \right\}
\end{aligned}
$$

$$= -\frac{m_{\prime}}{a}\left\{\frac{1}{2}b_{1,0} + b_{1,1}\cos\tau + b_{1,2}\cos 2\tau + \&c.\right\}$$

$$+ \frac{m_{\prime}a}{8a_{\prime}^2}(\gamma^2+\gamma_{\prime}^2)b_{3,1} + \frac{m_{\prime}a}{8a_{\prime}^2}(b_{3,0}+b_{3,2})(\gamma^2+\gamma_{\prime}^2)\cos\tau$$

$$- \frac{m_{\prime}a}{8a_{\prime}^2}b_{3,0}\gamma^2\cos(\tau-2\eta) - \frac{m_{\prime}a}{8a_{\prime}^2}b_{3,0}\gamma_{\prime}^2\cos(\tau+2\eta_{\prime})$$

$$- \frac{m_{\prime}a}{4a_{\prime}^2}b_{3,0}\gamma\gamma_{\prime}\cos(\eta-\eta_{\prime}) + \frac{m_{\prime}a}{4a_{\prime}^2}b_{3,0}\cos(\eta+\eta_{\prime}) - \frac{m_{\prime}a}{8a_{\prime}^2}b_{3,1}\gamma_{\prime}^2\cos(2\tau-2\eta)$$

$$- \frac{m_{\prime}a}{8a_{\prime}^2}b_{3,1}\gamma^2\cos 2\eta - \frac{m_{\prime}a}{8a_{\prime}^2}b_{3,1}\gamma_{\prime}^2\cos(2\tau+2\eta_{\prime}) - \frac{m_{\prime}a}{8a_{\prime}^2}b_{3,1}\gamma_{\prime}^2\cos 2\eta_{\prime}$$

$$- \frac{m_{\prime}a}{4a_{\prime}^2}b_{3,1}\gamma\gamma_{\prime}\cos(\tau-\eta+\eta_{\prime}) + \frac{m_{\prime}a}{4a_{\prime}^2}b_{3,1}\gamma\gamma_{\prime}\cos(\tau+\eta+\eta_{\prime})$$

$$+ \frac{m_{\prime}a}{4a_{\prime}^2}b_{3,1}\gamma\gamma_{\prime}\cos(\tau-\eta-\eta_{\prime}) - \frac{m_{\prime}a}{4a_{\prime}^2}b_{3,2}\gamma\gamma_{\prime}\cos(2\tau-\eta+\eta_{\prime}) + \&c.$$

As before, $\frac{dR}{de} = \frac{dR}{dr}\frac{dr}{de} + \frac{dR}{d\lambda}\frac{d\lambda}{de}$,

but in this form of development

$$\frac{dR}{d\lambda} = \frac{dR}{d\tau} + \frac{dR}{d\eta} \qquad \frac{dR}{d\lambda_{\prime}} = -\frac{dR}{d\tau} + \frac{dR}{d\eta_{\prime}}.$$

Calculation of the Term in R multiplied by $e^2\gamma^2$.

$$R_2 = -\frac{a\,dR_0}{da} \qquad R_0 = \frac{a}{8a_{\prime}^2}b_{3,1}$$

$$R_2 = -\frac{a}{8a_{\prime}^2}b_{3,1} + \frac{3a^2}{8a_{\prime}^3}\left(\frac{a}{a_{\prime}}b_{5,1} - \frac{1}{2}b_{5,0} - \frac{1}{2}b_{5,2}\right)$$

$$= -\frac{3a^2}{16a_{\prime}^3}b_{5,0} + \frac{3a^2}{16a_{\prime}^3}b_{5,2} + \frac{3a^3}{8a_{\prime}^4}b_{5,1} - \frac{3a^2}{16a_{\prime}^3}b_{5,0} - \frac{3a^2}{16a_{\prime}^3}b_{5,2}$$

$$= -\frac{3a^2}{8a_{\prime}^3}b_{5,0} + \frac{3a^3}{8a_{\prime}^4}b_{5,1}.$$

If the term in R_0 multiplied by $e^2\gamma^2$ be called R''_0,

$$2R''_0 = \frac{a\,dR_0}{2da} + \frac{a\,dR_2}{2da}$$

$$= \frac{3a^2}{16a_{\prime}^3}b_{5,0} - \frac{3a^3}{16a_{\prime}^4}b_{5,1} + \frac{3a^2}{8a_{\prime}^3}b_{5,0} - \frac{9a^3}{16a_{\prime}^4}b_{5,1}$$

$$- \frac{15}{16}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}}b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}}b_{7,1} - \frac{1}{2}b_{7,0} - \frac{1}{2}b_{7,2}\right)\right\}$$

$$= \frac{9a^2}{16a_{\prime}^3}b_{5,0} - \frac{3a^3}{4a_{\prime}^4}b_{5,1} - \frac{15}{16}\left\{-\frac{a^3}{a_{\prime}^4}\left(\frac{a^2+a_{\prime}^2}{a^2}b_{7,1} - \frac{a}{a_{\prime}}b_{7,0} - \frac{a}{a_{\prime}}b_{7,2}\right) + \frac{a^4}{2a_{\prime}^5}b_{7,0} - \frac{a^4}{2a_{\prime}^5}b_{7,2}\right\}$$

$$= \frac{9a^2}{16a_{\prime}^3}b_{5,0} - \frac{3a^3}{4a_{\prime}^4}b_{5,1} + \frac{15a^3}{16a_{\prime}^4}b_{5,1} - \frac{3a^3}{16a_{\prime}^4}b_{5,1}$$

$$= \frac{9a^2}{16a_{\prime}^3}b_{5,0}.$$

Hence R contains the term $\frac{9 m_{,} a^2}{32 a_{,}^3} b_{5,0} e^2 (\gamma^2 + \gamma_{,}^2)$. Putting for $b_{5,0}$ its value in series according to powers of $\frac{a}{a_{,}}$, neglecting $\gamma_{,}^2$, I find for the lunar theory $\frac{9 m_{,} a^2}{16 a_{,}^3} e^2 \gamma^2$, which agrees with the result I have arrived at elsewhere by other methods.

Calculation of the Term in R_0 multiplied by $e_{,}^2 \gamma^2$.

$$R_5 = -\frac{a_{,} \mathrm{d} R_0}{\mathrm{d} a_{,}} \qquad\qquad R_0 = \frac{a}{8 a_{,}^2} b_{3,1}$$

$$R_5 = \frac{a}{4 a_{,}^2} b_{3,1} - \frac{3 a^2}{8 a_{,}^3}\left(\frac{a}{a_{,}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$= \frac{a}{4 a_{,}^2} b_{3,1} - \frac{a}{8 a_{,}^2} b_{3,1} + \frac{3 a^2}{16 a_{,}^2} b_{5,0} - \frac{3 a^2}{16 a_{,}^3} b_{5,2} - \frac{3 a^3}{8 a_{,}^4} b_{5,1} + \frac{3 a^2}{16 a_{,}^3} b_{5,0} + \frac{3 a^2}{16 a_{,}^3} b_{5,2}$$

$$= \frac{a}{8 a_{,}^2} b_{3,1} + \frac{3 a^2}{8 a_{,}^3} b_{5,0} - \frac{3 a^3}{8 a_{,}^4} b_{5,1}.$$

If R''_0 denote the term in R_0 multiplied $e_{,}^2 \gamma_{,}^2$,

$$2 R''_0 = \frac{a_{,} \mathrm{d} R_0}{2 \mathrm{d} a_{,}} - \frac{a_{,} \mathrm{d} R_5}{2 \mathrm{d} a_{,}}$$

$$= -\frac{a}{16 a_{,}^2} b_{3,1} - \frac{3 a^2}{16 a_{,}^3} b_{5,0} + \frac{3 a^3}{16 a_{,}^4} b_{5,1} + \frac{a}{8 a_{,}^2} b_{3,1} + \frac{9 a^2}{16 a_{,}^3} b_{5,0} - \frac{3 a^3}{4 a_{,}^4} b_{5,1}$$

$$- \frac{3 a^2}{16 a_{,}^3}\left(\frac{a}{a_{,}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{,25}\right) + \frac{15 a^3}{16 a_{,}^4} b_{5,1} - \frac{3 a^3}{16 a_{,}^4} b_{5,1}$$

$$= \frac{9 a^2}{16 a_{,}^3} b_{5,0}.$$

Hence R contains the term $\frac{9 m_{,} a^2}{32 a_{,}^3} b_{5,0} e_{,}^2 (\gamma^2 + \gamma_{,}^2)$, or in the lunar theory $\frac{9 m_{,} a^2}{16 a_{,}^3} e_{,}^2 \gamma^2$.

Calculation of the Term in R_{15} or (R_1) multiplied by γ^2.

$$R_3 = -\frac{a \mathrm{d} R_1}{2 \mathrm{d} a} - R_1 \qquad\qquad R_1 = \frac{a}{8 a_{,}^2}(b_{3,0} + b_{3,2})$$

$$R_3 = -\frac{a}{16 a_{,}^2}(b_{3,0} + b_{3,2} + \frac{3 a^2}{16 a_{,}^3}\left\{\frac{a}{a_{,}} b_{5,0} - b_{5,1} + \frac{a}{a_{,}} b_{5,2} - \frac{1}{2} b_{5,1} - \frac{1}{2} b_{5,3}\right\}$$

$$- \frac{a}{8 a_{,}^2}(b_{3,0} + b_{3,2}$$

$$= -\frac{3 a}{16 a_{,}^2}\left(\frac{a^2 + a_{,}^2}{a_{,}^2} b_{5,0} - 2\frac{a}{a_{,}} b_{5,1}\right) - \frac{a}{16 a_{,}^2} b_{3,2} + \frac{3 a^3}{16 a_{,}^4} b_{5,0} - \frac{3 a^2}{16 a_{,}^3} b_{5,1}$$

$$+ \frac{3 a^3}{16 a_{,}^4} b_{5,2} - \frac{3 a^2}{32 a_{,}^3} b_{5,1} - \frac{3 a^2}{32 a_{,}^3} b_{5,3} - \frac{3 a^2}{32 a_{,}^3} b_{5,1} + \frac{3 a^2}{32 a_{,}^3} b_{5,3}$$

$$= -\frac{a}{16 a_{,}^2} b_{3,2} - \frac{3 a}{16 a_{,}^2} b_{5,0} + \frac{3 a^3}{16 a_{,}^4} b_{5,2}.$$

If the term in R_{15} multiplied by γ^2 be called R''_{15},

$$R''_{15} = -\frac{a_, \mathrm{d} R_3}{2 \mathrm{d} a_,} - R_3$$

$$= -\frac{a}{16 a_,^2} b_{3,2} - \frac{3 a}{16 a_,^2} b_{5,0} + \frac{3 a^3}{8 a_,^4} b_{5,2} + \frac{15}{32}\left\{\frac{a^2}{a_,^3}\left(\frac{a}{a_,} b_{7,0} - b_{7,1}\right)\right.$$

$$\left. - \frac{a^4}{a_,^5}\left(\frac{a}{a_,} b_{7,2} - \frac{1}{2} b_{7,1} - \frac{1}{2} b_{7,3}\right)\right\} + \frac{a}{16 a_,^2} b_{3,2} + \frac{3 a}{16 a_,^2} b_{5,0} - \frac{3 a^3}{16 a_,^4} b_{5,2}$$

$$= \frac{3 a^3}{16 a_,^4} b_{5,2} + \frac{15}{32}\left\{ - \frac{a^2}{a_,^3}\left(\frac{a^2 + a_,^2}{a_,^2} b_{7,1} - \frac{a}{a_,} b_{7,0} - \frac{a}{a_,} b_{7,2}\right)\right.$$

$$\left. - \frac{a^3}{a_,^4}\left(\frac{a^2 + a_,^2}{a_,^2} b_{7,2} - \frac{a}{a_,} b_{7,1} - \frac{a}{a_,} b_{7,3}\right) + \frac{a^4}{2 a_,^5} b_{7,1} - \frac{a^4}{2 a_,^5} b_{7,3}\right)\right\}$$

$$= \frac{3 a^3}{16 a_,^4} b_{5,2} - \frac{15}{32}\frac{a^2}{a_,^3} b_{5,1} - \frac{15 a^3}{32 a_,^4} b_{5,2} + \frac{3 a^3}{16 a_,^4} b_{5,2}$$

$$= -\frac{15 a^2}{32 a_,^3} b_{5,1} - \frac{3 a^3}{32 a_,^2} b_{5,2}.$$

Therefore R contains the term

$$m_,\left\{ - \frac{15 a_,^2}{32 a_,^3} b_{5,1} - \frac{3 a^3}{32 a_,^4} b_{5,2}\right\} e\, e_, (\gamma^2 + \gamma_,^2) \cos(\tau - \xi + \xi_,).$$

Calculation of R_{III}, or the Coefficient of $\cos(\tau + \xi + \xi_, - 2\eta)$, *in the Development of R.*

Distinguishing at present the argument $\tau + \xi_, - 2\eta$ by the index 7, and the argument $\tau - 2\eta$ by the index 1,

$$R_7 = -\frac{a_, \mathrm{d} R_,}{2 \mathrm{d} a_,} - R_1 \qquad\qquad R_1 = -\frac{a}{8 a_,^2} b_{3,0}$$

$$R_7 = -\frac{a}{8 a_,^2} b_{3,0} + \frac{3 a^2}{16 a_,^3}\left(\frac{a}{a_,} b_{5,0} - b_{5,1}\right) + \frac{a}{8 a_,^2} b_{3,0} = \frac{3 a^3}{16 a_,^4} b_{5,0} - \frac{3 a^2}{16 a_,^3} b_{5,1}$$

$$R_{\mathrm{III}} = -\frac{a \mathrm{d} R_7}{2 \mathrm{d} a} - R_7$$

$$= -\frac{9 a^3}{32 a_,^4} b_{5,0} + \frac{2 a^2}{16 a_,^3} b_{5,1} + \frac{15}{32}\left\{\frac{a^4}{a_,^5}\left(\frac{a}{a_,} b_{7,0} - b_{7,1}\right)\right.$$

$$\left. - \frac{a^3}{a_,^4}\left(\frac{a}{a_,} b_{7,1} - \frac{1}{2} b_{0,0} + \frac{1}{2} b_{7,2}\right)\right\} - \frac{3 a^3}{16 a_,^4} b_{5,0} + \frac{3 a^2}{16 a_,^3} b_{5,1}$$

$$= -\frac{15 a^3}{32 a_,^4} b_{5,0} + \frac{15}{32}\left\{\frac{a^3}{a_,^4}\left(\frac{a^2 + a_,^2}{a_,^2} b_{7,0} + 2\frac{a}{a_,} b_{7,1}\right) - \frac{a^3}{2 a_,^4} b_{7,0} + \frac{a^3}{2 a_,^4} b_{7,2}\right\} + \frac{3 a^2}{8 a_,^3} b_{5,1}$$

$$- \frac{15 a^3}{32 a_,^4} b_{5,0} + \frac{15 a^3}{32 a_,^4} b_{5,0} + \frac{3 a^2}{32 a_,^3} b_{5,1} + \frac{3 a^2}{8 a_,^3} b_{5,1} = \frac{9 a_,^2}{32 a_,^3} b_{5,1}.$$

Hence R contains the term $\frac{9 m_, a^2}{32 a_,^3} b_{5,1}\, e\, e_, \gamma^2 \cos(\tau + \xi + \xi_, - 2\eta)$.

Calculation of R_{IV}, or the Coefficient of $\cos(\tau - \xi - \xi_{,} + 2\eta_{,})$, *in the Development of R.*

Distinguishing at present the argument $\tau - \xi + 2\eta_{,}$ by the index 3, and $\tau + 2\eta_{,}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R}{2\,\mathrm{d}\,a} - R_1 \qquad\qquad R_1 = -\frac{a}{8\,a_{,}^2}\,b_{3,0}$$

$$R_3 = \frac{a}{16\,a_{,}^2}\,b_{3,0} - \frac{3\,a^2}{16\,a_{,}^3}\left(\frac{a}{a_{,}}\,b_{5,0} - b_{5,1}\right) + \frac{a}{8\,a_{,}^2}\,b_{3,0}$$

$$= \frac{3\,a}{16\,a_{,}^2}\,b_{3,0} - \frac{3\,a^3}{16\,a_{,}^4}\,b_{5,0} + \frac{3\,a^2}{16\,a_{,}^3}\,b_{5,1}$$

$$R_{\text{IV}} = -\frac{a_{,}\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a_{,}} - R_3$$

$$= \frac{3\,a}{16\,a_{,}^2}\,b_{3,0} - \frac{3\,a^3}{8\,a_{,}^4}\,b_{5,0} + \frac{9\,a^2}{32\,a_{,}^3}\,b_{5,1} - \frac{9\,a^2}{32\,a_{,}^3}\left(\frac{a}{a_{,}}\,b_{5,0} - b_{5,1}\right)$$

$$+ \frac{15\,a^3}{32\,a_{,}^4}\,b_{5,0} - \frac{3\,a^2}{32\,a_{,}^3}\,b_{5,1} - \frac{3\,a}{16\,a_{,}^2}\,b_{3,0} + \frac{3\,a^3}{16\,a_{,}^4}\,b_{5,0} - \frac{3\,a^2}{16\,a_{,}^3}\,b_{5,1}$$

$$= \frac{9\,a^2}{32\,a_{,}^3}\,b_{5,1}.$$

Hence R contains the term $\frac{9\,m_{,}\,a^2}{32\,a_{,}^3}\,b_{5,1}\,e\,e_{,}\,\gamma_{,}^2\cos(\tau - \xi - \xi_{,} + 2\eta_{,})$.

Calculation of R_{V}, or the Coefficient of $\cos(\xi - \xi_{,} - \eta + \eta_{,})$, *in the Development of R.*

Distinguishing the argument $\eta - \eta_{,} - \xi$ by the index 3, and $\eta - \eta_{,}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R_{,}}{2\,\mathrm{d}\,a_{,}} - R_1 \qquad\qquad R_1 = -\frac{a}{4\,a_{,}^2}\,b_{3,0}$$

$$R_{\text{V}} = -\frac{a_{,}\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a_{,}} - R_3 = \frac{9\,a^2}{16\,a_{,}^3}\,b_{5,1}$$

Hence R contains the term $\frac{9\,m_{,}\,a^2}{16\,a_{,}^3}\,b_{5,1}\,e\,e_{,}\,\gamma\,\gamma_{,}\cos(\xi - \xi_{,} - \eta + \eta_{,})$.

Calculation of R_{VI}, or the Coefficient of $\cos(\xi + \xi_{,} - \eta - \eta_{,})$, *in the Development of R.*

Distinguishing the argument $\eta + \eta_{,} - \xi$ by the index 3, and $\eta + \eta_{,}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} - R_1 \qquad\qquad R_1 = \frac{a}{4\,a_{,}^2}\,b_{3,0}$$

$$R_{\text{VI}} = -\frac{a_{,}\mathrm{d}\,R_3}{2\,\mathrm{d}\,a_{,}} - R_3 = -\frac{9\,a^2}{16\,a_{,}^3}\,b_{5,1}.$$

Hence R contains the term $-\frac{9\,m_{,}\,a^2}{16\,a_{,}^3}\,b_{5,1}\cos(\xi + \xi_{,} - \eta - \eta_{,})$.

Calculation of R_{VII}, or the Coefficient of $\cos(2\,\tau + 2\,\xi_{,} - 2\,\eta)$, *in the Development of R.*

Distinguishing the argument $2\,\tau - 2\,\eta + \xi_{,}$ by the index 7, and $2\,\tau - 2\,\eta$ by the index 1,

$$R_7 = -\frac{a_{\prime} \mathrm{d} R_1}{2 \mathrm{d} a_{\prime}} - 2 R_1 \qquad R_1 = -\frac{a}{8 a_{\prime}^2} b_{3,1}$$

$$= -\frac{a}{8 a_{\prime}^2} b_{3,1} + \frac{3 a^2}{16 a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) + \frac{a}{4 a_{\prime}^2} b_{3,1}$$

$$= \frac{a}{8 a_{\prime}^2} b_{3,1} + \frac{3 a^3}{16 a_{\prime}^4} b_{5,1} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,0} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,2}$$

$$= \frac{3 a^2}{16 a_{\prime}^4} b_{5,0} - \frac{3 a^2}{16 a_{\prime}^4} b_{5,2} + \frac{3 a^3}{16 a_{\prime}^4} b_{5,1} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,0} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,2}$$

$$= -\frac{3 a^2}{32 a_{\prime}^3} b_{5,0} + \frac{3 a^3}{16 a_{\prime}^4} b_{5,1} - \frac{9 a^2}{32 a_{\prime}^3} b_{5,2}$$

$$= \frac{3 a^2}{16 a_{\prime}^3} \text{ in the lunar theory.}$$

$$2 R_{\mathrm{VII}} = -\frac{a_{\prime} \mathrm{d} R_7}{2 \mathrm{d} a} - 2 R_7 - \frac{3}{4} \frac{a_{\prime} \mathrm{d} R_1}{\mathrm{d} a_{\prime}} - \frac{5}{2} R_1$$

$$-\frac{R_7}{2} - \frac{3}{4} R_1 = -\frac{3 a^2}{64 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{32 a_{\prime}^4} b_{5,1} + \frac{9 a^2}{64 a_{\prime}^3} b_{5,2} + \frac{9 a^2}{64 a_{\prime}^3} b_{5,0} - \frac{9 a^2}{64 a_{\prime}^3} b_{5,2}$$

$$= \frac{3 a^2}{32 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{32 a_{\prime}^4} b_{5,1}$$

$$-2 R_7 - \frac{5}{2} R_1 = -\frac{3 a^2}{16 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{8 a_{\prime}^4} b_{5,1} + \frac{9 a^2}{16 a_{\prime}^3} b_{5,2} + \frac{15 a^2}{32 a_{\prime}^3} b_{5,0} - \frac{15 a^2}{32 a_{\prime}^3} b_{5,2}$$

$$= \frac{9 a^2}{32 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{16 a_{\prime}^4} b_{5,1} + \frac{3 a^2}{32 a_{\prime}^3} b_{5,2}$$

$$2 R_{\mathrm{VII}} = -\frac{9 a^2}{32 a_{\prime}^3} b_{5,0} + \frac{3 a^3}{8 a_{\prime}^4} b_{5,1} + \frac{15}{32}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}} b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right)\right\}$$

$$+ \frac{9 a^2}{32 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{8 a_{\prime}^4} b_{5,1} + \frac{3 a^2}{32 a_{\prime}^3} b_{5,2}$$

$$= \frac{3 a^2}{32 a_{\prime}^3} b_{5,2} + \frac{15}{32}\left\{-\frac{a^3}{a_{\prime}^4}\left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{7,1} - \frac{a}{a_{\prime}} b_{7,0} - \frac{a}{a_{\prime}} b_{7,2}\right) + \frac{a^4}{2 a_{\prime}^5} b_{7,0} - \frac{a^4}{2 a_{\prime}^5} b_{7,2}\right\}$$

$$= \frac{3 a^2}{32 a_{\prime}^3} b_{5,2} - \frac{15 a^3}{32 a_{\prime}^4} b_{5,1} + \frac{3 a^3}{32 a_{\prime}^4} b_{5,1}$$

$$= -\frac{3 a^3}{8 a_{\prime}^4} b_{5,1} + \frac{3 a^2}{32 a_{\prime}^3} b_{5,2}.$$

Hence R contains the term $m_{\prime}\left\{-\frac{3 a^3}{16 a_{\prime}^4} b_{5,1} + \frac{3 a^2}{64 a_{\prime}^3} b_{5,2}\right\} e_{\prime}^2 \gamma^2 \cos(2\tau + 2\xi_{\prime} - 2\eta)$.

Calculation of R_{VII}, *or the Coefficient of* $\cos(2\xi - 2\eta)$, *in the Development of* R.

Distinguishing the argument $\xi - 2\eta$ by the index 65, and the argument 2η by the index 62,

$$R_{65} = -\frac{a \mathrm{d} R_{62}}{2 \mathrm{d} a} - 2 R_{62} \qquad R_{62} = -\frac{a}{8 a_{\prime}^2} b_{3,1}$$

$$R_{65} = \frac{a}{16\,a_{\prime}^2}\,b_{3,1} - \frac{3\,a^2}{16\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{a}{4\,a_{\prime}^2}\,b_{3,1}$$

$$= \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2} - \frac{3\,a^3}{16\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2} + \frac{a}{4\,a_{\prime}^2}\,b_{3,1}$$

$$= \frac{a}{4\,a_{\prime}^2}\,b_{3,1} + \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^3}{16\,a_{\prime}^4}\,b_{5,1}$$

$$= \frac{9\,a^2}{8\,a_{\prime}^3} \text{ in the lunar theory.}$$

$$2\,R_{\text{VIII}} = -\frac{a\,\mathrm{d}\,R_{65}}{2\,\mathrm{d}\,a} - 2\,R_{65} - \frac{3\,a\,\mathrm{d}\,R_{62}}{4\,\mathrm{d}\,a} - \frac{5}{2}\,R_{62}$$

$$= -\frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0} + \frac{9\,a^3}{32\,a_{\prime}^4}\,b_{5,1} + \frac{9\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{9\,a^3}{32\,a_{\prime}^4}\,b_{5,1}$$

$$- \frac{a}{8\,a_{\prime}^2}\,b_{3,1} + \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$+ \frac{15}{32}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}}\,b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)\right\}$$

$$- \frac{3\,a^2}{8\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{a}{2\,a_{\prime}^2}\,b_{3,1} + \frac{5\,a}{16\,a_{\prime}^2}\,b_{3,1}$$

$$= -\frac{15\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{15\,a^2}{32\,a_{\prime}^3}\,b_{5,0} + \frac{15\,a^2}{32\,a_{\prime}^3}\,b_{5,2} + \frac{3\,a^3}{4\,a_{\prime}^4}\,b_{5,1}$$

$$+ \frac{15}{32}\left\{-\frac{a^3}{a_{\prime}^4}\left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2}\,b_{7,1} - \frac{a}{a_{\prime}}\,b_{7,0} - \frac{a}{a_{\prime}}\,b_{7,2}\right) + \frac{a^4}{2\,a_{\prime}^5}\,b_{7,0} - \frac{a^4}{2\,a_{\prime}^5}\,b_{7,2}\right\}$$

$$= -\frac{15\,a^2}{16\,a_{\prime}^3}\,b_{5,0} + \frac{9\,a^2}{32\,a_{\prime}^3}\,b_{5,2} + \frac{3\,a^3}{4\,a_{\prime}^4}\,b_{5,1} - \frac{15\,a^3}{32\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^3}{32\,a_{\prime}^4}\,b_{5,1}$$

$$= -\frac{15\,a^2}{16\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} + \frac{9\,a^2}{32\,a_{\prime}^3}\,b_{5,2} = -\frac{3\,a}{8\,a_{\prime}^2}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}$$

$$= -\,{*}\,\frac{15\,a^2}{8\,a_{\prime}^3} \text{ in the lunar theory.}$$

Hence R contains the term $\left\{-\frac{3\,a}{16\,a_{\prime}^2}\,b_{5,1} + \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,2}\right\}e^2\,\gamma^2 \cos\,(2\,\xi - 2\,\eta)$.

Calculation of R_{IX}, or the Coefficient of $\cos\,(2\,\tau - 2\,\xi + 2\,\eta_{\prime})$, *in the Development of R.*

Distinguishing the argument $2\,\tau - \xi + 2\,\eta_{\prime}$ by the index 3, and $2\,\tau + 2\,\eta_{\prime}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}_2} - R_1 \qquad R_1 = -\frac{a}{8\,a_{\prime 2}}\,b_{3,1}$$

$$R_3 = \frac{a}{16\,a_{\prime}^2}\,b_{3,1} - \frac{3\,a^2}{16\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{a}{4\,a_{\prime}^2}\,b_{3,1}$$

* R_{VIII} (or R_{77}) $= -\frac{15\,a^2}{16\,a_{\prime}^3}$, this agrees with the result I arrived at formerly, since confirmed by M. POISSON.

$$= \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2} - \frac{3\,a^3}{16\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2} + \frac{a}{4\,a_{\prime}^2}\,b_{3,1}$$

$$= \frac{a}{4\,a_{\prime}^2}\,b_{3,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}$$

$$2\,R_{\text{IX}} = -\frac{a\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a} - 2\,R_3 - \frac{3}{4}\,\frac{a\,\mathrm{d}\,R_1}{\mathrm{d}\,a} - \frac{5}{2}\,R_1$$

$$-\frac{1}{2}\,R_3 - \frac{3}{4}\,R_1 = -\frac{a}{3\,a_{\prime}^2}\,b_{3,1} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{32\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a}{32\,a_{\prime}^2}\,b_{3,1}$$

$$= -\frac{a}{32\,a_{\prime}^2}\,b_{3,1} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{32\,a_{\prime}^4}\,b_{5,1}$$

$$-2\,R_3 - \frac{5}{2}\,R_1 = -\frac{3\,a^2}{8\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{a}{8\,a_{\prime}^2}\,b_{3,1} + \frac{5\,a}{16\,a_{\prime}^2}\,b_{3,1}$$

$$= -\frac{3\,a^2}{8\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a}{16\,a_{\prime}^2}\,b_{3,1}$$

$$2\,R_{\text{IX}} = -\frac{a}{32\,a_{\prime}^2}\,b_{3,1} + \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,3} + \frac{9\,a^3}{32\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$+\frac{15}{32}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}}\,b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)\right\}$$

$$-\frac{3\,a^2}{8\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a}{16\,a_{\prime}^2}\,b_{3,1}$$

$$= -\frac{7\,a}{32\,a_{\prime}^2}\,b_{3,1} - \frac{39\,a^2}{64\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{4\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a}{64\,a_{\prime}^2}\,b_{5,2}$$

$$+\frac{15}{32}\left\{-\frac{a^3}{a_{\prime}^4}\left(\frac{a^2+a_{\prime}^2}{a_{\prime}^2}\,b_{7,1} - \frac{a}{a_{\prime}}\,b_{7,0} - \frac{a}{a_{\prime}}\,b_{7,2}\right) + \frac{a^4}{2\,a_{\prime}^5}\,b_{7,0} - \frac{a^4}{2\,a_{\prime}^5}\,b_{7,2}\right\}$$

$$= -\frac{21\,a^2}{64\,a_{\prime}^3}\,b_{5,0} + \frac{21\,a^2}{64\,a_{\prime}^3}\,b_{5,2} - \frac{39\,a^2}{64\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{4\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a}{64\,a_{\prime}^2}\,b_{5,2} - \frac{15\,a^3}{32\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^3}{32\,a_{\prime}^4}\,b_{5,1}$$

$$= -\frac{3\,a}{8\,a_{\prime}^2}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}.$$

Hence R contains the term $\left\{-\frac{3\,a}{16\,a_{\prime}^2}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}\right\} e^2\,\gamma_{\prime}^2 \cos\left(2\,\tau - 2\,\xi + 2\,\eta_{\prime}\right)$.

Calculation of R_{x}, *or the Coefficient of* $\cos\left(2\,\xi_{\prime} - 2\,\eta_{\prime}\right)$, *in the Development of* R.

Distinguishing the argument $2\,\eta_{\prime} - \xi_{\prime}$ by the index 65, and the argument $2\,\eta_{\prime}$ by the index 62,

$$R_{65} = -\frac{a_{\prime}\mathrm{d}\,R_{62}}{2\,\mathrm{d}\,a_{\prime}} - 2\,R_{62} \qquad\qquad R_{62} = -\frac{a}{8\,a_{\prime}^2}\,b_{3,1}$$

$$R_{65} = -\frac{a}{8\,a_{\prime}^2}\,b_{3,1} + \frac{3\,a^2}{15\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{a}{4\,a_{\prime}^2}\,b_{3,1}$$

$$= \frac{3\,a}{16\,a_{\prime}^2}\,b_{3,1} - \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{16\,a_{\prime}^4}\,b_{5,1}$$

$$2\,R_{x} = -\frac{a_{,}\mathrm{d}R_{65}}{2\,\mathrm{d}\,a_{,}} - 2\,R_{65} - \frac{3\,a_{,}\,\mathrm{d}\,R_{62}}{4\,\mathrm{d}\,a_{,}} - \frac{5}{2}\,R_{62}$$

$$= \frac{3\,a}{16\,a_{,}^{2}}\,b_{3,1} - \frac{9\,a^{2}}{32\,a_{,}^{3}}\,b_{5,0} + \frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} - \frac{9\,a^{2}}{32\,a_{,}^{3}}\left(\frac{a}{a_{,}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$-{}^{*}\,\frac{15\,a^{3}}{32\,a_{,}^{4}}\,b_{5,1} + \frac{3\,a^{3}}{32\,a_{,}^{4}}\,b_{5,1} - \frac{3\,a}{8\,a_{,}^{2}}\,b_{3,1} + \frac{3\,a^{2}}{8\,a_{,}^{3}}\,b_{5,0} - \frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} - \frac{3\,a}{16\,a_{,}^{2}}\,b_{3,1}$$

$$+ \frac{9\,a^{2}}{32\,a_{,}^{3}}\left(\frac{a}{a_{,}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{5\,a}{16\,a_{,}^{2}}\,b_{3,1}$$

$$= \frac{3\,a^{2}}{32\,a_{,}^{3}}\,b_{5,0} - \frac{a}{16\,a_{,}^{2}}\,b_{3,1} - \frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1}$$

$$= -\frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} + \frac{3\,a^{2}}{32\,a_{,}^{3}}\,b_{5,2}.$$

Hence R contains the term $\left\{-\frac{3\,a^{3}}{16\,a_{,}^{4}}\,b_{5\,1} + \frac{3\,a^{2}}{64\,a_{,}^{3}}\,b_{5,2}\right\} e_{,}^{2}\,\gamma_{,}^{2}\cos\left(2\,\xi_{,} - 2\,\eta_{,}\right)$.

Calculation of the Term in R_{XI} multiplied by e^2.

Distinguishing the argument $\tau - \eta + \eta_{,}$ by the index 1, $\tau - \eta + \eta_{,} - \xi$ by the index 3, and $\tau - \eta + \eta_{,} + \xi$ by the index 4,

$$R_{3} = -\frac{a\,\mathrm{d}\,R_{1}}{2\,\mathrm{d}\,a} \qquad R_{4} = -\frac{a\mathrm{d}\,R_{1}}{2\,\mathrm{d}\,a} = R_{3} \qquad R_{1} = -\frac{a}{4\,a_{,}^{2}}\,b_{3,1}$$

$$R_{3} = \frac{a}{8\,a_{,}^{2}}\,b_{3,1} - \frac{3\,a^{2}}{8\,a_{,}^{3}}\left(\frac{a}{a_{,}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$= \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5\,0} - \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5,2} - \frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} + \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5,0} + \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5,2}$$

$$= \frac{3\,a^{2}}{8\,a_{,}^{3}}\,b_{5,0} - \frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1}$$

$$2\,R_{\text{XI}} = \frac{a\,\mathrm{d}\,R_{1}}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R_{3}}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R_{4}}{2\,\mathrm{d}\,a} = -\frac{a\,\mathrm{d}\,R_{1}}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R_{3}}{\mathrm{d}\,a}$$

$$= -\frac{3\,a^{2}}{4\,a_{,}^{3}}\,b_{5,0} + \frac{9\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} + \frac{15}{8}\left\{\frac{a}{a_{,}^{4}}\left(\frac{a}{a_{,}}\,b_{7,0} - b_{7,1}\right) - \frac{a^{4}}{a_{,}^{5}}\left(\frac{a}{a_{,}}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)\right\}$$

$$- \frac{a}{8\,a_{,}^{2}}\,b_{3,1} + \frac{3\,a^{2}}{8\,a_{,}^{3}}\left(\frac{a}{a_{,}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$= -\frac{3\,a^{2}}{4\,a_{,}^{3}}\,b_{5,0} + \frac{9\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} + \frac{15}{8}\left\{-\frac{a^{3}}{a_{,}^{4}}\left(\frac{a^{2}+a_{,}^{2}}{a_{,}^{2}}\,b_{7,1} - \frac{a}{a_{,}}\,b_{7,0} - \frac{a}{a_{,}}\,b_{7,2}\right) + \frac{a^{4}}{2\,a_{,}^{5}}\,b_{7,0} - \frac{a^{4}}{2\,a_{,}^{5}}\,b_{7,2}\right\}$$

$$- \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5\,0} + \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5,2} + \frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} - \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5,0} - \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5,2}$$

$$= -\frac{3\,a^{2}}{4\,a_{,}^{3}}\,b_{5,0} + \frac{9\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} - \frac{15\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} + \frac{3\,a^{3}}{8\,a_{,}^{4}}\,b_{5,1} - \frac{3\,a^{2}}{16\,a_{,}^{3}}\,b_{5,0}$$

* For certain reductions which occur here, see the calculation of R_{VIII}.

$$+ \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2}$$

$$= - \frac{9\,a^2}{8\,a_{\prime}^3} b_{5,0}.$$

Therefore R contains the term $- \frac{9\,a^2}{16\,a_{\prime}^3} b_{5,0}\, e^2 \gamma \gamma_{\prime} \cos (\tau - \eta + \eta_{\prime})$.

Calculation of the Term in R_{XI} multiplied by $e_{\prime}^2$.

Distinguishing the argument $\tau - \eta + \xi_{\prime}$ by the index 1, the argument $\tau - \eta + \eta_{\prime} - \xi_{\prime}$ by the index 6, and $\tau - \eta + \eta_{\prime} + \xi_{\prime}$ by the index 7,

$$R_6 = - \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} \qquad R_7 = - \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} = R_6 \qquad R_1 = - \frac{a}{4\,a_{\prime}^2} b_{3,1}$$

$$R_6 = - \frac{a}{4\,a_{\prime}^2} b_{3,1} + \frac{3\,a^2}{8\,a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2} \right)$$

$$= - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} + \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} - \frac{a}{8\,a_{\prime}^2} b_{3,1}$$

$$= - \frac{a}{8\,a_{\prime}^2} b_{3,1} - \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1}$$

$$2\,R_{\text{XI}} = \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} - \frac{a_{\prime}\,\mathrm{d}\,R_6}{2\,\mathrm{d}\,a_{\prime}} - \frac{a_{\prime}\,\mathrm{d}\,R_7}{2\,\mathrm{d}\,a_{\prime}} = \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} - \frac{a_{\prime}\,\mathrm{d}\,R_6}{\mathrm{d}\,a_{\prime}}$$

$$2\,R_{\text{XI}} = - \frac{9\,a^2}{8\,a_{\prime}^3} b_{5,0} + \frac{3\,a^3}{2\,a_{\prime}^4} b_{5,1} - \frac{a}{4\,a_{\prime}^2} b_{3,1}$$

$$+ \frac{15}{8} \left\{ \frac{a^3}{a_{\prime}^4} \left(\frac{a}{a_{\prime}} b_{7,0} - b_{7,1} \right) - \frac{a^4}{a_{\prime}^5} \left(\frac{a}{a_{\prime}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2} \right) \right\}$$

$$+ \frac{3\,a^2}{8\,a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2} \right) + \frac{a}{4\,a_{\prime}^2} b_{3,1} - \frac{3\,a^2}{8\,a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2} \right)$$

$$= - \frac{9\,a^2}{8\,a_{\prime}^3} b_{5,0} + \frac{3\,a^3}{2\,a_{\prime}^4} b_{5,1} - \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,0} + \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,2} - \frac{15\,a^3}{8\,a_{\prime}^4} b_{5,1} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1}$$

$$- \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} + \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,2} - \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} + \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} + \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2}$$

$$= - \frac{9\,a^2}{8\,a_{\prime}^3} b_{5,0}.$$

Therefore R contains the term $- \frac{9\,a^2}{16\,a_{\prime}^3} b_{5,0}\, e_{\prime}^2 \gamma \gamma_{\prime} \cos (\tau - \eta + \eta_{\prime})$.

Calculation of R_{XII} or the Coefficient of $\cos (\tau - 2\,\xi + \eta + \eta_{\prime})$ *in the Development of R.*

Distinguishing the argument $\tau + \eta + \eta_{\prime} - \xi$ by the index 3, and $\tau + \eta + \eta_{\prime}$ by the index 1,

$$R_3 = - \frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} - 2\,R_1 \qquad\qquad R_1 = \frac{a}{4\,a_{\prime}^2} b_{3,1}$$

$$2\,R_{\text{XII}} = -\frac{a\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a} - 2\,R_3 - \frac{3}{4}\frac{a\,\mathrm{d}\,R_1}{\mathrm{d}\,a} - \frac{5}{2}\,R_1.$$

It is evident from the calculation of R_{VIII} that R contains the term

$$\left\{\frac{3\,a}{8\,a_{,}^2}\,b_{5,1} - \frac{3\,a^2}{32\,a_{,}^3}\,b_{5,2}\right\} e^2\,\gamma\,\gamma_{,}\cos(\tau - 2\,\xi + \eta + \eta_{,}).$$

It is evident similarly, and from the calculation of R_{X}, that R contains the term

$$\left\{\frac{3\,a^3}{8\,a_{,}^4}\,b_{5,1} - \frac{3\,a^2}{32\,a_{,}^3}\,b_{5,2}\right\} e_{,}^2\,\gamma\,\gamma_{,}\cos(\tau + 2\,\xi_{,} - \eta + \eta_{,}).$$

Calculation of R_{XIV} or the Coefficient of $\cos(2\,\tau - \xi + \xi_{,} - \eta + \eta_{,})$.

Distinguishing the argument $2\,\tau - \eta + \eta_{,} + \xi_{,}$ by the index 7, and the argument $2\,\tau - \eta + \eta_{,}$ by the index 1,

$$R_7 = -\frac{a_{,}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{,}} - R_1 \qquad\qquad R_1 = -\frac{a}{4\,a_{,}^2}\,b_{3,2}$$

$$R_7 = \frac{a}{4\,a_{,}^2}\,b_{3,2} + \frac{3\,a^2}{8\,a_{,}^3}\left(\frac{a}{a_{,}}\,b_{5,2} - \frac{1}{2}\,b_{5,1} - \frac{1}{2}\,b_{5,3}\right) + \frac{a}{4\,a_{,}^2}\,b_{3,2}$$

$$= \frac{a}{8\,a_{,}^2}\,b_{3,2} - \frac{3\,a^3}{16\,a_{,}^4}\,b_{5,2} + \frac{3\,a^2}{16\,a_{,}^3}\,b_{5,2}$$

$$R_{\text{XIV}} = -\frac{a\,\mathrm{d}\,R_7}{2\,\mathrm{d}\,a} - R_7$$

$$= -\frac{a}{8\,a_{,}^4}\,b_{3,2} - \frac{9\,a^3}{16\,a_{,}^4}\,b_{5,2} + \frac{3\,a^2}{8\,a_{,}^3}\,b_{5,3} - \frac{3\,a^2}{8\,a_{,}^3}\left(\frac{a}{a_{,}}\,b_{5,2} - \frac{1}{2}\,b_{5,1} - \frac{1}{2}\,b_{5,3}\right)$$

$$+ \frac{15}{16}\left\{\frac{a^4}{a_{,}^5}\left(\frac{a}{a_{,}}\,b_{7,2} - \frac{1}{2}\,b_{7,1} - \frac{1}{2}\,b_{7,3}\right) - \frac{a^3}{a_{,}^4}\left(\frac{a}{a_{,}}\,b_{7,3} - \frac{1}{2}\,b_{7,2} - \frac{1}{2}\,b_{7,4}\right)\right\}$$

$$+ \frac{a}{4\,a_{,}^2}\,b_{3,2} - \frac{3\,a^3}{8\,a_{,}^4}\,b_{5,2} + \frac{3\,a^2}{8\,a_{,}^3}\,b_{3,5}$$

$$= -\frac{21\,a^3}{16\,a_{,}^4}\,b_{5,2} + \frac{15\,a^2}{8\,a_{,}^3}\,b_{5,3} + \frac{3\,a}{8\,a_{,}^2}\,b_{3,2}$$

$$+ \frac{15}{16}\left\{\frac{a^3}{a_{,}^4}\left(\frac{a^2 + a_{,}^2}{a_{,}^2}b_{7,2} - \frac{a}{a_{,}}b_{7,1} - \frac{a}{a_{,}}b_{7,3}\right) - \frac{a^3}{2\,a_{,}^2}b_{7,2} + \frac{a^3}{2\,a'^4}b_{7,4} + \frac{a^4}{2\,a_{,}^5}b_{7,1} - \frac{a^4}{2\,a_{,}^5}b_{7,3}\right\}$$

$$= -\frac{21\,a^3}{16\,a_{,}^4}b_{5,2} + \frac{15\,a^2}{8\,a_{,}^3}b_{5,3} + \frac{9\,a^2}{32\,a_{,}^3}b_{5,1} - \frac{9\,a^2}{32\,a_{,}^3}b_{5,3} + \frac{15\,a^3}{16\,a_{,}^4}b_{5,2} - \frac{9\,a^2}{16\,a_{,}^3}b_{5,3} + \frac{3\,a^3}{8\,a_{,}^4}\,b_{5,2}$$

$$= \frac{9\,a^2}{32\,a_{,}^3}\,b_{5,1} + \frac{33\,a^2}{32\,a_{,}^3}\,b_{5,3}.$$

Therefore R contains the term

$$\left\{\frac{9\,a^2}{32\,a_{,}^3}\,b_{5,1} + \frac{33\,a^2}{32\,a_{,}^3}\,b_{5,3}\right\} e\,e_{,}\,\gamma\,\gamma_{,}\cos(2\,\tau - \xi + \xi_{,} - \eta + \eta_{,})$$

$R =$ *terms independent of the quantities b

$$- \frac{m}{a}\left\{\frac{1}{2} b_{1,0} + b_{1,1}\cos\tau + b_{1,2}\cos 2\tau +, \&c.\right\}$$

$$- \frac{m_{,} a}{a_{,}^2}\left\{\frac{1}{2} b_{3,0} + b_{3,1}\cos\tau + b_{3,2}\cos 2\tau +, \&c.\right\}$$

$$\left\{-\frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4} + \frac{3}{16}\gamma^4 + \frac{\gamma^2\gamma_{,}^2}{16} + \frac{3}{16}\gamma_{,}^4\right\}\cos\tau + \frac{\gamma^2}{4}\left(1 - \frac{\gamma_{,}^2}{4}\right)(\cos\tau - 2\eta)$$

$$+ \frac{\gamma_{,}^2}{4}\left(1 - \frac{\gamma^2}{4}\right)\cos(\tau + 2\eta_{,}) + \frac{\gamma^2\gamma_{,}^2}{16}\cos(\tau - 2\eta + 2\eta_{,})$$

$$+ \frac{\gamma\gamma_{,}}{2}\left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4}\right)\cos(\eta - \eta_{,}) - \frac{\gamma\gamma_{,}}{2}\left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4}\right)\cos(\eta + \eta')\Big\}$$

$$- \frac{3a^2}{8a_{,}^3}\left\{\frac{1}{2} b_{5,0} + b_{5,1}\cos\tau + b_{5,2}\cos 2\tau +, \&c.\right\}$$

$$\left\{\left(\frac{\gamma^2}{2} + \frac{\gamma_{,}^2}{2}\right)\cos\tau - \frac{\gamma^2}{2}\cos(\tau - 2\eta) - \frac{\gamma_{,}^2}{2}\cos(\tau + 2\eta_{,}) - \gamma\gamma_{,}\cos(\eta - \eta_{,}) + \gamma\gamma_{,}\cos(\eta + \eta_{,})\right\}^2$$

$$\left\{\left(\frac{\gamma^2}{2} + \frac{\gamma_{,}^2}{2}\right)\cos\tau - \frac{\gamma^2}{2}\cos(\tau - 2\eta) - \frac{\gamma_{,}^2}{2}\cos(\tau + 2\eta_{,}) - \gamma\gamma_{,}\cos(\eta - \eta_{,}) + \gamma\gamma_{,}\cos(\eta + \eta_{,})\right\}^2$$

$$= \left\{\frac{\gamma^4}{8} + \frac{\gamma^2\gamma_{,}^2}{4} + \frac{\gamma_{,}^4}{8}\right\}\{1 + \cos 2\tau\} \quad + \left\{-\frac{\gamma^4}{4} - \frac{\gamma^2\gamma_{,}^2}{4}\right\}\{\cos(2\tau - 2\eta) + \cos 2\eta\}$$

$$+ \left\{-\frac{\gamma^2\gamma_{,}^2}{4} - \frac{\gamma_{,}^4}{4}\right\}\{\cos(2\tau + 2\eta_{,}) + \cos 2\eta_{,}\}$$

$$+ \left\{-\frac{\gamma^3\gamma_{,}}{2} - \frac{\gamma\gamma_{,}^3}{2}\right\}\{\cos(\tau + \eta - \eta_{,}) + \cos(\tau - \eta + \eta_{,})\}$$

$$+ \gamma\gamma_{,}\left\{\frac{\gamma^2}{2} + \frac{\gamma_{,}^2}{2}\right\}\{\cos(\tau + \eta + \eta_{,}) + \cos(\tau - \eta - \eta_{,})\}$$

$$+ \frac{\gamma^4}{8}\{1 + \cos(2\tau - 2\eta)\} + \frac{\gamma^2\gamma_{,}^2}{4}\{\cos(2\tau - 2\eta + 2\eta_{,}) + \cos(2\eta - 2\eta_{,})\}$$

$$+ \frac{\gamma^3\gamma_{,}}{2}\{\cos(\tau - \eta - \eta_{,}) + \cos(\tau - 3\eta + \eta_{,})\}$$

$$+ \frac{\gamma^3\gamma_{,}}{2}\{\cos(\tau - \eta + \eta_{,}) + \cos(\tau - 3\eta - \eta_{,})\} + \frac{\gamma_{,}^4}{8}\{1 + \cos(2\tau - 2\eta_{,})\}$$

$$+ \frac{\gamma\gamma_{,}^3}{2}\{\cos(\tau - \eta + \eta_{,}) + \cos(\tau - \eta + 3\eta_{,})\} + \frac{\gamma^2\gamma_{,}^2}{2}\{1 + \cos(2\eta - 2\eta_{,})\}$$

$$- \frac{\gamma^2\gamma_{,}^2}{2}\{\cos 2\eta + \cos 2\eta_{,}\} + \frac{\gamma^2\gamma_{,}^2}{2}\{1 + \cos(2\eta + 2\eta_{,})\}.$$

* It is useless to consider these terms, because as R contains no term multiplied by $\frac{a}{a_{,}^2}$, if the other part is found to contain any term multiplied by $\frac{a}{a_{,}^2}$ it must be neglected, that is to say, got rid of by adding a similar term independent of the quantities b, and with a contrary sign.

In order to obtain the term in R depending upon γ^4, γ^2, γ_i^2 and γ_i^4, it is sufficient to take

$$R = \frac{m\,a}{a_i^2}\left\{\frac{1}{2}\,b_{3,0} + b_{3,1}\cos\tau + b_{3,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{-\left(\frac{3}{8}\gamma^4 + \frac{\gamma^2\gamma_i^2}{8} + \frac{3}{8}\gamma_i^4\right)\cos\tau + \gamma\,\gamma_i\,(\gamma^2 + \gamma_i^2)\cos(\eta - \eta_i)\right.$$

$$\left. - \frac{\gamma^2\gamma_i^2}{8}\cos(\tau - 2\,\eta + 2\,\eta_i) +, \&c.\right\}$$

$$- \frac{3\,m_i\,a^2}{8\,a_i^3}\left\{\frac{1}{2}\,b_{5,0} + b_{5,1}\cos\tau + b_{5,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{\left(\frac{\gamma^2}{2} + \frac{\gamma_i^2}{2}\right)\cos\tau - \frac{\gamma^2}{2}\cos(\tau - 2\,\eta) - \frac{\gamma_i^2}{2}\cos(\tau + 2\,\eta_i)\right.$$

$$\left. - \gamma\,\gamma_i\cos(\eta - \eta_i) + \gamma\,\gamma_i\cos(\eta + \eta_i)\right\}^2$$

$$= \frac{m_i\,a}{a_i^2}\left\{\frac{1}{2}\,b_{3,0} + b_{3,1}\cos\tau + b_{3,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{-\left(\frac{3}{8}\gamma^4 + \frac{\gamma^2\gamma_i^2}{8} + \frac{3}{8}\gamma_i^4\right)\cos\tau + \gamma\,\gamma_i\,(\gamma^2 + \gamma_i^2)\cos(\eta - \eta_i)\right.$$

$$\left. - \frac{\gamma^2\gamma_i^2}{8}\cos(\tau - 2\,\eta + 2\,\eta_i) +, \&c.\right\}$$

$$- \frac{3\,m_i\,a^2}{8\,a_i^3}\left\{\frac{1}{2}\,b_{5,0} + b_{5,1}\cos\tau + b_{5,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{\frac{\gamma^4}{4} + \frac{5\,\gamma^2\gamma_i^2}{4} + \frac{\gamma_i^4}{4} - \frac{\gamma\,\gamma_i}{2}(\gamma^2 + \gamma_i^2)\cos(\tau + \eta - \eta_i) - \frac{\gamma\,\gamma_i}{2}(\gamma^2 + \gamma_i^2)\cos(\tau - \eta + \eta_i)\right.$$

$$\left. + \left(\frac{\gamma^4}{8} + \frac{\gamma^2\gamma_i^2}{4} + \frac{\gamma_i^4}{8}\right)\cos\tau + \frac{3}{4}\gamma^2\,\gamma_i^2\cos(2\,\eta - 2\,\eta_i) + \frac{\gamma^2\gamma_i^2}{4}\cos(2\,\tau - 2\,\eta + 2\,\eta_i)\right\}$$

$$= \left\{-\frac{9\,a^2}{64\,a_i^3}\,b_{5,0} + \frac{9\,a^2}{64\,a_i^3}\,b_{5,2} - \frac{3\,a^2}{64\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{64\,a_i^3}\,b_{5,2}\right\}\gamma^4$$

$$+ \left\{-\frac{3\,a^2}{64\,a_i^3}\,b_{5,0} + \frac{3\,a^2}{64\,a_i^3}\,b_{5,2} - \frac{15\,a^2}{64\,a_i^3}\,b_{5,0} - \frac{15\,a^2}{64\,a_i^3}\,b_{5,2}\right\}\gamma^2\,\gamma_i^2$$

$$+ \left\{-\frac{9\,a^2}{64\,a_i^3}\,b_{5,0} + \frac{9\,a^2}{64\,a_i^3}\,b_{5,2} - \frac{3\,a^2}{64\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{64\,a_i^3}\,b_{5,2}\right\}\gamma_i^4$$

$$+ \left\{\frac{3\,a^2}{32\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_i^3}\,b_{5,2} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,2}\right\}\gamma\,\gamma_i\,(\gamma^2 + \gamma_i^2)\cos(\tau - \eta + \eta_i)$$

$$+ \left\{-\frac{3\,a^2}{64\,a_i^3}\,b_{5,0} + \frac{3\,a^2}{64\,a_i^3}\,b_{5,2} - \frac{3\,a^2}{64\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{64\,a_i^3}\,b_{5,2}\right\}\gamma^2\,\gamma_i^2\cos(2\,\tau - 2\,\eta + 2\,\eta_i)$$

$$= \left\{-\frac{3\,a^2}{16\,a_i^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,2}\right\}\gamma^4 + \left\{-\frac{9\,a^2}{32\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{16\,a_i^3}\,b_{5,2}\right\}\gamma^2\,\gamma_i^2$$

$$+ \left\{-\frac{3\,a^2}{16\,a_i^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,2}\right\}\gamma_i^4 + \frac{3\,a^2}{16\,a_i^3}\,b_{5,0}\,\gamma\,\gamma_i\,(\gamma^2 + \gamma_i^2)\cos(\tau - \eta + \eta_i)$$

$$- \frac{3\,a^2}{32\,a_i^3}\,b_{5,0}\,\gamma^2\,\gamma_i^2\cos(2\,\tau - 2\,\eta + 2\,\eta_i).$$

In order to give another example of the employment of this method, I propose to calculate the coefficient of

$$e\,\gamma^4 \cos(13\,\tau - \xi - 4\,\eta),$$

the argument of which occurs in Professor AIRY's inequality of Venus, $n\,t$ and $n_{,}\,t$ being the mean motions of that planet and of the earth.

It is easily seen from the preceding pages that R contains the term

$$-\frac{3}{128}\frac{a^2}{a_{,}^3}\gamma^4\,b_{5,11}\cos(13\,\tau - 4\,\eta).$$

If the coefficient of $\gamma^4 \cos(13\,\tau - 4\,\eta)$ be denoted by R_1

$e\,\gamma^4 \cos(13\,\tau - \xi - 4\,\eta)$ . . . R_3

$$R_3 = -\frac{a\,\mathrm{d}R_1}{2\,\mathrm{d}a} - 9\,R_1$$

$$= -\frac{3}{128}\left\{-\frac{a^2}{a_{,}^3}b_{5,11} + \frac{5\,a^3}{a_{,}^4}\left(\frac{a}{a_{,}}b_{7,11} - \frac{1}{2}b_{7,10} - \frac{1}{2}b_{7,12}\right) - \frac{9\,a^2}{a_{,}^3}b_{5,11}\right\}$$

$$= \frac{3}{128}\left\{\frac{10\,a^2}{a_{,}^3}b_{5,11} - \frac{5\,a^3}{a_{,}^4}\left(\frac{a}{a_{,}}b_{7,11} - \frac{1}{2}b_{7,10} - \frac{1}{2}b_{7,12}\right)\right\}$$

And R contains the term

$$\frac{3}{128}\left\{\frac{10\,a^2}{a_{,}^3}b_{5,11} - \frac{5\,a^3}{a_{,}^4}\left(\frac{a}{a_{,}}b_{7,11} - \frac{1}{2}b_{7,10} - \frac{1}{2}b_{7,12}\right)\right\}e\,\gamma^4.$$

Professor AIRY has

$$\left\{\frac{27}{8}(0,0) + \frac{3}{16}(0,1)\right\} C^{11}_{\frac{5}{2}}\,e\,f^4.\,*$$

In Professor AIRY's notation

$$f = \sin\frac{\iota}{2} \qquad f^4 = \frac{\gamma^4}{16} \qquad (0,0)\ C^{11}_{\frac{5}{2}} = \frac{a^2}{a_{,}^3}b_{5,2} \qquad (0,1)\ C^{11}_{\frac{5}{2}} = \frac{2\,a^2}{a_{,}^3}b_{5,11} - \frac{a^2}{a_{,}^3}\frac{\mathrm{d}\,b_{5,11}}{\mathrm{d}\,a}$$

and substituting my notation in Professor AIRY's expression, that which I have found results.

The method I have given of developing the disturbing function in terms of the mean longitudes may also be employed with advantage in procuring the development in terms of the true longitudes. In this problem

$$\frac{\mathrm{d}R}{\mathrm{d}e} = \frac{r\,\mathrm{d}R\,\mathrm{d}r}{\mathrm{d}r\,r\,\mathrm{d}e} = \frac{a\,\mathrm{d}R\,\mathrm{d}r}{\mathrm{d}a\,r\,\mathrm{d}e}$$

$$\frac{\mathrm{d}r}{r\,\mathrm{d}e} = \frac{\mathrm{d}\,.\log.r}{\mathrm{d}e}$$

$$\log.r = \log.a + \log.(1 - e^2) - \log.(1 + e\cos(\lambda - \varpi)$$

$$= \log.a - e^2 - \frac{e^4}{2} - e\cos(\lambda - \varpi) + \frac{e^2}{2\,.\,2}\left\{1 + 2\cos(2\,\lambda - 2\,\varpi)\right\}$$

* See p. 89 of Professor AIRY's paper.

$$- \frac{83}{3 \cdot 4} \left\{ 3 \cos (\lambda - \varpi) + \cos (3 \lambda - 3 \varpi) \right\}$$

$$+ \frac{84}{4 \cdot 8} \left\{ 3 + \cos (2 \lambda - 2 \varpi) + \cos (4 \lambda - 4 \varpi) \right\}$$

$$= \log. a - \frac{3 e^2}{4} - \frac{13}{32} e^4 - e \left(1 + \frac{e^2}{4}\right) \cos (\lambda - \varpi) + \frac{e^2}{4} \left(1 + \frac{e^2}{2}\right) \cos (2 \lambda - 2 \varpi)$$

$$- \frac{e^3}{12} \cos (3 \lambda - 3 \varpi) + \frac{e^4}{32} \cos (4 \lambda - 4 \varpi)$$

$$\frac{d r}{r d e} = - \frac{3 e}{2} - \frac{13}{8} e^3 - e \left(1 + \frac{3}{4} e^2\right) \cos (\lambda - \varpi) + \frac{e^2}{2} (1 + e^2) \cos (2 \lambda - 2 \varpi)$$

$$- \frac{e^2}{4} \cos (3 \lambda - 3 \varpi) + \frac{e^3}{8} \cos (4 \lambda - 4 \varpi).$$

It follows from the analysis of M. Poisson, in his Mémoire sur le Mouvement de la Lune autour de la Terre, that the coefficient of $\cos (2 \varpi - 2 \varpi_{\prime})$ in the development of the quantity

$$\frac{r^2 r_{\prime}^2 R}{a^2 a_{\prime}^2 \sqrt{1 - e^2} \sqrt{1 - e_{\prime}^2}}$$

according to the *true longitudes*, is the same as that of $\cos (2 \varpi - 2 \varpi_{\prime})$ in the development of R according to the *mean longitudes*.

If $\dfrac{r^2 r_{\prime}^2}{a^2 a_{\prime}^2 \sqrt{1-e^2} \sqrt{1-e_{\prime}^2}}$ be called Q,

$$\frac{d Q}{d e} = \frac{a \, d Q}{d a} \frac{r \, d r}{d e} \qquad \frac{d Q}{d e_{\prime}} = \frac{a_{\prime} d Q}{d a_{\prime}} \frac{r_{\prime} d r_{\prime}}{d e_{\prime}}.$$

By means of these equations, and after reductions similar to those of which so many examples have been given in the course of this paper, I find the coefficient of

$\cos (2 \lambda - 2 \lambda_{\prime})$ . . . . in Q . . . . $= - a^2 a_{\prime} b_{1,2}$

$e_{\prime} \cos (2 \lambda - 2 \lambda_{\prime} + \lambda_{\prime} - \varpi_{\prime})$ . . . . $= - \dfrac{a^4}{2 a_{\prime}} b_{3,1} + \dfrac{a^3}{2} b_{3,2}$

$e e_{\prime} \cos (2 \lambda - 2 \lambda_{\prime} - \lambda + \varpi + \lambda_{\prime} - \varpi_{\prime})$ . . . . $= - \dfrac{a^3}{4} b_{3,2}$

$e e_{\prime}^2 \cos (2 \lambda - 2 \lambda_{\prime} - \lambda + \varpi + 2 \lambda_{\prime} - 2 \varpi_{\prime})$ . . . . $= - \dfrac{15 a^4}{32 a_{\prime}} b_{5,0} + \dfrac{3 a^3}{16} b_{5,1} + \dfrac{9 a^4}{32 a_{\prime}} b_{5,2}$

$e^2 e_{\prime}^2 \cos (2\lambda - 2\lambda_{\prime} - 2\lambda + 2\varpi + 2\lambda_{\prime} - 2\varpi_{\prime})$ or $e^2 e_{\prime}^2 \cos (2\varpi - 2\varpi_{\prime}) = - \dfrac{9 a^4}{64 a_{\prime}^2} b_{5,2}.$

V. *On the Results of Tide Observations made in June* 1834 *at the Coast Guard Stations in Great Britain and Ireland. By the Rev.* William Whewell, *F.R.S., Fellow of Trinity College, Cambridge.*

Received March 27,—Read April 2, 1835.

In the conclusion of " An Essay towards a first Approximation to a Map of Cotidal Lines," published in the Philosophical Transactions for 1833, I stated my opinion that simultaneous tide observations, made at the stations of the Preventive Service, and continued for a fortnight, would give us a clearer view of the progress of the tide along the coasts of this country than we could acquire from any records then extant. A representation to this effect being made to Captain Bowles, the Chief Commissioner of that Service, and to Captain Beaufort, the Hydrographer of the Admiralty, those gentlemen entered with great interest and activity into the proposal for promoting this branch of science by such a series of observations; and they undertook to give orders for carrying the plan into effect, and directions for its execution. Such observations were accordingly made at all the Preventive Service stations on the coasts of England, Ireland, and Scotland, from June 7 to June 22 inclusive, and the registers of the observations were sent to the Admiralty, where they now are.

I expected to be able to deduce from these returns the solution of several curious and important questions respecting the tides, and probably to obtain some new laws of their phenomena. For this purpose, however, it was necessary to perform a previous reduction of the registered observations, correcting the times as far as the methods employed would allow, and subtracting from each time of tide the time of the previous transit of the moon, in order to obtain the interval. Though this operation was very simple, the performance of it in so many cases (above 12,000) required more time than I could devote to it. Captain Beaufort kindly allowed it to be executed by Mr. Dessiou, of the Hydrographer's Office; and it was my intention to defer laying the account of the observations before the Society till the whole of them had been reduced, and their results investigated. But Mr. Dessiou, having executed this reduction for the whole of the south coast of England, has been prevented by illness and by more pressing employments, from proceeding to the remaining coasts. In the mean time, having examined the reduced observations, I have been led to some conclusions which appear to me interesting and important; and which, I think, considering the delay which may attend the reduction of the remaining returns, and the intention which is entertained of repeating the observations in the ensuing June, it may be worth while very briefly to announce. I shall defer the communication of

the details by which these results are established till I am able to include in them the east coast of England and the coasts of Ireland and Scotland.

1. In the first place I will observe, that I am convinced from the examination which I have given to the subject, that observations made in this manner may be depended upon for many extensive and important inferences. The returns of last June are more consistent and accurate than I could have anticipated. I have reason to believe that in much the greater part of the cases they were made with care and fidelity, and in many instances with ingenious and suitable contrivances. It is impossible not to take the opportunity of saying that they reflect great credit both upon the intelligence and the punctuality of the officers and men of the coast-guard service.

2. One of the reasons for wishing to obtain such simultaneous tide observations, was the hope of ascertaining by this means whether there are *general irregularities* which affect the tides at all places along an extensive line of coast. Such irregularities are beforehand very conceivable. The tide-wave which visits our coasts has been propagated up the Atlantic, and influenced at least, if not produced, by the tide of the Antarctic Ocean. If the causes which determine the velocity of this wave could in any case so far vary as to make it an hour behind its time in the Atlantic, that one tide would at all our ports take place an hour later than the regular time; and the existence of an influence of this kind would be detected by such an anomaly appearing in the observations of the whole or a large portion of the British coast.

I think I may venture to say that no such general irregularity affected any of the tides from the 7th to the 22nd of last June. Partial anomalies of greater or less extent occur in the observations, but nothing which can be considered as being of a general character, and indicating a distant origin, like what has been spoken of.

This result is, I conceive, important; for it appears to render it probable that we may, with care and perseverance, make our mathematical prediction of the time of the tide much more accurate than we might otherwise have hoped. Since the tide is not affected by distant and general irregularities, it is irregular only so far as it is influenced by causes which operate in the neighbourhood, and vary from one place to another; as, for instance, the effect of the wind in connexion with the form of the land. Now, not only will irregularities arising from such causes disappear in the *means* of long series of observations, but where such mean results have been obtained, the effect of the disturbing causes (as, for instance, the wind blowing at and near the place,) may be determined empirically. We should thus have a *local* meteorological correction to apply to the prediction of the tide, in addition to the astronomical corrections; our tide tables would be much improved, and our knowledge of the tides rendered more correct and complete.

3. My examination of the results of the observations of the time of high water has been conducted for the most part by the method already so often employed by Mr. Lubbock; namely, by erecting a series of equidistant ordinates to represent the

intervals of the moon's transit and high water, and drawing a continuous line through the extremities of these ordinates. The curves present, in general, the form of that deduced by Mr. Lubbock from the London Observations, though of course in the rude observations of a single fortnight there are great irregularities. But the means of several places, and even, in most instances, the tides of a single place, present the features of agreement with theory, which Mr. Lubbock has shown to obtain with such remarkable exactness in the London tides; that is, the ordinate of the curve has in the course of a fortnight a minimum and a maximum magnitude, so that the curve assumes the form ∽. Moreover, it is not symmetrical on the two sides of the minimum and maximum, the slope being greater after the minimum than before it. The curve descends from the 7th to about the 13th of June; it then ascends till the 18th or 19th, and ascends more rapidly than it had descended, and then descends again less rapidly. All this agrees with the form given by theory.

4. But though the general course of the curves has this resemblance, the amount of flexure is not the same at different places. This result had already been obtained by the comparison of previous observations, especially those made at Brest, Plymouth, and London; it is confirmed so clearly by the observations here referred to, that I think it may now be assumed to be a general fact.

The inferences from this fact are very important; for in the first place it puts an end to all attempts to deduce the mass of the moon from the phenomena of tides, or to correct the tables of the tides by means of the mass of the moon. The approximate agreement of the mass of the moon deduced by Laplace from the tides of Brest, with the mass obtained from other phenomena, cannot be considered as otherwise than accidental. If he had employed the tides of London, he would have obtained a mass very different, as Mr. Lubbock has shown; if he had taken those of Plymouth, or of Brighton, the mass would have been again very different.

This evidence of the inapplicability of this part of the theory will not surprise any one who recollects how remote the hypotheses of the theory are from the case of nature. Such a theory may point out the general features of the phenomena, but any assumption of the actual correctness of the magnitudes determined by means of it is altogether gratuitous. The force of the moon *determines* the amount of the semimenstrual inequality; but probably we shall never be able to ascertain otherwise than empirically, *by what rule* this force, producing oscillations in an ocean of irregular form and depth, as the actual ocean is, determines the semimenstrual inequality at each point.

5. But since the semimenstrual inequality is thus determined in general by the force of the moon, and has a common form at different places, and yet is different in amount (and in other circumstances) at each place, we may represent it by resolving it into two parts, one of which shall be common to the whole ocean, or to a large portion of it, and the other part shall be a smaller and *local* correction, also following a cycle of half a month.

By the introduction of a *local semimenstrual inequality*, in addition to the general semimenstrual inequality, we should be able to reconcile the discrepancies of the curve which represents this inequality for different places, as London and Plymouth; discrepancies which have hitherto been a source of perplexity to those who have studied the subject.

The existence of these discrepancies, and their general prevalence, which is shown by our observations, make it clear that we cannot correctly use the tide table of one place to determine the tides of another, by adding or subtracting a constant interval, as is often done. For such a purpose the difference of the local semimenstrual inequalities of the two places requires also to be applied.

I have not attempted to determine the amount or form of the local semimenstrual inequalities of different places, not thinking our present materials sufficient; but a comparison of the curve of the semimenstrual inequality of different places is the way in which it must be obtained, and on this subject I have some remarks to make.

6. By what causes is the semimenstrual inequality at one place made to be different from that at another? There are some circumstances which we can readily imagine may produce such an effect, though we should probably not succeed in guessing what the effect would be; as, for instance, the form of the coast, the distance which the tide wave has travelled over, and the meeting of tides proceeding different ways. I think I can discover in the observations of last June indications of the effect of all these circumstances.

In the first place it appears that the curves (by which I mean here, and in what follows, the curves of the semimenstrual inequality) are *flatter* when the observations are made at promontories than they are for the general line of coast. I speak here of the promontories of the first order, which divide the south coast of England into large or primary bays, as the Lizard, the Rame Head, Prawle Point, Portland Bill, St. Alban's Head, St. Catherine's Point, Beachy Head. At such places the amount of the semimenstrual inequality appears from the observations to be less than it is in the intervening bays.

7. In the next place it seems to follow from the observations that the curves are flatter and flatter as the tide wave proceeds further and further. Thus the curve is flatter in Brighton Bay than it is in Mount's Bay on the coast of Cornwall, the tide having travelled further from west to east.

I do not consider this point as quite firmly established, because, though the curves do exhibit such a modification in going eastward, when we get as far east as the Isle of Wight we seem to perceive the influence of another cause which has been mentioned, the meeting of the two tides, which may produce this apparent modification.

8. This subject, the meeting of the tides, appears to be often misunderstood. For instance, in a paper published in the Philosophical Transactions for 1819, it is taken for granted, that when the two tides meet which come up the British Channel and down the German Ocean there must be a visible and marked conflict of opposite cur-

rents of the water. But this supposition is altogether gratuitous. The place of the meeting of the two *tide-waves*, which come in opposite directions, is the part of the coast where the tide is later than it is on either side of that part. For example, we know that on the coast of Dorsetshire and Hampshire the tide-hours are VII. VIII. IX. going eastward, and that on the coast of Norfolk the tide-hours are VII. VIII. IX. proceeding southwards. The tide-waves, therefore, move in an opposite direction along these coasts, and must *meet* at some intermediate point, as, for instance, on the coast of Kent; and at this point the tide is later than it is if we go along the coast either to the east and north, or to the south and west. But these motions of the *tide-waves* must be distinguished from the motions of the *streams* which bring the tide, as will be obvious when it is recollected that the tide-wave travels from the Land's End to the Isle of Wight in six hours. At the place where the tides meet there will not necessarily be anything more marked in the stream of flood and ebb than at any other point. The tendencies to opposite tide-streams may partially balance each other during a part of the flow and of the ebb, and leave only the difference of tendency in actual operation. There may be strong and conflicting tide-streams produced under certain circumstances; but these results will depend much more upon the local conditions of the ground than upon the general course of the tide-wave.

The meeting of the tides, however, will not be a single point; for by the laws of fluids the two opposite undulations, which we term the tide-waves, will be propagated independently of each other, and the fluid will be affected by both. If they were thus propagated without any loss of magnitude, we could easily trace the consequences. Let the tide-wave on the south coast move eastward so as to bring high water to certain places,

	A.	B.	C.	D.	E.	F.	G.	H.	K.	L.	M.,
at the hours	VI.	VII.	VIII.	IX.	X.	XI.	XII.	I.	II.	III.	IV.,

and let the tide coming from the north and east in the opposite direction arrive at the same places at the hours

IV.	III.	II.	I.	XII.	XI.	X.	IX.	VIII.	VII.	VI.

It is then manifest that the tide at F. will still take place at XI.; also the tides at E. and D. will occur about XI., the hour intermediate between X. and XII., and between IX. and I. In the same way the tide at G. and H. will be about XI. I do not say *exactly* at XI., because each tide may diminish in amount as it advances; and for this reason each tide may, at a certain distance after their meeting, less affect the other. From these considerations we may expect the tide-hours along such a coast to be as follows:

A.	B.	C.	D.	E.	F.	G.	H.	K.	L.	M.
VI.	VII.	IX.	XI.	XI.	XI.	XI.	XI.	IX.	VII.	VI.

The question now remains to be answered, Do we find any such succession of tide-hours as this on the coast of Britain? And to this the coast-guard observations

on the south coast enable us to reply, that the hours do follow an order of this kind. Along a great extent of coast (from the Land's End to the Isle of Wight) the tide-hours increase in order from $4^h\ 30^m$ to $11^h\ 30^m$. But from the Isle of Wight eastward, the tide-hour continues to be about $11^h\ 30^m$, with small and irregular changes only: and this is true as far along the coast eastward as the observations have been reduced. The examination of the eastern-coast observations will show how far this peculiarity extends.

Thus "the tides meet," in reality, along the whole coast, from the Isle of Wight to the Downs, and perhaps to the coast of Suffolk; that is, along the whole of this tract the water is affected by the tide-waves which arrive in the two opposite directions.

It may appear strange that the influence of the eastern tide should cease suddenly when it reaches the Isle of Wight, not extending any further to the west. If, however, we look at the map, and observe the sudden widening of the channel to the west of Cape La Hogue, we shall be at no loss to conceive that the tide-wave may be extremely diminished by this rapid diffusion, as we know that the tides are greatly increased by the gradual contraction of their beds in estuaries and rivers. But whether or not this be the cause, the fact is indisputable in the observations, that to the east of this point the tide-hour changes very little, while to the west it diminishes with comparative rapidity.

If it were at all doubtful that this difference arises from the interference of the eastern tide as far as this point, the question would, I conceive, be settled by the two following articles.

9. The heights, as well as the times of high water, were observed; these heights I have hitherto examined for a particular purpose only, namely, to ascertain the existence of a diurnal difference of height, which follows from the theory, as I have observed in a former paper*. From this examination it appears that this diurnal difference manifests itself with remarkable constancy along a large portion of the coast now under consideration. From the Scilly Islands to Portland Bill, most of the stations exhibit this inequality operating upon the greater part of the tides. The law is, as is well known, that at a certain season of the year the morning tide is greater than the afternoon tide, and at a certain other season it is less. In June the evening tide was the greater; this appears clearly in the early part of the observations. As the morning tide approaches noon, the difference diminishes; and when the morning tide is become the afternoon tide, the diurnal difference has *skipped* one tide, so as still to be found conforming to the rule. The diurnal difference of height is variable, ranging from two or three inches to one foot.

10. I have said that this diurnal difference may be traced as far as Portland Bill. Eastward from this point the tides do not appear to be affected by it, the morning and evening tide not having any steady relation of greater or less.

This change is remarkable, and the more so when we observe that it takes place

* Philosophical Transactions, 1833, p. 221.

at the limit of the influence of the eastern tide, according to what was said in article 8. Will the interference of the tides explain such a change?

It obviously will do so; for the two tides at their meeting differ by twelve hours in the extent of their course, the one which has come round the northern extremity of Scotland and down the east coast being so much older than the channel tide. If, therefore, one of the two be a morning tide (when referred to its origin), the other must be an afternoon tide; and each compound tide being made up of such a pair, will show no peculiar character of either one or the other. Thus we may expect that, as far as the interference of the tides extends, the diurnal difference will disappear.

Taking the two considerations of Article 8. and this article together, I think it cannot be doubted that the sea, from the Isle of Wight to the Downs, and probably further, is affected by both the western and the northern tide.

11. It is natural to inquire whether we can, from our observations, discover the nature of the effect which the form of the coast produces on the time and height of the tide. On this subject I can offer some reply, though a more complete discussion of the existing returns, and of future observations, is desirable to confirm and extend our views.

The principal feature which appears in the observations of June is, that the tide-hour varies very rapidly in rounding the main promontories of the coast, and very slowly in passing along the shores of the intervening bays. Thus, along the whole of the great bay formed by the coast of Devonshire and Dorsetshire, from Prawle Point to Portland Bill, the tide hour is nearly the same, ranging only from about $6^h\ 5^m$ to $6^h\ 20^m$. But in passing round into Weymouth Bay the hour becomes 7^h, and on going round St. Alban's Head into Swanage Bay, it becomes suddenly 9^h.

If we draw the cotidal lines so as to correspond with these conditions, it is clear that the ends of these lines will be brought close together at the promontories, and that the lines will run along nearly parallel to the shore. Thus, the extremity of the 6^h cotidal line is near Prawle Head, the line itself following nearly the coast of the bay to Portland Bill. The 7^h cotidal line ends at Portland Bill, and the 8^h and 9^h lines end at St. Alban's Head. The 10^h and 11^h lines appear to meet the coast near St. Catherine's Head in the Isle of Wight; and, agreeably with what has already been stated, the 11^h line runs at a little distance from the coast through the straits of Dover. The cotidal lines drawn in my Essay printed in 1833 require to be modified according to these remarks.

12. At points of the coast where the cotidal lines are brought near together, the place of high water moves slowly; so that it is high water at one point, while at another point not far off, the water is still considerably deficient from its greatest height. Hence there will be a difference of level and a rapid tide-stream in such cases. Thus the peculiarity just noticed in the reference of cotidal lines to promontories is con-

nected with the occurrence of strong currents governed by the tide, like the Race of Portland and the similar current which is found off St. Alban's Head.

I abstain from making any further remarks till the reduction of the whole of the returns of last June shall give me more complete materials. I am the more desirous to draw attention to the results which such observations may supply, on account of its being intended to repeat the observations at the Coast Guard stations in the ensuing June, from the 9th to the 27th. I am also glad to be able to state, that the subject having been laid before the Lords of the Admiralty by Captain Beaufort, Their Lordships expressed their wish that application should be made to foreign maritime states, with a view to induce them to make contemporaneous observations on their coasts; and that such applications are now in the course of being made. The extension of such results as have been stated in the present paper to other coasts, and the discovery of other similar laws, cannot but be looked upon as a valuable and interesting addition to our knowledge.

VI. *On certain Peculiarities in the Double Refraction and Absorption of Light exhibited in the Oxalate of Chromium and Potash. By Sir* DAVID BREWSTER, *K.H. LL.D. F.R.S.*

Received January 27,—Read February 12, 1835.

THIS remarkable salt was put into my hands about the end of the year 1832, by Dr. WILLIAM GREGORY, of Edinburgh, to whom I have been indebted for much kind assistance in carrying on my inquiries respecting the action of coloured bodies in absorbing definite rays of the spectrum. A very brief examination of its optical properties was sufficient to indicate its more obvious peculiarities, and a short notice of these was published at the time. Having received, however, from Dr. GREGORY a very fine group of well formed crystals, and having had an opportunity in the spring of 1833 of observing their action upon the spectrum, both in their solid state and in the state of aqueous solution, I am now able to present to the Society a general view of the results which I obtained.

The oxalate of chromium and potash occurs in flat, irregular, six-sided prisms. The two broadest faces are inclined to each other like the faces of a wedge, whose sharp edge is the summit of the crystal. These faces are considerably rounded, being parallel near the base, and inclined to each other about three degrees at the apex of the prism. The incidence of the broad faces upon the adjacent faces of the prism is about 140°, and therefore these faces are inclined to one another at an angle of $180° - 148° \times 2 = 64°$. The crystal is terminated by four minute planes equally inclined to the broad face and the axis of the prism, but two of these faces often disappear, and the crystal terminates in an oblique edge in place of a triangular apex.

If we call A A′ the broad faces of the crystal, *m*, *m′*, *m*, *m′* the other four faces of the prism, and *o*, *o′*, *p*, *p′* the faces on the summit, the following are the angles which they form with each other.

Incidence of A upon A in a line passing through the axis of the prism	5° 10′
———— A upon *m*, and A′ upon *m′*	148 0
———— *m* upon *m*	64 0
———— A upon *o*, and A′ upon *o′*	112 10
———— A upon *p*, and A′ upon *p′*	112 10
———— *o* upon *o′*, and *p′* upon *p′*	50 10
———— A upon A′ over *o*, *o′* or *p*, *p′*	4 36

The crystals of oxalate of chromium and potash are, generally speaking, opake; and at thicknesses not much greater than the twenty-fifth of an inch they are abso-

lutely impervious to the sun's rays. In this state their colour, seen by reflected light, is nearly black; but their powder is *green* in daylight, and of a *French grey* colour by candlelight. In the smaller crystals, which are generally the best formed, the colour both of reflected and transmitted daylight is *blue*, but that of candlelight is *purple*. I have not been able to find any distinct traces of cleavage.

This salt possesses a powerful double refraction, which is no doubt related to two axes. In reference to the axis of the prism the double refraction is *negative*, like that of calcareous spar. The greatest refractive index is about 1·605, and the least about 1·506, reckoning from a line near the boundary of the blue and green rays.

One of the most remarkable properties of this salt is the difference of colour in the two images formed by double refraction. At a certain small thickness the *least* refracted image is *bright blue*, and the *most* refracted image *bright green*, in daylight, or *bright pink* in candlelight. The *blue* contains an admixture of green when analysed by the prism, and the green an admixture of red, the red predominating over the green in candlelight. At greater thicknesses the blue becomes purer and fainter, and the green passes into red; and at a certain thickness the least refracted blue image disappears altogether, and the most refracted image is olive green. At still greater thicknesses this image disappears also, and absolute opacity ensues.

When the crystal is exposed to polarized light, with its axis in the plane of polarization, the transmitted light is *green*; but when the axis of the crystal is perpendicular to that plane, the transmitted light is *blue*.

When the oxalate of chromium and potash is dissolved in water its double refraction disappears, in consequence of the particles being released from the force of aggregation by which they are held together in the solid state, and by which double refraction is produced. The solution, however, exhibits the same general action upon light as the solid. At moderate thicknesses its colour is a dark blueish green by daylight, and a bright blood red by candlelight; but when we increase the thickness of the fluid it becomes of a *blueish pink* by daylight, and of a deeper *blood red* by candlelight, the *red* rays continuing to increase both in day- and candle-light, as we lengthen the path of the ray through the solution.

The most remarkable property of the oxalate of chromium and potash, and the one on account of which I have submitted this paper to the Royal Society, is its specific action upon a definite red ray lying near the extremity of the red portion of the spectrum. This is a property which is not possessed by any solid or fluid body with which I am acquainted, although I have submitted some hundreds of coloured bodies to direct experiment. Like all coloured bodies, the oxalate under our consideration exercises a general absorbent action on the spectrum. The smallest thickness of it, in which colour is scarcely discernible, attacks the *yellow* rays of the spectrum on the more refrangible side of the line D of Fraunhofer. As the thickness of colour of the solution increases, the *violet* rays are absorbed, and also all the *yellow, orange*, and less refrangible *green*, till the whole space D E, and part of the spaces on the other

side of the lines D, E, are wholly destroyed. In this state the prism gives two distinct images of objects, viz. a *red* and a *greenish blue* image, which are considerably separated. As the absorption advances, the *green* on the blue side of E, and the *blue* on the violet side of F, gradually disappear, till a *pure blue* image about F alone remains, and this too wholly vanishes by an increased thickness of the solution, leaving the red rays unabsorbed.

While these changes are going on throughout the spectrum, a specific action is exerted upon a red ray between A and B of FRAUNHOFER, and in that very part of the spectrum over which the solution exercises no general absorptive action. The sharp and narrow black band which is thus formed constitutes a *fixed line in all artificial lights*, and also in solar and day light, which will enable philosophers to measure the refractive powers of all bodies in reference to this line with an accuracy which could not otherwise be obtained, unless by the use of fine prisms of the refracting substances, which in most cases are unattainable.

In order to render this line or band of real use in practical optics, I have endeavoured to fix its place with as great accuracy as possible. Between the lines A, B of FRAUNHOFER there is a group of lines nearly bisecting the space A B, which he has marked *a* in his map. The dark band lies in the space B *a*; and if we designate it by the letter X, its position is such that $BX = \frac{1}{3} Ba$, or the index of refraction in the *Water spectrum*, of the rays which are absorbed at the band X is almost exactly 1·330701, the temperature of the water being 65° of FAHRENHEIT.

The relations of this salt to common and polarized light may be readily examined and finely exhibited by placing upon a plate of glass a few drops of a saturated solution of it in water. If the crystals are slowly formed they will be found of various thicknesses, each thickness exhibiting a different colour, varying from perfect transparency, through all shades of *pale yellow*, *green*, and *blue*, in daylight, and through all shades of *pale yellow*, *pale orange*, *red*, and *blue*, in candlelight.

BELLEVILLE, *by Kingussie*,
March 21*st*, 1835.

VII. *Second Essay on a General Method in Dynamics.* By William Rowan Hamilton, *Member of several Scientific Societies in Great Britain and in Foreign Countries, Andrews' Professor of Astronomy in the University of Dublin, and Royal Astronomer of Ireland. Communicated by Captain* Beaufort, *R.N. F.R.S.*

Received October 29, 1834,—Read January 15, 1835.

Introductory Remarks.

THE former Essay* contained a general method for reducing all the most important problems of dynamics to the study of one characteristic function, one central or radical relation. It was remarked at the close of that Essay, that many eliminations required by this method in its first conception, might be avoided by a general transformation, introducing the time explicitly into a part S of the whole characteristic function V; and it is now proposed to fix the attention chiefly on this part S, and to call it the *Principal Function.* The properties of this part or function S, which were noticed briefly in the former Essay, are now more fully set forth; and especially its uses in questions of perturbation, in which it dispenses with many laborious and circuitous processes, and enables us to express accurately the disturbed configuration of a system by the rules of undisturbed motion, if only the initial components of velocities be changed in a suitable manner. Another manner of extending rigorously to disturbed motion the rules of undisturbed, by the gradual variation of elements, in number double the number of the coordinates or other marks of position of the system, which was first invented by Lagrange, and was afterwards improved by Poisson, is considered in this Second Essay under a form perhaps a little more general; and the general method of calculation which has already been applied to other analogous questions in optics and in dynamics by the author of the present Essay, is now applied to the integration of the equations which determine these elements. This general method is founded chiefly on a combination of the principles of variations with those of partial differentials, and may furnish, when it shall be matured by the labours of other analysts, a separate branch of algebra, which may be called perhaps the *Calculus of Principal Functions*; because, in all the chief applications of algebra to physics, and in a very extensive class of purely mathematical questions, it reduces the determination of many mutually connected functions to the search and study of one principal or central relation. When applied to the integration of the equations of varying elements, it suggests, as is now shown, the consideration

* Philosophical Transactions for the year 1834, Second Part.

of a certain *Function of Elements*, which may be variously chosen, and may either be rigorously determined, or at least approached to, with an indefinite accuracy, by a corollary of the general method. And to illustrate all these new general processes, but especially those which are connected with problems of perturbation, they are applied in this Essay to a very simple example, suggested by the motions of projectiles, the parabolic path being treated as the undisturbed. As a more important example, the problem of determining the motions of a ternary or multiple system, with any laws of attraction or repulsion, and with one predominant mass, which was touched upon in the former Essay, is here resumed in a new way, by forming and integrating the differential equations of a new set of varying elements, entirely distinct in theory (though little differing in practice) from the elements conceived by LAGRANGE, and having this advantage, that the differentials of all the new elements for *both* the disturbed and disturbing masses may be expressed by the coefficients of *one* disturbing function.

Transformations of the Differential Equations of Motion of an Attracting or Repelling System.

1. It is well known to mathematicians, that the differential equations of motion of any system of free points, attracting or repelling one another according to any functions of their distances, and not disturbed by any foreign force, may be comprised in the following formula:

$$\Sigma \,.\, m\,(x''\,\delta x + y''\,\delta y + z''\,\delta z) = \delta \mathrm{U}: \quad \ldots \ldots \ldots \quad (1.)$$

the sign of summation Σ extending to all the points of the system; m being, for any one such point, the constant called its mass, and $x\,y\,z$ being its rectangular coordinates; while $x''\,y''\,z''$ are the accelerations, or second differential coefficients taken with respect to the time, and δx, δy, δz are any arbitrary infinitesimal variations of those coordinates, and U is a certain *force-function*, introduced into dynamics by LAGRANGE, and involving the masses and mutual distances of the several points of the system. If the number of those points be n, the formula (1.) may be decomposed into $3\,n$ ordinary differential equations of the second order, between the coordinates and the time,

$$m_i\,x''_i = \frac{\delta \mathrm{U}}{\delta x_i}; \quad m_i\,y''_i = \frac{\delta \mathrm{U}}{\delta y_i}; \quad m_i\,z''_i = \frac{\delta \mathrm{U}}{\delta z_i}: \quad \ldots \ldots \ldots \quad (2.)$$

and to integrate these differential equations of motion of an attracting or repelling system, or some transformations of these, is the chief and perhaps ultimately the only problem of mathematical dynamics.

2. To facilitate and generalize the solution of this problem, it is useful to express previously the $3\,n$ rectangular coordinates $x\,y\,z$ as functions of $3\,n$ other and more general marks of position $\eta_1\,\eta_2\,\ldots\,\eta_{3n}$; and then the differential equations of motion take this more general form, discovered by LAGRANGE,

$$\frac{d}{dt}\frac{\delta T}{\delta \eta'_i} - \frac{\delta T}{\delta \eta_i} = \frac{\delta U}{\delta \eta_i}, \quad \ldots \ldots \quad (3.)$$

in which

$$T = \tfrac{1}{2} \Sigma . m (x'^2 + y'^2 + z'^2). \quad \ldots \ldots \quad (4.)$$

For, from the equations (2.) or (1.),

$$\left.\begin{aligned} \frac{\delta U}{\delta \eta_i} &= \Sigma . m \left(x'' \frac{\delta x}{\delta \eta_i} + y'' \frac{\delta y}{\delta \eta_i} + z'' \frac{\delta z}{\delta \eta_i} \right) \\ &= \frac{d}{dt} \Sigma . m \left(x' \frac{\delta x}{\delta \eta_i} + y' \frac{\delta y}{\delta \eta_i} + z' \frac{\delta z}{\delta \eta_i} \right) \\ &- \Sigma . m \left(x' \frac{d}{dt}\frac{\delta x}{\delta \eta_i} + y' \frac{d}{dt}\frac{\delta y}{\delta \eta_i} + z' \frac{d}{dt}\frac{\delta z}{\delta \eta_i} \right); \end{aligned}\right\} \quad \ldots \ldots \quad (5.)$$

in which

$$\left.\begin{aligned} &\Sigma . m \left(x' \frac{\delta x}{\delta \eta_i} + y' \frac{\delta y}{\delta \eta_i} + z' \frac{\delta z}{\delta \eta_i} \right) \\ &= \Sigma . m \left(x' \frac{\delta x'}{\delta \eta'_i} + y' \frac{\delta y'}{\delta \eta'_i} + z' \frac{\delta z'}{\delta \eta'_i} \right) = \frac{\delta T}{\delta \eta'_i}, \end{aligned}\right\} \quad \ldots \ldots \quad (6.)$$

and

$$\left.\begin{aligned} &\Sigma . m \left(x' \frac{d}{dt}\frac{\delta x}{\delta \eta_i} + y' \frac{d}{dt}\frac{\delta y}{\delta \eta_i} + z' \frac{d}{dt}\frac{\delta z}{\delta \eta_i} \right) \\ &= \Sigma . m \left(x' \frac{\delta x'}{\delta \eta_i} + y' \frac{\delta y'}{\delta \eta_i} + z' \frac{\delta z'}{\delta \eta_i} \right) = \frac{\delta T}{\delta \eta_i}, \end{aligned}\right\} \quad \ldots \ldots \quad (7.)$$

T being here considered as a function of the $6n$ quantities of the forms η' and η, obtained by introducing into its definition (4.), the values

$$x' = \eta'_1 \frac{\delta x}{\delta \eta_1} + \eta'_2 \frac{\delta x}{\delta \eta_2} + \ldots + \eta'_{3n} \frac{\delta x}{\delta \eta_{3n}}, \text{ \&c.} \quad \ldots \ldots \quad (8.)$$

A different proof of this important transformation (3.) is given in the Mécanique Analytique.

3. The function T being homogeneous of the second dimension with respect to the quantities η', must satisfy the condition

$$2T = \Sigma . \eta' \frac{\delta T}{\delta \eta'}; \quad \ldots \ldots \quad (9.)$$

and since the variation of the same function T may evidently be expressed as follows,

$$\delta T = \Sigma \left(\frac{\delta T}{\delta \eta'} \delta \eta' + \frac{\delta T}{\delta \eta} \delta \eta \right), \quad \ldots \ldots \quad (10.)$$

we see that this variation may be expressed in this other way,

$$\delta T = \Sigma \left(\eta' \delta \frac{\delta T}{\delta \eta'} - \frac{\delta T}{\delta \eta} \delta \eta \right). \quad \ldots \ldots \quad (11.)$$

If then we put, for abridgement,

$$\frac{\delta T}{\delta \eta'_1} = \varpi_1, \ldots \frac{\delta T}{\delta \eta'_{3n}} = \varpi_{3n}, \quad \ldots \ldots \quad (12.)$$

and consider T (as we may) as a function of the following form,

$$T = F(\varpi_1, \varpi_2, \ldots \varpi_{3n}, \eta_1, \eta_2, \ldots \eta_{3n}), \quad \ldots \quad (13.)$$

we see that

$$\frac{\delta F}{\delta \varpi_1} = \eta'_1, \ldots \frac{\delta F}{\delta \varpi_{3n}} = \eta'_{3n}, \quad \ldots \quad (14.)$$

and

$$\frac{\delta F}{\delta \eta_1} = -\frac{\delta T}{\delta \eta_1}, \ldots \frac{\delta F}{\delta \eta_{3n}} = -\frac{\delta T}{\delta \eta_{3n}}; \quad \ldots \quad (15.)$$

and therefore that the general equation (3.) may receive this new transformation,

$$\frac{d\varpi_i}{dt} = \frac{\delta (U - F)}{\delta \eta_i}. \quad \ldots \quad (16.)$$

If then we introduce, for abridgement, the following expression H,

$$H = F - U = F(\varpi_1, \varpi_2, \ldots \varpi_{3n}, \eta_1, \eta_2, \ldots \eta_{3n}) - U(\eta_1, \eta_2, \ldots \eta_{3n}), \quad \cdot \quad (17.)$$

we are conducted to this new manner of presenting the differential equations of motion of a system of n points, attracting or repelling one another:

$$\left.\begin{array}{l} \dfrac{d\eta_1}{dt} = \dfrac{\delta H}{\delta \varpi_1}; \quad \dfrac{d\varpi_1}{dt} = -\dfrac{\delta H}{\delta \eta_1}; \\ \dfrac{d\eta_2}{dt} = \dfrac{\delta H}{\delta \varpi_2}; \quad \dfrac{d\varpi_2}{dt} = -\dfrac{\delta H}{\delta \eta_2}; \\ \cdots\cdots\cdots \\ \dfrac{d\eta_{3n}}{dt} = \dfrac{\delta H}{\delta \varpi_{3n}}; \quad \dfrac{d\varpi_{3n}}{dt} = -\dfrac{\delta H}{\delta \eta_{3n}}. \end{array}\right\} \quad \ldots \quad (A.)$$

In this view, the problem of mathematical dynamics, for a system of n points, is to integrate a system (A.) of $6n$ ordinary differential equations of the first order, between the $6n$ variables $\eta_i\ \varpi_i$ and the time t; and the solution of the problem must consist in assigning these $6n$ variables as functions of the time, and of their own initial values, which we may call $e_i\ p_i$. And all these $6n$ functions, or $6n$ relations to determine them, may be expressed, with perfect generality and rigour, by the method of the former Essay, or by the following simplified process.

Integration of the Equations of Motion, by means of one Principal Function.

4. If we take the variation of the definite integral

$$S = \int_0^t \left(\Sigma \,.\, \varpi \frac{\delta H}{\delta \varpi} - H \right) dt \quad \ldots \quad (18.)$$

without varying t or dt, we find, by the Calculus of Variations,

$$\delta S = \int_0^t \delta S' \,.\, dt, \quad \ldots \quad (19.)$$

in which

$$S' = \Sigma \,.\, \varpi \frac{\delta H}{\delta \varpi} - H, \quad \ldots \quad (20.)$$

and therefore

$$\delta S' = \Sigma \left(\varpi \delta \frac{\delta H}{\delta \varpi} - \frac{\delta H}{\delta \eta} \delta \eta \right), \quad \ldots \ldots \quad (21.)$$

that is, by the equations of motion (A.),

$$\delta S' = \Sigma \left(\varpi \delta \frac{d \eta}{d t} + \frac{d \varpi}{d t} \delta \eta \right) = \frac{d}{d t} \Sigma . \varpi \delta \eta ; \quad \ldots \ldots \quad (22.)$$

the variation of the integral S is therefore

$$\delta S = \Sigma (\varpi \delta \eta - p \delta e), \quad \ldots \ldots \quad (23.)$$

(p and e being still initial values,) and it decomposes itself into the following $6n$ expressions, when S is considered as a function of the $6n$ quantities η_i e_i, (involving also the time,)

$$\left.\begin{array}{l} \varpi_1 = \frac{\delta S}{\delta \eta_1}; \ p_1 = - \frac{\delta S}{\delta e_1}; \\ \varpi_2 = \frac{\delta S}{\delta \eta_2}; \ p_2 = - \frac{\delta S}{\delta e_2}; \\ \ldots\ldots \\ \varpi_{3n} = \frac{\delta S}{\delta \eta_{3n}}; \ p_{3n} = - \frac{\delta S}{\delta e_{3n}}; \end{array}\right\} \quad \ldots \ldots \quad (B.)$$

which are evidently forms for the sought integrals of the $6n$ differential equations of motion (A.), containing only one unknown function S. The difficulty of mathematical dynamics is therefore reduced to the search and study of this one function S, which may for that reason be called the PRINCIPAL FUNCTION of motion of a system.

This function S was introduced in the first Essay under the form

$$S = \int_0^t (T + U) \, d t,$$

the symbols T and U having in this form their recent meanings; and it is worth observing, that when S is expressed by this definite integral, the conditions for its variation vanishing (if the final and initial coordinates and the time be given) are precisely the differential equations of motion (3.), under the forms assigned by LAGRANGE. The variation of this definite integral S has therefore the double property, of giving the differential equations of motion for any transformed coordinates when the extreme positions are regarded as fixed, and of giving the integrals of those differential equations when the extreme positions are treated as varying.

5. Although the function S seems to deserve the name here given it of *Principal Function*, as serving to express, in what appears the simplest way, the integrals of the equations of motion, and the differential equations themselves; yet the same analysis conducts to other functions, which also may be used to express the integrals of the same equations. Thus, if we put

$$Q = \int_0^t \left(- \Sigma . \eta \frac{\delta H}{\delta \eta} + H \right) d t, \quad \ldots \ldots \quad (24.)$$

and take the variation of this integral Q without varying t or $d t$, we find, by a similar process,

$$\delta Q = \Sigma (\eta \delta \varpi - e \delta p) ; \quad \ldots \ldots \quad (25.)$$

so that if we consider Q as a function of the $6n$ quantities ϖ_i p_i and of the time, we shall have $6n$ expressions

$$\eta_i = + \frac{\delta Q}{\delta \varpi_i}, \quad e_i = - \frac{\delta Q}{\delta p_i}, \quad \ldots \ldots \ldots \quad (26.)$$

which are other forms for the integrals of the equations of motion (A.), involving the function Q instead of S. We might also employ the integral

$$V = \int_0^t \Sigma \,.\, \varpi \frac{\delta H}{\delta \varpi} dt = \Sigma \int_e^\eta \varpi \, d\eta, \quad \ldots \ldots \ldots \quad (27.)$$

which was called the *Characteristic Function* in the former Essay, and of which, when considered as a function of the $6n + 1$ quantities η_i e_i H, the variation is

$$\delta V = \Sigma (\varpi \delta \eta - p \delta e) + t \delta H. \quad \ldots \ldots \ldots \quad (28.)$$

And all these functions S, Q, V, are connected in such a way, that the forms and properties of any one may be deduced from those of any other.

Investigation of a Pair of Partial Differential Equations of the first Order, which the Principal Function must satisfy.

6. In forming the variation (23.), or the partial differential coefficients (B.), of the Principal Function S, the variation of the time was omitted; but it is easy to calculate the coefficient $\frac{\delta S}{\delta t}$ corresponding to this variation, since the evident equation

$$\frac{dS}{dt} = \frac{\delta S}{\delta t} + \Sigma \frac{\delta S}{\delta \eta} \frac{d\eta}{dt} \quad \ldots \ldots \ldots \quad (29.)$$

gives, by (20.), and by (A.), (B.),

$$\frac{\delta S}{\delta t} = S' - \Sigma \,.\, \varpi \frac{\delta H}{\delta \varpi} = - H. \quad \ldots \ldots \ldots \quad (30.)$$

It is evident also that this coefficient, or the quantity $-$ H, is constant, so as not to alter during the motion of the system; because the differential equations of motion (A.) give

$$\frac{dH}{dt} = \Sigma \left(\frac{\delta H}{\delta \eta} \frac{d\eta}{dt} + \frac{\delta H}{\delta \varpi} \frac{d\varpi}{dt} \right) = 0. \quad \ldots \ldots \ldots \quad (31.)$$

If, therefore, we attend to the equation (17.), and observe that the function F is necessarily rational and integer and homogeneous of the second dimension with respect to the quantities ϖ_i, we shall perceive that the principal function S must satisfy the two following equations between its partial differential coefficients of the first order, which offer the chief means of discovering its form:

$$\left.\begin{aligned} \frac{\delta S}{\delta t} + F\left(\frac{\delta S}{\delta \eta_1}, \frac{\delta S}{\delta \eta_2}, \ldots \frac{\delta S}{\delta \eta_{3n}}, \eta_1, \eta_2, \ldots \eta_{3n}\right) = U(\eta_1, \eta_2, \ldots \eta_{3n}), \\ \frac{\delta S}{\delta t} + F\left(\frac{\delta S}{\delta e_1}, \frac{\delta S}{\delta e_2}, \ldots \frac{\delta S}{\delta e_{3n}}, e_1, e_2, \ldots e_{3n}\right) = U(e_1, e_2, \ldots e_{3n}). \end{aligned}\right\} \text{(C.)}$$

Reciprocally, if the form of S be known, the forms of these equations (C.) can be deduced from it, by elimination of the quantities e or η between the expressions of its partial differential coefficients; and thus we can return from the principal function S to the functions F and U, and consequently to the expression H, and the equations of motion (A.).

Analogous remarks apply to the functions Q and V, which must satisfy the partial differential equations,

$$\left.\begin{aligned}
-\frac{\delta Q}{\delta t}+F\left(\varpi_1,\varpi_2,\ldots\varpi_{3n},\frac{\delta Q}{\delta\varpi_1},\frac{\delta Q}{\delta\varpi_2},\ldots\frac{\delta Q}{\delta\varpi_{3n}}\right)&=U\left(\frac{\delta Q}{\delta\varpi_1},\frac{\delta Q}{\delta\varpi_2},\ldots\frac{\delta Q}{\delta\varpi_{3n}}\right),\\
-\frac{\delta Q}{\delta t}+F\left(p_1,p_2,\ldots p_{3n},-\frac{\delta Q}{\delta p_1},-\frac{\delta Q}{\delta p_2},\ldots-\frac{\delta Q}{\delta p_{3n}}\right)&=U\left(-\frac{\delta Q}{\delta p_1},-\frac{\delta Q}{\delta p_2},\ldots-\frac{\delta Q}{\delta p_{3n}}\right),
\end{aligned}\right\}\quad(32.)$$

and

$$\left.\begin{aligned}
F\left(\frac{\delta V}{\delta\eta_1},\frac{\delta V}{\delta\eta_2},\ldots\frac{\delta V}{\delta\eta_{3n}},\eta_1,\eta_2,\ldots\eta_{3n}\right)&=H+U\left(\eta_1,\eta_2,\ldots\eta_{3n}\right),\\
F\left(\frac{\delta V}{\delta e_1},\frac{\delta V}{\delta e_2},\ldots\frac{\delta V}{\delta e_{3n}},e_1,e_2,\ldots e_{3n}\right)&=H+U\left(e_1,e_2,\ldots e_{3n}\right).
\end{aligned}\right\}\quad\ldots\quad(33.)$$

General Method of improving an approximate Expression for the Principal Function in any Problem of Dynamics.

7. If we separate the principal function S into any two parts,

$$S_1+S_2=S,\quad\ldots\ldots\quad(34.)$$

and substitute their sum for S in the first equation (C.), the function F, from its rational and integer and homogeneous form and dimension, may be expressed in this new way,

$$\left.\begin{aligned}
F\left(\frac{\delta S}{\delta\eta_1},\ldots\frac{\delta S}{\delta\eta_{3n}},\eta_1,\ldots\eta_{3n}\right)&=F\left(\frac{\delta S_1}{\delta\eta_1},\ldots\frac{\delta S_1}{\delta\eta_{3n}},\eta_1,\ldots\eta_{3n}\right)\\
&+F'\left(\frac{\delta S_1}{\delta\eta_1}\right)\frac{\delta S_2}{\delta\eta_1}+\ldots+F'\left(\frac{\delta S_1}{\delta\eta_{3n}}\right)\frac{\delta S_2}{\delta\eta_{3n}}+F\left(\frac{\delta S_2}{\delta\eta_1},\ldots\frac{\delta S_2}{\delta\eta_{3n}},\eta_1,\ldots\eta_{3n}\right)\\
&=F\left(\frac{\delta S_1}{\delta\eta_1},\ldots\frac{\delta S_1}{\delta\eta_{3n}},\eta_1,\ldots\eta_{3n}\right)-F\left(\frac{\delta S_2}{\delta\eta_1},\ldots\frac{\delta S_2}{\delta\eta_{3n}},\eta_1,\ldots\eta_{3n}\right)\\
&+F'\left(\frac{\delta S}{\delta\eta_1}\right)\frac{\delta S_2}{\delta\eta_1}+\ldots+F'\left(\frac{\delta S}{\delta\eta_{3n}}\right)\frac{\delta S_2}{\delta\eta_{3n}},
\end{aligned}\right\}.\quad(35.)$$

because

$$F'\left(\frac{\delta S_1}{\delta\eta_i}\right)=F'\left(\frac{\delta S}{\delta\eta_i}\right)-F'\left(\frac{\delta S_2}{\delta\eta_i}\right),\quad\ldots\ldots\quad(36.)$$

and

$$\Sigma\,.\,F'\left(\frac{\delta S_2}{\delta\eta}\right)\frac{\delta S_2}{\delta\eta}=2\,F\left(\frac{\delta S_2}{\delta\eta_1},\ldots\frac{\delta S_2}{\delta\eta_{3n}},\eta_1,\ldots\eta_{3n}\right);\quad\ldots\ldots\quad(37.)$$

and since, by (A.) and (B.),

$$F\left(\frac{\delta S}{\delta \eta_i}\right) = F'(\varpi_i) = \frac{\delta H}{\delta \varpi_i} = \frac{d\eta_i}{dt}, \quad \ldots \ldots \ldots \ldots \ldots \quad (38.)$$

we easily transform the first equation (C.) to the following,

$$\frac{dS_2}{dt} = -\frac{\delta S_1}{\delta t} + U(\eta_1, .. \eta_{3n}) - F\left(\frac{\delta S_1}{\delta \eta_1}, .. \frac{\delta S_1}{\delta \eta_{3n}}, \eta_1, .. \eta_{3n}\right) + F\left(\frac{\delta S_2}{\delta \eta_1}, .. \frac{\delta S_2}{\delta \eta_{3n}}, \eta_1, .. \eta_{3n}\right), \quad (D.)$$

which gives rigorously

$$\left.\begin{array}{l} S_2 = \int_0^t \left\{ -\frac{\delta S_1}{\delta t} + U(\eta_1, .. \eta_{3n}) - F\left(\frac{\delta S_1}{\delta \eta_1}, .. \frac{\delta S_1}{\delta \eta_{3n}}, \eta_1, .. \eta_{3n}\right) \right\} dt \\ \quad + \int_0^t F\left(\frac{\delta S_2}{\delta \eta_1}, .. \frac{\delta S_2}{\delta \eta_{3n}}, \eta_1, .. \eta_{3n}\right) dt, \end{array}\right\} \quad . \quad (E.)$$

supposing only that the two parts S_1, S_2, like the whole principal function S, are chosen so as to vanish with the time.

This general and rigorous transformation offers a general method of improving an approximate expression for the principal function S, in any problem of dynamics. For if the part S_1 be such an approximate expression, then the remaining part S_2 will be small; and the homogeneous function F involving the squares and products of the coefficients of this small part, in the second definite integral (E.), will be in general also small, and of a higher order of smallness; we may therefore in general neglect this second definite integral, in passing to a second approximation, and may in general improve a first approximate expression S_1 by adding to it the following correction,

$$\Delta S_1 = \int_0^t \left\{ -\frac{\delta S_1}{\delta t} + U(\eta_1, .. \eta_{3n}) - F\left(\frac{\delta S_1}{\delta \eta_1}, .. \frac{\delta S_1}{\delta \eta_{3n}}, \eta_1, .. \eta_{3n}\right) \right\} dt; \quad (F.)$$

in calculating which definite integral we may employ the following approximate forms for the integrals of the equations of motion,

$$p_1 = -\frac{\delta S_1}{\delta e_1}, p_2 = -\frac{\delta S_1}{\delta e_2}, .. p_{3n} = -\frac{\delta S_1}{\delta e_{3n}}, \quad \ldots \ldots \ldots \quad (39.)$$

expressing first, by these, the variables η_i as functions of the time and of the $6n$ constants $e_i\ p_i$, and then eliminating, after the integration, the $3n$ quantities p_i, by the same approximate forms. And when an improved expression, or second approximate value $S_1 + \Delta S_1$, for the principal function S, has been thus obtained, it may be substituted in like manner for the first approximate value S_1, so as to obtain a still closer approximation, and the process may be repeated indefinitely.

An analogous process applies to the indefinite improvement of a first approximate expression for the function Q or V.

Rigorous Theory of Perturbations, founded on the Properties of the Disturbing Part of the whole Principal Function.

8. If we separate the expression H (17.) into any two parts of the same kind,

$$H_1 + H_2 = H, \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (40.)$$

in which

$$H_1 = F_1 (\varpi_1, \varpi_2, \ldots \varpi_{3n}, \eta_1, \eta_2, \ldots \eta_{3n}) - U_1 (\eta_1, \eta_2, \ldots \eta_{3n}), \quad (41.)$$

and

$$H_2 = F_2 (\varpi_1, \varpi_2, \ldots \varpi_{3n}, \eta_1, \eta_2, \ldots \eta_{3n}) - U_2 (\eta_1, \eta_2, \ldots \eta_{3n}), \quad (42.)$$

the functions F_1 F_2 U_1 U_2 being such that

$$F_1 + F_2 = F, \; U_1 + U_2 = U; \quad (43.)$$

the differential equations of motion (A.) will take this form,

$$\frac{d \eta_i}{d t} = \frac{\delta H_1}{\delta \varpi_i} + \frac{\delta H_2}{\delta \varpi_i}, \quad \frac{d \varpi_i}{d t} = - \frac{\delta H_1}{\delta \eta_i} - \frac{\delta H_2}{\delta \eta_i}, \quad (G.)$$

and if the part H_2 and its coefficients be small, they will not differ much from these other differential equations,

$$\frac{d \eta_i}{d t} = \frac{\delta H_1}{\delta \varpi_i}, \quad \frac{d \varpi_i}{d t} = - \frac{\delta H_1}{\delta \eta_i}; \quad (H.)$$

so that the rigorous integrals of the latter system will be approximate integrals of the former. Whenever then, by a proper choice of the predominant term H_1, a system of $6n$ equations such as (H.) has been formed and rigorously integrated, giving expressions for the $6n$ variables η_i ϖ_i as functions of the time t, and of their own initial values e_i p_i, which may be thus denoted:

$$\eta_i = \varphi_i (t, e_1, e_2, \ldots e_{3n}, p_1, p_2, \ldots p_{3n}), \quad (44.)$$

and

$$\varpi_i = \psi_i (t, e_1, e_2, \ldots e_{3n}, p_1, p_2, \ldots p_{3n}); \quad (45.)$$

the simpler motion thus defined by the rigorous integrals of (H.) may be called the *undisturbed motion* of the proposed system of n points, and the more complex motion expressed by the rigorous integrals of (G.) may be called by contrast the *disturbed motion* of that system; and to pass from the one to the other, may be called a *Problem of Perturbation.*

9. To accomplish this passage, let us observe that the differential equations of undisturbed motion (H.), being of the same form as the original equations (A.), may have their integrals similarly expressed, that is, as follows:

$$\varpi_i = \frac{\delta S_1}{\delta \eta_i}, \quad p_i = - \frac{\delta S_1}{\delta e_i}, \quad (I.)$$

S_1 being here the *principal function of undisturbed motion*, or the definite integral

$$S_1 = \int_0^t \left(\Sigma \, . \, \varpi \frac{\delta H_1}{\delta \varpi} - H_1 \right) d t, \quad (46.)$$

considered as a function of the time and of the quantities η_i e_i. In like manner if we represent by $S_1 + S_2$ the whole principal function of disturbed motion, the rigorous integrals of (G.) may be expressed by (B.), as follows:

$$\varpi_i = \frac{\delta S_1}{\delta \eta_i} + \frac{\delta S_2}{\delta \eta_i}, \quad p_i = - \frac{\delta S_1}{\delta e_i} - \frac{\delta S_2}{\delta e_i}. \quad (K.)$$

Comparing the forms (44.) with the second set of equations (I.) for the integrals of undisturbed motion, we find that the following relations between the functions φ_i S_1 must be rigorously and *identically* true:

$$\eta_i = \varphi_i \left(t, e_1, e_2, \ldots e_{3n}, -\frac{\delta S_1}{\delta e_1}, -\frac{\delta S_1}{\delta e_2}, \ldots -\frac{\delta S_1}{\delta e_{3n}} \right); \quad \ldots \ldots \quad (47.)$$

and therefore, by (K.), that the integrals of disturbed motion may be put under the following forms,

$$\eta_i = \varphi_i \left(t, e_1, e_2, \ldots e_{3n}, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, \ldots p_{3n} + \frac{\delta S_2}{\delta e_{3n}} \right). \quad \ldots \quad (L.)$$

We may therefore calculate rigorously the disturbed variables η_i by the rules of undisturbed motion (44.), if without altering the time t, or the initial values e_i of those variables, which determine the initial configuration, we alter (in general) the initial velocities and directions, by adding to the elements p_i the following perturbational terms,

$$\Delta p_1 = \frac{\delta S_2}{\delta e_1}, \; \Delta p_2 = \frac{\delta S_2}{\delta e_2}, \ldots \; \Delta p_{3n} = \frac{\delta S_2}{\delta e_{3n}}: \quad \ldots \ldots \quad (M.)$$

a remarkable result, which includes the whole theory of perturbation. We might deduce from it the differential coefficients η'_i, or the connected quantities ϖ_i, which determine the disturbed directions and velocities of motion at any time t; but a similar reasoning gives at once the general expression,

$$\varpi_i = \frac{\delta S_2}{\delta \eta_i} + \psi_i \left(t, e_1, e_2, \ldots e_{3n}, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, \ldots p_{3n} + \frac{\delta S_2}{\delta e_{3n}} \right), \quad . \quad (N.)$$

implying, that after altering the initial velocities and directions or the elements p_i as before, by the perturbational terms (M.), we may then employ the rules of undisturbed motion (45.) to calculate the velocities and directions at the time t, or the varying quantities ϖ_i, if we finally apply to these quantities thus calculated the following new corrections for perturbation:

$$\Delta \varpi_1 = \frac{\delta S_2}{\delta \eta_1}, \; \Delta \varpi_2 = \frac{\delta S_2}{\delta \eta_2}, \; \ldots \Delta \varpi_{3n} = \frac{\delta S_2}{\delta \eta_{3n}}. \quad \ldots \ldots \quad (O.)$$

Approximate expressions deduced from the foregoing rigorous Theory.

10. The foregoing theory gives indeed rigorous expressions for the perturbations, in passing from the simpler motion (H.) or (I.) to the more complex motion (G.) or (K.): but it may seem that these expressions are of little use, because they involve an unknown *disturbing function* S_2, (namely, the perturbational part of the whole principal function S,) and also unknown or disturbed coordinates or marks of position η_i. However, it was lately shown that whenever a first approximate form for the principal function S, such as here the principal function S_1 of undisturbed motion, has been found, the correction S_2 can in general be assigned, with an indefinitely increasing

accuracy; and since the perturbations (M.) and (O.) involve the disturbed coordinates η_i only as they enter into the coefficients of this small disturbing function S_2, it is evidently permitted to substitute for these coordinates, at first, their undisturbed values, and then to correct the results by substituting more accurate expressions.

11. The function S_1 of undisturbed motion must satisfy rigorously two partial differential equations of the form (C.), namely,

$$\left.\begin{aligned} \frac{\delta S_1}{\delta t} + F_1\left(\frac{\delta S_1}{\delta \eta_1}, \ldots \frac{\delta S_1}{\delta \eta_{3n}}, \eta_1, \ldots \eta_{3n}\right) &= U_1(\eta_1, \ldots \eta_{3n}), \\ \frac{\delta S_1}{\delta t} + F_1\left(\frac{\delta S_1}{\delta e_1}, \ldots \frac{\delta S_1}{\delta e_{3n}}, e_1, \ldots e_{3n}\right) &= U_1(e_1, \ldots e_{3n}); \end{aligned}\right\} \quad . \quad . \quad \text{(P.)}$$

and therefore, by (D.), the disturbing function S_2 must satisfy rigorously the following other condition:

$$\frac{d S_2}{d t} = U_2(\eta_1, \ldots \eta_{3n}) - F_2\left(\frac{\delta S_1}{\delta \eta_1}, \ldots \frac{\delta S_1}{\delta \eta_{3n}}, \eta_1, \ldots \eta_{3n}\right) + F\left(\frac{\delta S_2}{\delta \eta_1}, \ldots \frac{\delta S_2}{\delta \eta_{3n}}, \eta_1, \ldots \eta_{3n}\right), \text{(Q.)}$$

and may, on account of the homogeneity and dimension of F, be approximately expressed as follows:

$$S_2 = \int_0^t \left\{ U_2(\eta_1, \ldots \eta_{3n}) - F_2\left(\frac{\delta S_1}{\delta \eta_1}, \ldots \frac{\delta S_1}{\delta \eta_{3n}}, \eta_1, \ldots \eta_{3n}\right)\right\} dt, \quad . \quad . \quad . \quad \text{(R.)}$$

or thus, by (I.),

$$S_2 = \int_0^t \left\{ U_2(\eta_1, \ldots \eta_{3n}) - F_2(\varpi_1, \ldots \varpi_{3n}, \eta_1, \ldots \eta_{3n})\right\} dt, \quad . \quad . \quad . \quad . \quad \text{(S.)}$$

that is, by (42.),

$$S_2 = -\int_0^t H_2\, dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(T.)}$$

In this expression, H_2 is given immediately as a function of the varying quantities $\eta_i\ \varpi_i$, but it may be considered in the same order of approximation as a known function of their initial values $e_i\ p_i$ and of the time t, obtained by substituting for $\eta_i\ \varpi_i$ their undisturbed values (44.) (45.) as functions of those quantities; its variation may therefore be expressed in either of the two following ways:

$$\delta H_2 = \Sigma\left(\frac{\delta H_2}{\delta \eta}\delta \eta + \frac{\delta H_2}{\delta \varpi}\delta \varpi\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(48.)}$$

or

$$\delta H_2 = \Sigma\left(\frac{\delta H_2}{\delta e}\delta e + \frac{\delta H_2}{\delta p}\delta p\right) + \frac{\delta H_2}{\delta t}\delta t. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(49.)}$$

Adopting the latter view, and effecting the integration (T.) with respect to the time, by treating the elements $e_i\ p_i$ as constant, we are afterwards to substitute for the quantities p_i their undisturbed expressions (39.) or (I.), and then we find for the variation of the disturbing function S_2 the expression

$$\delta S_2 = -H_2\,\delta t + \Sigma\left(-\delta e \,.\int_0^t \frac{\delta H_2}{\delta e}dt + \delta\frac{\delta S_1}{\delta e}\,.\int_0^t \frac{\delta H_2}{\delta p}dt\right), \quad . \quad \text{(50.)}$$

which enables us to transform the perturbational terms (M.) (O.) into the following approximate forms:

$$\Delta p_i = -\int_0^t \frac{\delta H_2}{\delta e_i} dt + \Sigma \, . \, \frac{\delta^2 S_1}{\delta e \, \delta e_i} \int_0^t \frac{\delta H_2}{\delta p} dt, \quad \ldots \ldots \quad (U.)$$

and

$$\Delta \varpi_i = \Sigma \, . \, \frac{\delta^2 S_1}{\delta e \, \delta \eta_i} \int_0^t \frac{\delta H_2}{\delta p} dt, \quad \ldots \ldots \quad (V.)$$

containing only functions and quantities which may be regarded as given, by the theory of undisturbed motion.

12. In the same order of approximation, if the variation of the expression (44.) for an undisturbed coordinate η_i be thus denoted,

$$\delta \eta_i = \frac{\delta \eta_i}{\delta t} \delta t + \Sigma \left(\frac{\delta \eta_i}{\delta e} \delta e + \frac{\delta \eta_i}{\delta p} \delta p \right), \quad \ldots \ldots \quad (51.)$$

the perturbation of that coordinate may be expressed as follows:

$$\Delta \eta_i = \Sigma \, . \, \frac{\delta \eta_i}{\delta p} \Delta p ; \quad \ldots \ldots \quad (W.)$$

that is, by (U.),

$$\left. \begin{aligned} \Delta \eta_i = & - \frac{\delta \eta_i}{\delta p_1} \int_0^t \frac{\delta H_2}{\delta e_1} dt - \frac{\delta \eta_i}{\delta p_2} \int_0^t \frac{\delta H_2}{\delta e_2} dt - \ldots - \frac{\delta \eta_i}{\delta p_{3n}} \int_0^t \frac{\delta H_2}{\delta e_{3n}} dt \\ & + \left(\frac{\delta \eta_i}{\delta p_1} \frac{\delta^2 S_1}{\delta e_1^2} + \frac{\delta \eta_i}{\delta p_2} \frac{\delta^2 S_1}{\delta e_1 \, \delta e_2} + \ldots + \frac{\delta \eta_i}{\delta p_{3n}} \frac{\delta^2 S_1}{\delta e_1 \, \delta e_{3n}} \right) \int_0^t \frac{\delta H_2}{\delta p_1} dt \\ & + \ldots\ldots \\ & + \left(\frac{\delta \eta_i}{\delta p_1} \frac{\delta^2 S_1}{\delta e_{3n} \, \delta e_1} + \frac{\delta \eta_i}{\delta p_2} \frac{\delta^2 S_1}{\delta e_{3n} \, \delta e_2} + \ldots + \frac{\delta \eta_i}{\delta p_{3n}} \frac{\delta^2 S_1}{\delta e_{3n}^2} \right) \int_0^t \frac{\delta H_2}{\delta p_{3n}} dt. \end{aligned} \right\} \quad . \ (52.)$$

Besides, the identical equation (47.) gives

$$\frac{\delta \eta_i}{\delta e_k} = \frac{\delta \eta_i}{\delta p_1} \frac{\delta^2 S_1}{\delta e_k \, \delta e_1} + \frac{\delta \eta_i}{\delta p_2} \frac{\delta^2 S_1}{\delta e_k \, \delta e_2} + \ldots + \frac{\delta \eta_i}{\delta p_{3n}} \frac{\delta^2 S_1}{\delta e_k \, \delta e_{3n}} ; \quad \ldots \ldots \quad (53.)$$

the expression (52.) may therefore be thus abridged,

$$\left. \begin{aligned} \Delta \eta_i = & - \frac{\delta \eta_i}{\delta p_1} \int_0^t \frac{\delta H_2}{\delta e_1} dt - \ldots - \frac{\delta \eta_i}{\delta p_{3n}} \int_0^t \frac{\delta H_2}{\delta e_{3n}} dt \\ & + \frac{\delta \eta_i}{\delta e_1} \int_0^t \frac{\delta H_2}{\delta p_1} dt + \ldots + \frac{\delta \eta_i}{\delta e_{3n}} \int_0^t \frac{\delta H_2}{\delta p_{3n}} dt, \end{aligned} \right\} \quad \ldots \quad (X.)$$

and shows that instead of the rigorous perturbational terms (M.) we may approximately employ the following,

$$\Delta p_i = - \int_0^t \frac{\delta H_2}{\delta e_i} dt, \quad \ldots \ldots \quad (Y.)$$

in order to calculate the disturbed configuration at any time t by the rules of undis-

turbed motion, provided that besides thus altering the initial velocities and directions we alter also the initial configuration, by the formula

$$\Delta e_i = \int_0^t \frac{\delta H_2}{\delta p_i} d t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(Z.)}$$

It would not be difficult to calculate, in like manner, approximate expressions for the disturbed directions and velocities at any time t; but it is better to resume, in another way, the rigorous problem of perturbation.

Other Rigorous Theory of Perturbation, founded on the properties of the disturbing part of the constant of living force, and giving formulæ for the Variation of Elements more analogous to those already known.

13. Suppose that the theory of undisturbed motion has given the $6n$ constants $e_i\ p_i$ or any combinations of these, $\varkappa_1, \varkappa_2, \ldots \varkappa_{6n}$, as functions of the $6n$ variables $\eta_i\ \varpi_i$ and of the time t, which may be thus denoted:

$$\varkappa_i = \chi_i (t, \eta_1, \eta_2, \ldots \eta_{3n}, \varpi_1, \varpi_2, \ldots \varpi_{3n}), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54.)$$

and which give reciprocally expressions for the variables $\eta_i\ \varpi_i$ in terms of these elements and of the time, analogous to (44.) and (45.), and capable of being denoted similarly,

$$\eta_i = \varphi_i (t, \varkappa_1, \varkappa_2, \ldots \varkappa_{6n}), \quad \varpi_i = \psi_i (t, \varkappa_1, \varkappa_2, \ldots \varkappa_{6n}); \quad . \quad . \quad . \quad . \quad (55.)$$

then, the total differential coefficient of every such *element* or function $\varkappa_i$, taken with respect to the time, (both as it enters explicitly and implicitly into the expressions (54.),) must vanish in the undisturbed motion; so that, by the differential equations of such motion (H.), the following general relation must be rigorously and *identically* true:

$$0 = \frac{\delta \varkappa_i}{\delta t} + \Sigma \left(\frac{\delta \varkappa_i}{\delta \eta} \frac{\delta H_1}{\delta \varpi} - \frac{\delta \varkappa_i}{\delta \varpi} \frac{\delta H_1}{\delta \eta} \right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56.)$$

In passing to disturbed motion, if we retain the equation (54.) as a *definition* of the quantity $\varkappa_i$, that quantity will no longer be constant, but it will continue to satisfy the inverse relations (55.), and may be called, by analogy, a *varying element* of the motion; and its total differential coefficient, taken with respect to the time, may, by the identical equation (56.), and by the differential equations of disturbed motion (G.), be rigorously expressed as follows:

$$\frac{d \varkappa_i}{d t} = \Sigma \left(\frac{\delta \varkappa_i}{\delta \eta} \frac{\delta H_2}{\delta \varpi} - \frac{\delta \varkappa_i}{\delta \varpi} \frac{\delta H_2}{\delta \eta} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A^1.)$$

14. This result (A^1.) contains the whole theory of the gradual variation of the elements of disturbed motion of a system; but it may receive an advantageous transformation, by the substitution of the expressions (55.) for the variables $\eta_i\ \varpi_i$ as functions of the time and of the elements; since it will thus conduct to a system of $6n$

rigorous and ordinary differential equations of the first order between those varying elements and the time. Expressing, therefore, the quantity H_2 as a function of these latter variables, its variation δH_2 takes this new form,

$$\delta H_2 = \Sigma \, . \frac{\delta H_2}{\delta \varkappa} \delta \varkappa + \frac{\delta H_2}{\delta t} \delta t, \quad \ldots \ldots \ldots \quad (57.)$$

and gives, by comparison with the form (48.), and by (54.),

$$\frac{\delta H_2}{\delta \eta_r} = \Sigma \, . \frac{\delta H_2}{\delta \varkappa} \frac{\delta \varkappa}{\delta \eta_r}; \quad \frac{\delta H_2}{\delta \varpi_r} = \Sigma \, . \frac{\delta H_2}{\delta \varkappa} \frac{\delta \varkappa}{\delta \varpi_r}; \quad \ldots \ldots \quad (58.)$$

and thus the general equation (A^1.) is transformed to the following,

$$\frac{d \varkappa_i}{d t} = a_{i,1} \frac{\delta H_2}{\delta \varkappa_1} + a_{i,2} \frac{\delta H_2}{\delta \varkappa_2} + \ldots + a_{i,6n} \frac{\delta H_2}{\delta \varkappa_{6n}}; \quad \ldots \ldots \quad (B^1.)$$

in which

$$a_{i,s} = \Sigma \left(\frac{\delta \varkappa_i}{\delta \eta} \frac{\delta \varkappa_s}{\delta \varpi} - \frac{\delta \varkappa_i}{\delta \varpi} \frac{\delta \varkappa_s}{\delta \eta} \right) : \quad \ldots \ldots \ldots \quad (C^1.)$$

so that it only remains to eliminate the variables η ϖ from the expressions of these latter coefficients. Now it is remarkable that this elimination removes the symbol t also, and leaves the coefficients $a_{i,s}$ expressed as functions of the elements $\varkappa$ alone, not explicitly involving the time. This general theorem of dynamics, which is, perhaps, a little more extensive than the analogous results discovered by LAGRANGE and by POISSON, since it does not limit the disturbing terms in the differential equations of motion to depend on the configuration only, may be investigated in the following way.

15. The sign of summation Σ in (C^1.), like the same sign in those other analogous equations in which it has already occurred without an index in this Essay, refers not to the expressed indices, such as here i, s, in the quantity to be summed, but to an index which is not expressed, and which may be here called r; so that if we introduce for greater clearness this variable index and its limits, the expression (C^1.) becomes

$$a_{i,s} = \Sigma_{(r)\,1}^{\;3n} \left(\frac{\delta \varkappa_i}{\delta \eta_r} \frac{\delta \varkappa_s}{\delta \varpi_r} - \frac{\delta \varkappa_i}{\delta \varpi_r} \frac{\delta \varkappa_s}{\delta \eta_r} \right) : \quad \ldots \ldots \ldots \quad (59.)$$

and its total differential coefficient, taken with respect to the time, may be separated into the two following parts,

$$\left.\begin{aligned} \frac{d}{dt} a_{i,s} = {} & \Sigma_{(r)\,1}^{\;3n} \left(\frac{\delta \varkappa_i}{\delta \eta_r} \frac{d}{dt} \frac{\delta \varkappa_s}{\delta \varpi_r} - \frac{\delta \varkappa_s}{\delta \eta_r} \frac{d}{dt} \frac{\delta \varkappa_i}{\delta \varpi_r} \right) \\ & + \Sigma_{(r)\,1}^{\;3n} \left(\frac{\delta \varkappa_s}{\delta \varpi_r} \frac{d}{dt} \frac{\delta \varkappa_i}{\delta \eta_r} - \frac{\delta \varkappa_i}{\delta \varpi_r} \frac{d}{dt} \frac{\delta \varkappa_s}{\delta \eta_r} \right), \end{aligned}\right\} \quad \ldots \ldots \quad (60.)$$

which we shall proceed to calculate separately, and then to add them together. By the definition (54.), and the differential equations of disturbed motion (G.),

$$\frac{d}{dt} \frac{\delta \varkappa_i}{\delta \varpi_r} = \frac{\delta^2 \varkappa_i}{\delta t \, \delta \varpi_r} + \Sigma_{(u)\,1}^{\;3n} \left\{ \frac{\delta^2 \varkappa_i}{\delta \eta_u \, \delta \varpi_r} \left(\frac{\delta H_1}{\delta \varpi_u} + \frac{\delta H_2}{\delta \varpi_u} \right) - \frac{\delta^2 \varkappa_i}{\delta \varpi_u \, \delta \varpi_r} \left(\frac{\delta H_1}{\delta \eta_u} + \frac{\delta H_2}{\delta \eta_u} \right) \right\}, \quad (61.)$$

in which, by the identical equation (56.),

$$\frac{\delta^2 \varkappa_i}{\delta t\, \delta \varpi_r} = -\frac{\delta}{\delta \varpi_r} \Sigma_{(u)\,1}^{3n} \left(\frac{\delta \varkappa_i}{\delta \eta_u} \frac{\delta \mathrm{H}_1}{\delta \varpi_u} - \frac{\delta \varkappa_i}{\delta \varpi_u} \frac{\delta \mathrm{H}_1}{\delta \eta_u} \right); \quad \dots \quad (62.)$$

we have therefore

$$\frac{d}{dt} \frac{\delta \varkappa_i}{\delta \varpi_r} = \Sigma_{(u)\,1}^{3n} \left(\frac{\delta^2 \varkappa_i}{\delta \eta_u \delta \varpi_r} \frac{\delta \mathrm{H}_2}{\delta \varpi_u} - \frac{\delta^2 \varkappa_i}{\delta \varpi_u \delta \varpi_r} \frac{\delta \mathrm{H}_2}{\delta \eta_u} + \frac{\delta \varkappa_i}{\delta \varpi_u} \frac{\delta^2 \mathrm{H}_1}{\delta \eta_u \delta \varpi_r} - \frac{\delta \varkappa_i}{\delta \eta_u} \frac{\delta^2 \mathrm{H}_1}{\delta \varpi_u \delta \varpi_r} \right), \quad (63.)$$

and $\frac{d}{dt} \frac{\delta \varkappa_s}{\delta \varpi_r}$ may be found from this, by merely changing i to s: so that

$$\left.\begin{aligned}
&\Sigma_{(r)\,1}^{3n} \left(\frac{\delta \varkappa_i}{\delta \eta_r} \frac{d}{dt} \frac{\delta \varkappa_s}{\delta \varpi_r} - \frac{\delta \varkappa_s}{\delta \eta_r} \frac{d}{dt} \frac{\delta \varkappa_i}{\delta \varpi_r} \right) = \\
&\Sigma_{(r,u)\,1,1}^{3n,3n} \left\{ \left(\frac{\delta \varkappa_s}{\delta \eta_r} \frac{\delta^2 \varkappa_i}{\delta \varpi_u \delta \varpi_r} - \frac{\delta \varkappa_i}{\delta \eta_r} \frac{\delta^2 \varkappa_s}{\delta \varpi_u \delta \varpi_r} \right) \frac{\delta \mathrm{H}_2}{\delta \eta_u} + \left(\frac{\delta \varkappa_i}{\delta \eta_r} \frac{\delta^2 \varkappa_s}{\delta \eta_u \delta \varpi_r} - \frac{\delta \varkappa_s}{\delta \eta_r} \frac{\delta^2 \varkappa_i}{\delta \eta_u \delta \varpi_r} \right) \frac{\delta \mathrm{H}_2}{\delta \varpi_u} \right. \\
&\qquad \left. + \left(\frac{\delta \varkappa_i}{\delta \eta_r} \frac{\delta \varkappa_s}{\delta \varpi_u} - \frac{\delta \varkappa_s}{\delta \eta_r} \frac{\delta \varkappa_i}{\delta \varpi_u} \right) \frac{\delta^2 \mathrm{H}_1}{\delta \eta_u \delta \varpi_r} + \left(\frac{\delta \varkappa_s}{\delta \eta_r} \frac{\delta \varkappa_i}{\delta \eta_u} - \frac{\delta \varkappa_i}{\delta \eta_r} \frac{\delta \varkappa_s}{\delta \eta_u} \right) \frac{\delta^2 \mathrm{H}_1}{\delta \varpi_u \delta \varpi_r} \right\},
\end{aligned}\right\} (64.)$$

and similarly,

$$\left.\begin{aligned}
&\Sigma_{(r)\,1}^{3n} \left(\frac{\delta \varkappa_s}{\delta \varpi_r} \frac{d}{dt} \frac{\delta \varkappa_i}{\delta \eta_r} - \frac{\delta \varkappa_i}{\delta \varpi_r} \frac{d}{dt} \frac{\delta \varkappa_s}{\delta \eta_r} \right) = \\
&\Sigma_{(r,u)\,1,1}^{3n,3n} \left\{ \left(\frac{\delta \varkappa_s}{\delta \varpi_r} \frac{\delta^2 \varkappa_i}{\delta \eta_u \delta \eta_r} - \frac{\delta \varkappa_i}{\delta \varpi_r} \frac{\delta^2 \varkappa_s}{\delta \eta_u \delta \eta_r} \right) \frac{\delta \mathrm{H}_2}{\delta \varpi_u} + \left(\frac{\delta \varkappa_i}{\delta \varpi_r} \frac{\delta^2 \varkappa_s}{\delta \varpi_u \delta \eta_r} - \frac{\delta \varkappa_s}{\delta \varpi_r} \frac{\delta^2 \varkappa_i}{\delta \varpi_u \delta \eta_r} \right) \frac{\delta \mathrm{H}_2}{\delta \eta_u} \right. \\
&\qquad \left. + \left(\frac{\delta \varkappa_i}{\delta \varpi_r} \frac{\delta \varkappa_s}{\delta \eta_u} - \frac{\delta \varkappa_s}{\delta \varpi_r} \frac{\delta \varkappa_i}{\delta \eta_u} \right) \frac{\delta^2 \mathrm{H}_1}{\delta \varpi_u \delta \eta_r} + \left(\frac{\delta \varkappa_s}{\delta \varpi_r} \frac{\delta \varkappa_i}{\delta \varpi_u} - \frac{\delta \varkappa_i}{\delta \varpi_r} \frac{\delta \varkappa_s}{\delta \varpi_u} \right) \frac{\delta^2 \mathrm{H}_1}{\delta \eta_u \delta \eta_r} \right\}.
\end{aligned}\right\} (65.)$$

Adding, therefore, the two last expressions, and making the reductions which present themselves, we find, by (60.),

$$\frac{d}{dt} a_{i,s} = \Sigma_{(u)\,1}^{3n} \left(\mathrm{A}_{i,s}^{(u)} \frac{\delta \mathrm{H}_2}{\delta \eta_u} + \mathrm{B}_{i,s}^{(u)} \frac{\delta \mathrm{H}_2}{\delta \varpi_u} \right), \quad \dots \dots \quad (\mathrm{D}^1.)$$

in which

$$\left.\begin{aligned}
\mathrm{A}_{i,s}^{(u)} &= \Sigma_{(r)\,1}^{3n} \left(\frac{\delta \varkappa_s}{\delta \eta_r} \frac{\delta^2 \varkappa_i}{\delta \varpi_u \delta \varpi_r} - \frac{\delta \varkappa_i}{\delta \eta_r} \frac{\delta^2 \varkappa_s}{\delta \varpi_u \delta \varpi_r} + \frac{\delta \varkappa_i}{\delta \varpi_r} \frac{\delta^2 \varkappa_s}{\delta \varpi_u \delta \eta_r} - \frac{\delta \varkappa_s}{\delta \varpi_r} \frac{\delta^2 \varkappa_i}{\delta \varpi_u \delta \eta_r} \right), \\
\mathrm{B}_{i,s}^{(u)} &= \Sigma_{(r)\,1}^{3n} \left(\frac{\delta \varkappa_s}{\delta \varpi_r} \frac{\delta^2 \varkappa_i}{\delta \eta_u \delta \eta_r} - \frac{\delta \varkappa_i}{\delta \varpi_r} \frac{\delta^2 \varkappa_s}{\delta \eta_u \delta \eta_r} + \frac{\delta \varkappa_i}{\delta \eta_r} \frac{\delta^2 \varkappa_s}{\delta \eta_u \delta \varpi_r} - \frac{\delta \varkappa_s}{\delta \eta_r} \frac{\delta^2 \varkappa_i}{\delta \eta_u \delta \varpi_r} \right);
\end{aligned}\right\} \quad . \quad (66.)$$

and since this general form (D^1.) for $\frac{d}{dt} a_{i,s}$ contains no term independent of the disturbing quantities $\frac{\delta \mathrm{H}_2}{\delta \eta}$, $\frac{\delta \mathrm{H}_2}{\delta \varpi}$, it is easy to infer from it the important consequence already mentioned, namely, that the coefficients $a_{i,s}$, in the differentials (B^1.) of the elements, may be expressed as functions of those elements alone, not explicitly involving the time.

It is evident also, that these coefficients $a_{i,s}$ have the property

$$a_{s,i} = -a_{i,s}, \quad \ldots \quad (67.)$$

and

$$a_{i,i} = 0; \quad \ldots \quad (68.)$$

the term proportional to $\frac{\delta H_2}{\delta \varkappa_i}$ disappears therefore from the expression (B[1].) for $\frac{d \varkappa_i}{d t}$; and the term

$$\frac{\delta H_2}{\delta \varkappa_i} \cdot a_{i,s} \cdot \frac{\delta H_2}{\delta \varkappa_s} \text{ in } \frac{\delta H_2}{\delta \varkappa_i} \frac{d \varkappa_i}{d t},$$

destroys the term

$$\frac{\delta H_2}{\delta \varkappa_s} \cdot a_{s,i} \cdot \frac{\delta H_2}{\delta \varkappa_i} \text{ in } \frac{\delta H_2}{\delta \varkappa_s} \frac{d \varkappa_s}{d t},$$

when these terms are added together; we have, therefore,

$$\Sigma \cdot \frac{\delta H_2}{\delta \varkappa} \frac{d \varkappa}{d t} = 0, \quad \ldots \quad (E^1.)$$

or

$$\frac{d H_2}{d t} = \frac{\delta H_2}{\delta t}; \quad \ldots \quad (F^1.)$$

that is, in taking the first total differential coefficient of the disturbing expression H_2 with respect to the time, the elements may be treated as constant.

Simplification of the differential equations which determine these gradually varying elements, in any problem of Perturbation; and Integration of the simplified equations by means of certain Functions of Elements.

16. The most natural choice of these elements is that which makes them correspond, in undisturbed motion, to the initial quantities $e_i\ p_i$. These quantities, by the differential equations (H.), may be expressed in undisturbed motion as follows,

$$e_i = \eta_i - \int_0^t \frac{\delta H_1}{\delta \varpi_i} d t, \quad p_i = \varpi_i + \int_0^t \frac{\delta H_1}{\delta \eta_i} d t; \quad \ldots \quad (69.)$$

and if we suppose them found, by elimination, under the forms

$$\left.\begin{aligned} e_i &= \eta_i + \Phi_i (t, \eta_1, \eta_2, \ldots \eta_{3n}, \varpi_1, \varpi_2, \ldots \varpi_{3n}), \\ p_i &= \varpi_i + \Psi_i (t, \eta_1, \eta_2, \ldots \eta_{3n}, \varpi_1, \varpi_2, \ldots \varpi_{3n}), \end{aligned}\right\} \quad \ldots \quad (70.)$$

it is easy to see that the following equations must be rigorously and identically true, for all values of $\eta_i\ \varpi_i$,

$$\left.\begin{aligned} 0 &= \Phi_i (0, \eta_1, \eta_2, \ldots \eta_{3n}, \varpi_1, \varpi_2, \ldots \varpi_{3n}), \\ 0 &= \Psi_i (0, \eta_1, \eta_2, \ldots \eta_{3n}, \varpi_1, \varpi_2, \ldots \varpi_{3n}). \end{aligned}\right\} \quad \ldots \quad (71.)$$

When, therefore, in passing to disturbed motion, we establish the equations of definition,

$$\left.\begin{aligned} \varkappa_i &= \eta_i + \Phi_i\,(t, \eta_1, \eta_2, \ldots \eta_{3n}, \varpi_1, \varpi_2, \ldots \varpi_{3n}), \\ \lambda_i &= \varpi_i + \Psi_i\,(t, \eta_1, \eta_2, \ldots \eta_{3n}, \varpi_1, \varpi_2, \ldots \varpi_{3n}), \end{aligned}\right\} \quad \ldots \ldots \quad (72.)$$

introducing $6n$ varying elements $\varkappa_i\,\lambda_i$, of which the set λ_i would have been represented in our recent notation as follows:

$$\lambda_i = \varkappa_{3n+i}; \quad \ldots \ldots \quad (73.)$$

we see that all the partial differential coefficients of the forms $\frac{\delta \varkappa_i}{\delta \eta_r}, \frac{\delta \varkappa_i}{\delta \varpi_r}, \frac{\delta \lambda_i}{\delta \eta_r}, \frac{\delta \lambda_i}{\delta \varpi_r}$, vanish when $t = 0$, except the following:

$$\frac{\delta \varkappa_i}{\delta \eta_i} = 1, \quad \frac{\delta \lambda_i}{\delta \varpi_i} = 1; \quad \ldots \ldots \quad (74.)$$

and, therefore, that when t is made $= 0$, in the coefficients $a_{i,s}$, (59.), all those coefficients vanish, except the following:

$$a_{r,3n+r} = 1, \quad a_{3n+r,r} = -1. \quad \ldots \ldots \quad (75.)$$

But it has been proved that these coefficients $a_{i,s}$, when expressed as functions of the elements, do not contain the time explicitly; and the supposition $t = 0$ introduces no relation between those $6n$ elements $\varkappa_i\,\lambda_i$, which still remain independent: the coefficients $a_{i,s}$, therefore, could not acquire the values $1, 0, -1$, by the supposition $t = 0$, unless they had those values constantly, and independently of that supposition. The differential equations of the forms (B^1.), may therefore be expressed, for the present system of varying elements, in the following simpler way:

$$\frac{d\varkappa_i}{dt} = \frac{\delta H_2}{\delta \lambda_i}; \quad \frac{d\lambda_i}{dt} = -\frac{\delta H_2}{\delta \varkappa_i}; \quad \ldots \ldots \quad (G^1.)$$

and an easy verification of these expressions is offered by the formula (E^1.), which takes now this form,

$$\Sigma\left(\frac{\delta H_2}{\delta \varkappa}\frac{d\varkappa}{dt} + \frac{\delta H_2}{\delta \lambda}\frac{d\lambda}{dt}\right) = 0. \quad \ldots \ldots \quad (H^1.)$$

17. The initial values of the varying elements $\varkappa_i\,\lambda_i$ are evidently $e_i\,p_i$, by the definitions (72.), and by the identical equations (71.); the problem of integrating rigorously the equations of disturbed motion (G.), between the variables $\eta_i\,\varpi_i$ and the time, or of determining these variables as functions of the time and of their own initial values $e_i\,p_i$, is therefore rigorously transformed into the problem of integrating the equations (G^1.), or of determining the $6n$ elements $\varkappa_i\,\lambda_i$ as functions of the time and of the same initial values. The chief advantage of this transformation is, that if the perturbations be small, the new variables (namely, the elements,) alter but little: and that, since the new differential equations are of the same form as the old, they may be integrated by a similar method. Considering, therefore, the definite integral

$$E = \int_0^t \left(\Sigma\,.\,\lambda\frac{\delta H_2}{\delta \lambda} - H_2\right) dt, \quad \ldots \ldots \quad (76.)$$

as a function of the time and of the $6n$ quantities $\varkappa_1, \varkappa_2, \ldots \varkappa_{3n}, e_1, e_2, \ldots e_{3n}$, and observing that its variation, taken with respect to the latter quantities, may be shown by a process similar to that of the fourth number of this Essay to be

$$\delta \mathrm{E} = \Sigma (\lambda \delta \varkappa - p \delta e), \quad \ldots \quad (\mathrm{I}^1.)$$

we find that the rigorous integrals of the differential equations (G^1.) may be expressed in the following manner:

$$\lambda_i = \frac{\delta \mathrm{E}}{\delta \varkappa_i}, \quad p_i = - \frac{\delta \mathrm{E}}{\delta e_i}, \quad \ldots \quad (\mathrm{K}^1.)$$

in which there enters only one unknown *function of elements* E, to the search and study of which single function the problem of perturbation is reduced by this new method.

We might also have put

$$\mathrm{C} = \int_0^t \left(- \Sigma . \varkappa \frac{\delta \mathrm{H}_2}{\delta \varkappa} + \mathrm{H}_2 \right) dt, \quad \ldots \quad (77.)$$

and have considered this definite integral C as a function of the time and of the $6n$ quantities $\lambda_i\, p_i$; and then we should have found the following other forms for the integrals of the differential equations of varying elements,

$$\varkappa_i = + \frac{\delta \mathrm{C}}{\delta \lambda_i}, \quad e_i = - \frac{\delta \mathrm{C}}{\delta p_i}. \quad \ldots \quad (\mathrm{L}^1.)$$

And each of these *functions of elements*, C and E, must satisfy a certain partial differential equation, analogous to the first equation of each pair mentioned in the sixth number of this Essay, and deduced on similar principles.

18. Thus, it is evident, by the form of the function E, and by the equations (K^1.), (G^1.), and (76.), that the partial differential coefficient of this function, taken with respect to the time, is

$$\frac{\delta \mathrm{E}}{\delta t} = \frac{d \mathrm{E}}{d t} - \Sigma . \frac{\delta \mathrm{E}}{\delta \varkappa} \frac{d \varkappa}{d t} = - \mathrm{H}_2; \quad \ldots \quad (\mathrm{M}^1.)$$

and therefore that if we separate this function E into any two parts

$$\mathrm{E}_1 + \mathrm{E}_2 = \mathrm{E}, \quad \ldots \quad (\mathrm{N}^1.)$$

and if, for greater clearness, we put the expression H_2 under the form

$$\mathrm{H}_2 = \mathrm{H}_2 (t, \varkappa_1, \varkappa_2, \ldots \varkappa_{3n}, \lambda_1, \lambda_2, \ldots \lambda_{3n}), \quad \ldots \quad (\mathrm{O}^1.)$$

we shall have rigorously the partial differential equation

$$0 = \frac{\delta \mathrm{E}_1}{\delta t} + \frac{\delta \mathrm{E}_2}{\delta t} + \mathrm{H}_2 \left(t, \varkappa_1, \ldots \varkappa_{3n}, \frac{\delta \mathrm{E}_1}{\delta \varkappa_1} + \frac{\delta \mathrm{E}_2}{\delta \varkappa_1}, \ldots \frac{\delta \mathrm{E}_1}{\delta \varkappa_{3n}} + \frac{\delta \mathrm{E}_2}{\delta \varkappa_{3n}} \right) : \quad (\mathrm{P}^1.)$$

which gives, approximately, by (G^1.) and (K^1.), when the part E_2 is small, and when we neglect the squares and products of its partial differential coefficients,

$$0 = \frac{d \mathrm{E}_2}{d t} + \frac{\delta \mathrm{E}_1}{\delta t} + \mathrm{H}_2 \left(t, \varkappa_1, \ldots \varkappa_{3n}, \frac{\delta \mathrm{E}_1}{\delta \varkappa_1}, \ldots \frac{\delta \mathrm{E}_1}{\delta \varkappa_{3n}} \right). \quad \ldots \quad (\mathrm{Q}^1.)$$

Hence, in the same order of approximation, if the part E_1, like the whole function E, be chosen so as to vanish with the time, we shall have

$$E_2 = -\int_0^t \left\{ \frac{\delta E_1}{\delta t} + H_2\left(t, x_1, \ldots x_{3n}, \frac{\delta E_1}{\delta x_1}, \ldots \frac{\delta E_1}{\delta x_{3n}}\right) \right\} : \quad . \quad . \quad . \quad . \quad (R^1.)$$

and thus a first approximate expression E_1 can be successively and indefinitely corrected.

Again, by (L^1.) and (G^1.), and by the definition (77.),

$$\frac{\delta C}{\delta t} = \frac{d C}{d t} - \Sigma . \frac{\delta C}{\delta \lambda} \frac{d \lambda}{d t} = H_2 ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (S^1.)$$

the function C must therefore satisfy rigorously the partial differential equation,

$$\frac{\delta C}{\delta t} = H_2\left(t, \frac{\delta C}{\delta \lambda_1}, \ldots \frac{\delta C}{\delta \lambda_{3n}}, \lambda_1, \ldots \lambda_{3n}\right) : \quad . \quad . \quad . \quad . \quad . \quad . \quad (T^1.)$$

and if we put

$$C = C_1 + C_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (U^1.)$$

and suppose that the part C_2 is small, then the rigorous equation

$$\frac{\delta C_1}{\delta t} + \frac{\delta C_2}{\delta t} = H_2\left(t, \frac{\delta C_1}{\delta \lambda_1} + \frac{\delta C_2}{\delta \lambda_1}, \ldots \frac{\delta C_1}{\delta \lambda_{3n}} + \frac{\delta C_2}{\delta \lambda_{3n}}, \lambda_1, \ldots \lambda_{3n}\right) \quad . \quad . \quad . \quad . \quad (V^1.)$$

becomes approximately, by (G^1.) and (L^1.),

$$\frac{d C_2}{d t} = -\frac{\delta C_1}{\delta t} + H_2\left(t, \frac{\delta C_1}{\delta \lambda_1}, \ldots \frac{\delta C_1}{\delta \lambda_{3n}}, \lambda_1, \ldots \lambda_{3n}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (W^1.)$$

and gives by integration,

$$C_2 = \int_0^t \left\{ -\frac{\delta C_1}{\delta t} + H_2\left(t, \frac{\delta C_1}{\delta \lambda_1}, \ldots \frac{\delta C_1}{\delta \lambda_{3n}}, \lambda_1, \ldots \lambda_{3n}\right) \right\} d t, \quad . \quad . \quad . \quad . \quad (X^1.)$$

the parts C_1 and C_2 being supposed to vanish separately when $t = 0$, like the whole function of elements C.

And to obtain such a first approximation, E_1 or C_1, to either of these two functions of elements E, C, we may change, in the definitions (76.) (77.), the varying elements x, λ, to their initial values e, p, and then eliminate one set of these initial values by the corresponding set of the following approximate equations, deduced from the formulæ (G^1.):

$$x_i = e_i + \int_0^t \frac{\delta H_2}{\delta p_i} d t ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (Y^1.)$$

and

$$\lambda_i = p_i - \int_0^t \frac{\delta H_2}{\delta e_i} d t . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (Z^1.)$$

It is easy also to see that these two functions of elements C and E are connected with each other, and with the disturbing function S_2, so that the form of any one may be deduced from that of any other, when the function S_1 of undisturbed motion is known.

Analogous formulæ for the motion of a Single Point.

19. Our general method in dynamics, though intended chiefly for the study of attracting and repelling systems, is not confined to such, but may be used in all questions to which the law of living forces applies. And all the analysis of this Essay, but especially the theory of perturbations, may usefully be illustrated by the following analogous reasonings and results respecting the motion of a single point.

Imagine then such a point, having for its three rectangular coordinates $x\ y\ z$, and moving in an orbit determined by three ordinary differential equations of the second order of forms analogous to the equations (2.), namely,

$$x'' = \frac{\delta U}{\delta x};\ y'' = \frac{\delta U}{\delta y};\ z'' = \frac{\delta U}{\delta z}; \quad \ldots \quad (78.)$$

U being any given function of the coordinates not expressly involving the time: and let us establish the following definition, analogous to (4.),

$$T = \frac{1}{2}\,(x'^2 + y'^2 + z'^2), \quad \ldots \quad (79.)$$

$x'\ y'\ z'$ being the first, and $x''\ y''\ z''$ being the second differential coefficients of the coordinates, considered as functions of the time t. If we express, for greater generality or facility, the rectangular coordinates $x\ y\ z$ as functions of three other marks of position $\eta_1\ \eta_2\ \eta_3$, T will become a homogeneous function of the second dimension of their first differential coefficients $\eta'_1\ \eta'_2\ \eta'_3$ taken with respect to the time; and if we put, for abridgement,

$$\varpi_1 = \frac{\delta T}{\delta \eta'_1},\ \varpi_2 = \frac{\delta T}{\delta \eta'_2},\ \varpi_3 = \frac{\delta T}{\delta \eta'_3}, \quad \ldots \quad (80.)$$

T may be considered also as a function of the form

$$T = F\,(\varpi_1, \varpi_2, \varpi_3, \eta_1, \eta_2, \eta_3), \quad \ldots \quad (81.)$$

which will be homogeneous of the second dimension with respect to $\varpi_1\ \varpi_2\ \varpi_3$. We may also put, for abridgement,

$$F\,(\varpi_1, \varpi_2, \varpi_3, \eta_1, \eta_2, \eta_3) - U\,(\eta_1, \eta_2, \eta_3) = H; \quad \ldots \quad (82.)$$

and then, instead of the three differential equations of the second order (78.), we may employ the six following of the first order, analogous to the equations (A.), and obtained by a similar reasoning,

$$\left.\begin{aligned} \frac{d\eta_1}{dt} &= +\frac{\delta H}{\delta \varpi_1},\ \frac{d\eta_2}{dt} = +\frac{\delta H}{\delta \varpi_2},\ \frac{d\eta_3}{dt} = +\frac{\delta H}{\delta \varpi_3}, \\ \frac{d\varpi_1}{dt} &= -\frac{\delta H}{\delta \eta_1},\ \frac{d\varpi_2}{dt} = -\frac{\delta H}{\delta \eta_2},\ \frac{d\varpi_3}{dt} = -\frac{\delta H}{\delta \eta_3}. \end{aligned}\right\} \quad \ldots \quad (83.)$$

20. The rigorous integrals of these six differential equations may be expressed under the following forms, analogous to (B.),

$$\left.\begin{aligned} \varpi_1 &= \frac{\delta S}{\delta \eta_1},\ \varpi_2 = \frac{\delta S}{\delta \eta_2},\ \varpi_3 = \frac{\delta S}{\delta \eta_3}, \\ p_1 &= -\frac{\delta S}{\delta e_1},\ p_2 = -\frac{\delta S}{\delta e_2},\ p_3 = -\frac{\delta S}{\delta e_3}, \end{aligned}\right\} \quad \ldots \quad (84.)$$

in which $e_1\, e_2\, e_3\, p_1\, p_2\, p_3$ are the initial values, or values at the time 0, of $\eta_1\, \eta_2\, \eta_3\, \varpi_1\, \varpi_2\, \varpi_3$; and S is the definite integral

$$S = \int_0^t \left(\varpi_1 \frac{\delta H}{\delta \varpi_1} + \varpi_2 \frac{\delta H}{\delta \varpi_2} + \varpi_3 \frac{\delta H}{\delta \varpi_3} - H \right) d\,t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85.)$$

considered as a function of $\eta_1\, \eta_2\, \eta_3\, e_1\, e_2\, e_3$ and t. The quantity H does not change in the course of the motion, and the function S must satisfy the following pair of partial differential equations of the first order, analogous to the pair (C.),

$$\left.\begin{array}{l} \dfrac{\delta S}{\delta t} + F\left(\dfrac{\delta S}{\delta \eta_1}, \dfrac{\delta S}{\delta \eta_2}, \dfrac{\delta S}{\delta \eta_3}, \eta_1, \eta_2, \eta_3\right) = U\,(\eta_1, \eta_2, \eta_3)\,; \\ \dfrac{\delta S}{\delta t} + F\left(\dfrac{\delta S}{\delta e_1}, \dfrac{\delta S}{\delta e_2}, \dfrac{\delta S}{\delta e_3}, e_1, e_2, e_3\right) = U\,(e_1, e_2, e_3). \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86.)$$

This important function S, which may be called the *principal function* of the motion, may hence be rigorously expressed under the following form, obtained by reasonings analogous to those of the seventh number of this Essay:

$$\left.\begin{array}{l} S = S_1 + \displaystyle\int_0^t \left\{ - \frac{\delta S_1}{\delta t} + U\,(\eta_1, \eta_2, \eta_3) - F\left(\frac{\delta S_1}{\delta \eta_1}, \frac{\delta S_1}{\delta \eta_2}, \frac{\delta S_1}{\delta \eta_3}, \eta_1, \eta_2, \eta_3\right)\right\} d\,t \\ \qquad + \displaystyle\int_0^t F\left(\frac{\delta S}{\delta \eta_1} - \frac{\delta S_1}{\delta \eta_1}, \frac{\delta S}{\delta \eta_2} - \frac{\delta S_1}{\delta \eta_2}, \frac{\delta S}{\delta \eta_3} - \frac{\delta S_1}{\delta \eta_3}, \eta_1, \eta_2, \eta_3\right) d\,t\,; \end{array}\right\} \quad (87.)$$

S_1 being any arbitrary function of the same quantities $\eta_1\, \eta_2\, \eta_3\, e_1\, e_2\, e_3\, t$, so chosen as to vanish with the time. And if this arbitrary function S_1 be chosen so as to be a first approximate value of the principal function S, we may neglect, in a second approximation, the second definite integral in (87.).

21. A first approximation of this kind can be obtained, whenever, by separating the expression H, (82.), into a predominant and a smaller part, H_1 and H_2, and by neglecting the part H_2, we have changed the differential equations (83.) to others, namely,

$$\left.\begin{array}{lll} \dfrac{d\,\eta_1}{d\,t} = \dfrac{\delta H_1}{\delta \varpi_1}, & \dfrac{d\,\eta_2}{d\,t} = \dfrac{\delta H_1}{\delta \varpi_2}, & \dfrac{d\,\eta_3}{d\,t} = \dfrac{\delta H_1}{\delta \varpi_3}, \\ \dfrac{d\,\varpi_1}{d\,t} = - \dfrac{\delta H_1}{\delta \eta_1}, & \dfrac{d\,\varpi_2}{d\,t} = - \dfrac{\delta H_1}{\delta \eta_2}, & \dfrac{d\,\varpi_3}{d\,t} = - \dfrac{\delta H_1}{\delta \eta_3}, \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88.)$$

and have succeeded in integrating rigorously these simplified equations, belonging to a simpler motion, which may be called the *undisturbed motion* of the point. For the principal function of such undisturbed motion, namely, the definite integral

$$S_1 = \int_0^t \left(\varpi_1 \frac{\delta H_1}{\delta \varpi_1} + \varpi_2 \frac{\delta H_1}{\delta \varpi_2} + \varpi_3 \frac{\delta H_1}{\delta \varpi_3} - H_1 \right) d\,t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89.)$$

considered as a function of $\eta_1\, \eta_2\, \eta_3\, e_1\, e_2\, e_3\, t$, will then be an approximate value for the original function of disturbed motion S, which original function corresponds to the more complex differential equations,

$$\left.\begin{array}{l} \frac{d\eta_1}{dt} = \frac{\delta H_1}{\delta \varpi_1} + \frac{\delta H_2}{\delta \varpi_1}, \frac{d\eta_2}{dt} = \frac{\delta H_1}{\delta \varpi_2} + \frac{\delta H_2}{\delta \varpi_2}, \frac{d\eta_3}{dt} = \frac{\delta H_1}{\delta \varpi_3} + \frac{\delta H_2}{\delta \varpi_3}, \\ \frac{d\varpi_1}{dt} = -\frac{\delta H_1}{\delta \eta_1} - \frac{\delta H_2}{\delta \eta_1}, \frac{d\varpi_2}{dt} = -\frac{\delta H_1}{\delta \eta_2} - \frac{\delta H_2}{\delta \eta_2}, \frac{d\varpi_3}{dt} = -\frac{\delta H_1}{\delta \eta_3} - \frac{\delta H_2}{\delta \eta_3}. \end{array}\right\} \quad (90.)$$

The function S_1 of undisturbed motion must satisfy a pair of partial differential equations of the first order, analogous to the pair (86.); and the integrals of undisturbed motion may be represented thus,

$$\left.\begin{array}{l} \varpi_1 = \frac{\delta S_1}{\delta \eta_1}, \varpi_2 = \frac{\delta S_1}{\delta \eta_2}, \varpi_3 = \frac{\delta S_1}{\delta \eta_3}, \\ p_1 = -\frac{\delta S_1}{\delta e_1}, p_2 = -\frac{\delta S_1}{\delta e_2}, p_3 = -\frac{\delta S_1}{\delta e_3}: \end{array}\right\} \quad \ldots\ldots \quad (91.)$$

while the integrals of disturbed motion may be expressed with equal rigour under the following analogous forms,

$$\left.\begin{array}{l} \varpi_1 = \frac{\delta S_1}{\delta \eta_1} + \frac{\delta S_2}{\delta \eta_1}, \quad \varpi_2 = \frac{\delta S_1}{\delta \eta_2} + \frac{\delta S_2}{\delta \eta_2}, \quad \varpi_3 = \frac{\delta S_1}{\delta \eta_3} + \frac{\delta S_2}{\delta \eta_3}, \\ p_1 = -\frac{\delta S_1}{\delta e_1} - \frac{\delta S_2}{\delta e_1}, \quad p_2 = -\frac{\delta S_1}{\delta e_2} - \frac{\delta S_2}{\delta e_2}, \quad p_3 = -\frac{\delta S_1}{\delta e_3} - \frac{\delta S_2}{\delta e_3}, \end{array}\right\} \quad \ldots \quad (92.)$$

if S_2 denote the rigorous correction of S_1, or the disturbing part of the whole principal function S. And by the foregoing general theory of approximation, this disturbing part or function S_2 may be approximately represented by the definite integral (T.),

$$S_2 = -\int_0^t H_2\, dt; \quad \ldots\ldots \quad (93.)$$

in calculating which definite integral the equations (91.) may be employed.

22. If the integrals of undisturbed motion (91.) have given

$$\left.\begin{array}{l} \eta_1 = \varphi_1(t, e_1, e_2, e_3, p_1, p_2, p_3), \\ \eta_2 = \varphi_2(t, e_1, e_2, e_3, p_1, p_2, p_3), \\ \eta_3 = \varphi_3(t, e_1, e_2, e_3, p_1, p_2, p_3), \end{array}\right\} \quad \ldots\ldots \quad (94.)$$

and

$$\left.\begin{array}{l} \varpi_1 = \psi_1(t, e_1, e_2, e_3, p_1, p_2, p_3), \\ \varpi_2 = \psi_2(t, e_1, e_2, e_3, p_1, p_2, p_3), \\ \varpi_3 = \psi_3(t, e_1, e_2, e_3, p_1, p_2, p_3), \end{array}\right\} \quad \ldots\ldots \quad (95.)$$

then the integrals of disturbed motion (92.) may be rigorously transformed as follows,

$$\left.\begin{array}{l} \eta_1 = \varphi_1\left(t, e_1, e_2, e_3, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, p_3 + \frac{\delta S_2}{\delta e_3}\right), \\ \eta_2 = \varphi_2\left(t, e_1, e_2, e_3, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, p_3 + \frac{\delta S_2}{\delta e_3}\right), \\ \eta_3 = \varphi_3\left(t, e_1, e_2, e_3, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, p_3 + \frac{\delta S_2}{\delta e_3}\right), \end{array}\right\} \quad \ldots \quad (96.)$$

and

$$\left.\begin{aligned}\varpi_1 &= \frac{\delta S_2}{\delta \eta_1} + \psi_1\left(t, e_1, e_2, e_3, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, p_3 + \frac{\delta S_2}{\delta e_3}\right),\\ \varpi_2 &= \frac{\delta S_2}{\delta \eta_2} + \psi_2\left(t, e_1, e_2, e_3, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, p_3 + \frac{\delta S_2}{\delta e_3}\right),\\ \varpi_3 &= \frac{\delta S_2}{\delta \eta_3} + \psi_3\left(t, e_1, e_2, e_3, p_1 + \frac{\delta S_2}{\delta e_1}, p_2 + \frac{\delta S_2}{\delta e_2}, p_3 + \frac{\delta S_2}{\delta e_3}\right),\end{aligned}\right\} \quad (97.)$$

S_2 being here the rigorous disturbing function. And the perturbations of position, at any time t, may be approximately expressed by the following formula,

$$\left.\begin{aligned}\Delta \eta_1 = \frac{\delta \eta_1}{\delta e_1}\int_0^t \frac{\delta H_2}{\delta p_1} dt + \frac{\delta \eta_1}{\delta e_2}\int_0^t \frac{\delta H_2}{\delta p_2} dt + \frac{\delta \eta_1}{\delta e_3}\int_0^t \frac{\delta H_2}{\delta p_3} dt\\ - \frac{\delta \eta_1}{\delta p_1}\int_0^t \frac{\delta H_2}{\delta e_1} dt - \frac{\delta \eta_1}{\delta p_2}\int_0^t \frac{\delta H_2}{\delta e_2} dt - \frac{\delta \eta_1}{\delta p_3}\int_0^t \frac{\delta H_2}{\delta e_3} dt,\end{aligned}\right\} \quad (98.)$$

together with two similar formulæ for the perturbations of the two other coordinates, or marks of position η_2, η_3. In these formulæ, the coordinates and H_2 are supposed to be expressed, by the theory of undisturbed motion, as functions of the time t, and of the constants $e_1\ e_2\ e_3\ p_1\ p_2\ p_3$.

23. Again, if the integrals of undisturbed motion have given, by elimination, expressions for these constants, of the forms

$$\left.\begin{aligned}e_1 &= \eta_1 + \Phi_1(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ e_2 &= \eta_2 + \Phi_2(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ e_3 &= \eta_3 + \Phi_3(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\end{aligned}\right\} \quad (99).$$

and

$$\left.\begin{aligned}p_1 &= \varpi_1 + \Psi_1(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ p_2 &= \varpi_2 + \Psi_2(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ p_3 &= \varpi_3 + \Psi_3(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3);\end{aligned}\right\} \quad (100.)$$

and if, for disturbed motion, we establish the definitions

$$\left.\begin{aligned}\varkappa_1 &= \eta_1 + \Phi_1(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ \varkappa_2 &= \eta_2 + \Phi_2(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ \varkappa_3 &= \eta_3 + \Phi_3(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\end{aligned}\right\} \quad (101.)$$

and

$$\left.\begin{aligned}\lambda_1 &= \varpi_1 + \Psi_1(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ \lambda_2 &= \varpi_2 + \Psi_2(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3),\\ \lambda_3 &= \varpi_3 + \Psi_3(t, \eta_1, \eta_2, \eta_3, \varpi_1, \varpi_2, \varpi_3);\end{aligned}\right\} \quad (102.)$$

we shall have, for such disturbed motion, the following rigorous equations, of the forms (94.) and (95.),

$$\left.\begin{aligned}\eta_1 &= \varphi_1(t, \varkappa_1, \varkappa_2, \varkappa_3, \lambda_1, \lambda_2, \lambda_3),\\ \eta_2 &= \varphi_2(t, \varkappa_1, \varkappa_2, \varkappa_3, \lambda_1, \lambda_2, \lambda_3),\\ \eta_3 &= \varphi_3(t, \varkappa_1, \varkappa_2, \varkappa_3, \lambda_1, \lambda_2, \lambda_3),\end{aligned}\right\} \quad (103.)$$

and

$$\left.\begin{aligned}\varpi_1 &= \psi_1(t, \varkappa_1, \varkappa_2, \varkappa_3, \lambda_1, \lambda_2, \lambda_3),\\ \varpi_2 &= \psi_2(t, \varkappa_1, \varkappa_2, \varkappa_3, \lambda_1, \lambda_2, \lambda_3),\\ \varpi_3 &= \psi_3(t, \varkappa_1, \varkappa_2, \varkappa_3, \lambda_1, \lambda_2, \lambda_3);\end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \quad (104.)$$

and may call the quantities $\varkappa_1\ \varkappa_2\ \varkappa_3\ \lambda_1\ \lambda_2\ \lambda_3$ the 6 *varying elements* of the motion. To determine these six varying elements, we may employ the six following rigorous equations in ordinary differentials of the first order, in which H_2 is supposed to have been expressed by (103.) and (104.) as a function of the elements and of the time:

$$\left.\begin{aligned}\frac{d\varkappa_1}{dt} &= \frac{\delta H_2}{\delta \lambda_1}, \quad \frac{d\varkappa_2}{dt} = \frac{\delta H_2}{\delta \lambda_2}, \quad \frac{d\varkappa_3}{dt} = \frac{\delta H_2}{\delta \lambda_3},\\ \frac{d\lambda_1}{dt} &= -\frac{\delta H_2}{\delta \varkappa_1}, \quad \frac{d\lambda_2}{dt} = -\frac{\delta H_2}{\delta \varkappa_2}, \quad \frac{d\lambda_3}{dt} = -\frac{\delta H_2}{\delta \varkappa_3};\end{aligned}\right\} \quad \ldots \ldots \quad (105.)$$

and the rigorous integrals of these 6 equations may be expressed in the following manner,

$$\left.\begin{aligned}\lambda_1 &= \frac{\delta E}{\delta \varkappa_1}, \quad \lambda_2 = \frac{\delta E}{\delta \varkappa_2}, \quad \lambda_3 = \frac{\delta E}{\delta \varkappa_3},\\ p_1 &= -\frac{\delta E}{\delta e_1}, \quad p_2 = -\frac{\delta E}{\delta e_2}, \quad p_3 = -\frac{\delta E}{\delta e_3},\end{aligned}\right\} \quad \ldots \ldots \quad (106.)$$

the constants $e_1\ e_2\ e_3\ p_1\ p_2\ p_3$ retaining their recent meanings, and being therefore the initial values of the elements $\varkappa_1\ \varkappa_2\ \varkappa_2\ \lambda_1\ \lambda_2\ \lambda_3$; while the function E, which may be called the *function of elements*, because its form determines the laws of their variations, is the definite integral

$$E = \int_0^t \left(\lambda_1 \frac{\delta H_2}{\delta \lambda_1} + \lambda_2 \frac{\delta H_2}{\delta \lambda_2} + \lambda_3 \frac{\delta H_2}{\delta \lambda_3} - H_2\right) dt, \quad \ldots \ldots \quad (107.)$$

considered as depending on $\varkappa_1\ \varkappa_2\ \varkappa_3\ e_1\ e_2\ e_3$ and t. The integrals of the equations (105.) may also be expressed in this other way,

$$\left.\begin{aligned}\varkappa_1 &= +\frac{\delta C}{\delta \lambda_1}, \quad \varkappa_2 = +\frac{\delta C}{\delta \lambda_2}, \quad \varkappa_3 = +\frac{\delta C}{\delta \lambda_3},\\ e_1 &= -\frac{\delta C}{\delta p_1}, \quad e_2 = -\frac{\delta C}{\delta p_2}, \quad e_3 = -\frac{\delta C}{\delta p_3},\end{aligned}\right\} \quad \ldots \ldots \ldots \quad (108.)$$

C being the definite integral

$$C = -\int_0^t \left(\varkappa_1 \frac{\delta H_2}{\delta \varkappa_1} + \varkappa_2 \frac{\delta H_2}{\delta \varkappa_2} + \varkappa_3 \frac{\delta H_2}{\delta \varkappa_3} - H_2\right) dt, \quad \ldots \ldots \quad (109.)$$

regarded as a function of $\lambda_1\ \lambda_2\ \lambda_3\ p_1\ p_2\ p_3$ and t: and it is easy to prove that each of these two *functions of elements*, C and E, must satisfy a partial differential equation of the first order, which can be previously assigned, and which may assist in discovering the forms of these two functions, and especially in improving an approximate expression for either. All these results for the motion of a single point, are analogous to the results already deduced in this Essay, for an attracting or repelling system.

Mathematical Example, suggested by the motion of Projectiles.

24. If the three marks of position $\eta_1\ \eta_2\ \eta_3$ of the moving point are the rectangular coordinates themselves, and if the function U has the form

$$U = -g\,\eta_3 - \tfrac{1}{2}\{\mu^2(\eta_1^2 + \eta_2^2) + \nu^2\eta_3^2\}, \quad \ldots \quad (110.)$$

g, μ, ν, being constants; then the expression

$$H = \tfrac{1}{2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) + g\,\eta_3 + \tfrac{1}{2}\{\mu^2(\eta_1^2 + \eta_2^2) + \nu^2\eta_3^2\}, \quad \ldots \quad (111.)$$

is that which must be substituted in the general forms (83.), in order to form the 6 differential equations of motion of the first order, namely,

$$\left.\begin{aligned} &\frac{d\eta_1}{dt} = \varpi_1, \quad \frac{d\eta_2}{dt} = \varpi_2, \quad \frac{d\eta_3}{dt} = \varpi_3, \\ &\frac{d\varpi_1}{dt} = -\mu^2\eta_1, \quad \frac{d\varpi_2}{dt} = -\mu^2\eta_2, \quad \frac{d\varpi_3}{dt} = -g - \nu^2\eta_3. \end{aligned}\right\} \quad \ldots \quad (112.)$$

These differential equations have for their rigorous integrals the six following,

$$\left.\begin{aligned} \eta_1 &= e_1\cos\mu t + \frac{p_1}{\mu}\sin\mu t, \\ \eta_2 &= e_2\cos\mu t + \frac{p_2}{\mu}\sin\mu t, \\ \eta_3 &= e_3\cos\nu t + \frac{p_3}{\nu}\sin\nu t - \frac{g}{\nu^2}\operatorname{vers}\nu t, \end{aligned}\right\} \quad \ldots \quad (113.)$$

and

$$\left.\begin{aligned} \varpi_1 &= p_1\cos\mu t - \mu e_1\sin\mu t, \\ \varpi_2 &= p_2\cos\mu t - \mu e_2\sin\mu t, \\ \varpi_3 &= p_3\cos\nu t - \left(\nu e_3 + \frac{g}{\nu}\right)\sin\nu t; \end{aligned}\right\} \quad \ldots \quad (114.)$$

$e_1\ e_2\ e_3\ p_1\ p_2\ p_3$ being still the initial values of $\eta_1\ \eta_2\ \eta_3\ \varpi_1\ \varpi_2\ \varpi_3$.

Employing these rigorous integral equations to calculate the function S, that is, by (85.) and (110.) (111.), the definite integral

$$S = \int_0^t \left(\frac{\varpi_1^2 + \varpi_2^2 + \varpi_3^2}{2} + U\right) dt, \quad \ldots \quad (115.)$$

we find

$$\left.\begin{aligned} \tfrac{1}{2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) &= \tfrac{1}{4}\left\{p_1^2 + p_2^2 + p_3^2 + \mu^2(e_1^2 + e_2^2) + \left(\nu e_3 + \frac{g}{\nu}\right)^2\right\} \\ &+ \tfrac{1}{4}\{p_1^2 + p_2^2 - \mu^2(e_1^2 + e_2^2)\}\cos 2\mu t - \tfrac{1}{2}\mu(e_1p_1 + e_2p_2)\sin 2\mu t \\ &+ \tfrac{1}{4}\left\{p_3^2 - \left(\nu e_3 + \frac{g}{\nu}\right)^2\right\}\cos 2\nu t - \tfrac{1}{2}\left(\nu e_3 + \frac{g}{\nu}\right)p_3\sin 2\nu t, \end{aligned}\right\} (116.)$$

and

$$\left.\begin{aligned} U = \frac{g^2}{2\nu^2} &- \tfrac{1}{4}\left\{p_1^2 + p_2^2 + p_3^2 + \mu^2(e_1^2 + e_2^2) + \left(\nu e_3 + \frac{g}{\nu}\right)^2\right\} \\ &+ \tfrac{1}{4}\{p_1^2 + p_2^2 - \mu^2(e_1^2 + e_2^2)\}\cos 2\mu t - \tfrac{1}{2}\mu(e_1p_1 + e_2p_2)\sin 2\mu t \\ &+ \tfrac{1}{4}\left\{p_3^2 - \left(\nu e_3 + \frac{g}{\nu}\right)^2\right\}\cos 2\nu t - \tfrac{1}{2}\left(\nu e_3 + \frac{g}{\nu}\right)p_3\sin 2\nu t; \end{aligned}\right\} (117.)$$

and therefore,

$$\left.\begin{aligned} \mathrm{S} = \frac{g^2 t}{2\nu^2} + \{p_1^2 + p_2^2 - \mu^2(e_1^2 + e_2^2)\}\frac{\sin 2\mu t}{4\mu} - \tfrac{1}{2}(e_1 p_1 + e_2 p_2)\,\mathrm{vers}\, 2\mu t \\ + \left\{p_3^2 - \left(\nu e_3 + \frac{g}{\nu}\right)^2\right\}\frac{\sin 2\nu t}{4\nu} - \tfrac{1}{2} p_3\left(e_3 + \frac{g}{\nu^2}\right)\mathrm{vers}\, 2\nu t. \end{aligned}\right\} \text{(118.)}$$

In order, however, to express this function S, as supposed by our general method, in terms of the final and initial coordinates and of the time, we must employ the analogous expressions for the constants $p_1\, p_2\, p_3$, deduced from the integrals (113.), namely, the following:

$$\left.\begin{aligned} p_1 &= \frac{\mu\eta_1 - \mu e_1 \cos\mu t}{\sin\mu t}, \\ p_2 &= \frac{\mu\eta_2 - \mu e_2 \cos\mu t}{\sin\mu t}, \\ p_3 &= \frac{\nu\eta_3 + \frac{g}{\nu} - \left(\nu e_3 + \frac{g}{\nu}\right)\cos\nu t}{\sin\nu t}; \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (119.)$$

and then we find

$$\left.\begin{aligned} \mathrm{S} = \frac{g^2 t}{2\nu^2} + \frac{\mu}{2}\cdot\frac{(\eta_1 - e_1)^2 + (\eta_2 - e_2)^2}{\tan\mu t} + \frac{\nu}{2}\cdot\frac{(\eta_3 - e_3)^2}{\tan\nu t} \\ - \mu(\eta_1 e_1 + \eta_2 e_2)\tan\frac{\mu t}{2} - \nu\left(\eta_3 + \frac{g}{\nu^2}\right)\left(e_3 + \frac{g}{\nu^2}\right)\tan\frac{\nu t}{2}. \end{aligned}\right\} \quad \ldots \quad (120.)$$

This *principal function* S satisfies the following pair of partial differential equations of the first order, of the kind (86.),

$$\left.\begin{aligned} \frac{\delta \mathrm{S}}{\delta t} + \frac{1}{2}\left\{\left(\frac{\delta \mathrm{S}}{\delta\eta_1}\right)^2 + \left(\frac{\delta \mathrm{S}}{\delta\eta_2}\right)^2 + \left(\frac{\delta \mathrm{S}}{\delta\eta_3}\right)^2\right\} = -g\eta_3 - \frac{\mu^2}{2}(\eta_1^2 + \eta_2^2) - \frac{\nu^2}{2}\eta_3^2, \\ \frac{\delta \mathrm{S}}{\delta t} + \frac{1}{2}\left\{\left(\frac{\delta \mathrm{S}}{\delta e_1}\right)^2 + \left(\frac{\delta \mathrm{S}}{\delta e_2}\right)^2 + \left(\frac{\delta \mathrm{S}}{\delta e_3}\right)^2\right\} = -g e_3 - \frac{\mu^2}{2}(e_1^2 + e_2^2) - \frac{\nu^2}{2}e_3^2; \end{aligned}\right\} \text{(121.)}$$

and *if its form had been previously found*, by the help of this pair, or in any other way, *the integrals of the equations of motion might* (*by our general method*) *have been deduced from it*, under the forms,

$$\left.\begin{aligned} \varpi_1 &= \frac{\delta \mathrm{S}}{\delta\eta_1} = \mu(\eta_1 - e_1)\,\mathrm{cotan}\,\mu t - \mu e_1 \tan\frac{\mu t}{2}, \\ \varpi_2 &= \frac{\delta \mathrm{S}}{\delta\eta_2} = \mu(\eta_2 - e_2)\,\mathrm{cotan}\,\mu t - \mu e_2 \tan\frac{\mu t}{2}, \\ \varpi_3 &= \frac{\delta \mathrm{S}}{\delta\eta_3} = \nu(\eta_3 - e_3)\,\mathrm{cotan}\,\nu t - \left(\nu e_3 + \frac{g}{\nu}\right)\tan\frac{\nu t}{2}, \end{aligned}\right\} \quad \ldots \quad (122.)$$

and

$$\left.\begin{aligned} p_1 &= -\frac{\delta \mathrm{S}}{\delta e_1} = \mu(\eta_1 - e_1)\,\mathrm{cotan}\,\mu t + \mu\eta_1 \tan\frac{\mu t}{2}, \\ p_2 &= -\frac{\delta \mathrm{S}}{\delta e_2} = \mu(\eta_2 - e_2)\,\mathrm{cotan}\,\mu t + \mu\eta_2 \tan\frac{\mu t}{2}, \\ p_3 &= -\frac{\delta \mathrm{S}}{\delta e_3} = \nu(\eta_3 - e_3)\,\mathrm{cotan}\,\nu t + \left(\nu\eta_3 + \frac{g}{\nu}\right)\tan\frac{\nu t}{2}: \end{aligned}\right\} \quad \ldots \quad (123.)$$

the last of these two sets of equations coinciding with the set (119.), or (113.), and conducting, when combined with the first set, (122.), to the other former set of integrals, (114.).

25. Suppose now, to illustrate the theory of perturbation, that the constants μ, ν are small, and that, after separating the expression (111.) for H into the two parts,

$$H_1 = \tfrac{1}{2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) + g\,\eta_3, \quad \ldots \quad (124.)$$

and

$$H_2 = \tfrac{1}{2}\{\mu^2(\eta_1^2 + \eta_2^2) + \nu^2\eta_3^2\}, \quad \ldots \quad (125.)$$

we suppress at first the small part H_2, and so form, by (88.), these other and simpler differential equations of a motion which we shall call *undisturbed*:

$$\left.\begin{aligned} &\frac{d\eta_1}{dt} = \varpi_1, \quad \frac{d\eta_2}{dt} = \varpi_2, \quad \frac{d\eta_3}{dt} = \varpi_3, \\ &\frac{d\varpi_1}{dt} = 0, \quad \frac{d\varpi_2}{dt} = 0, \quad \frac{d\varpi_3}{dt} = -g. \end{aligned}\right\} \quad \ldots \quad (126.)$$

These new equations have for their rigorous integrals, of the forms (94.) and (95.),

$$\eta_1 = e_1 + p_1 t, \; \eta_2 = e_2 + p_2 t, \; \eta_3 = e_3 + p_3 t - \tfrac{1}{2} g t^2, \quad \ldots \quad (127.)$$

and

$$\varpi_1 = p_1, \; \varpi_2 = p_2, \; \varpi_3 = p_3 - g\,t; \quad \ldots \quad (128.)$$

and the *principal function* S_1 of the same undisturbed motion is, by (89.),

$$\left.\begin{aligned} S_1 &= \int_0^t \left(\frac{\varpi_1^2 + \varpi_2^2 + \varpi_3^2}{2} - g\,\eta_3\right) dt \\ &= \int_0^t \left(\frac{p_1^2 + p_2^2 + p_3^2}{2} - g\,e_3 - 2\,g\,p_3\,t + g^2 t^2\right) dt \\ &= \left(\frac{p_1^2 + p_2^2 + p_3^2}{2} - g\,e_3\right) t - g\,p_3\,t^2 + \tfrac{1}{3} g^2 t^3, \end{aligned}\right\} \quad \ldots \quad (129.)$$

or finally, by (127.),

$$S_1 = \frac{(\eta_1 - e_1)^2 + (\eta_2 - e_2)^2 + (\eta_3 - e_3)^2}{2\,t} - \frac{1}{2} g\,t\,(\eta_3 + e_3) - \frac{1}{24} g^2 t^3. \quad \ldots \quad (130.)$$

This function satisfies, as it ought, the following pair of partial differential equations,

$$\left.\begin{aligned} &\frac{\delta S_1}{\delta t} + \frac{1}{2}\left\{\left(\frac{\delta S_1}{\delta \eta_1}\right)^2 + \left(\frac{\delta S_1}{\delta \eta_2}\right)^2 + \left(\frac{\delta S_1}{\delta \eta_3}\right)^2\right\} = -g\,\eta_3, \\ &\frac{\delta S_1}{\delta t} + \frac{1}{2}\left\{\left(\frac{\delta S_1}{\delta e_1}\right)^2 + \left(\frac{\delta S_1}{\delta e_2}\right)^2 + \left(\frac{\delta S_1}{\delta e_3}\right)^2\right\} = -g\,e_3; \end{aligned}\right\} \quad \ldots \quad (131.)$$

And if by the help of this pair, or in any other way, the form (130.) of this *principal function* S_1 had been found, the integral equations (127.) and (128.) might have been deduced from it, by our general method, as follows:

$$\left.\begin{aligned} \varpi_1 &= \frac{\delta S_1}{\delta \eta_1} = \frac{\eta_1 - e_1}{t}, \\ \varpi_2 &= \frac{\delta S_1}{\delta \eta_2} = \frac{\eta_2 - e_2}{t}, \\ \varpi_3 &= \frac{\delta S_1}{\delta \eta_3} = \frac{\eta_3 - e_3}{t} - \tfrac{1}{2} g\,t, \end{aligned}\right\} \quad \ldots \quad (132.)$$

and

$$\left.\begin{aligned} p_1 &= -\frac{\delta S_1}{\delta e_1} = \frac{\eta_1 - e_1}{t}, \\ p_2 &= -\frac{\delta S_1}{\delta e_2} = \frac{\eta_2 - e_2}{t}, \\ p_3 &= -\frac{\delta S_1}{\delta e_3} = \frac{\eta_3 - e_3}{t} + \tfrac{1}{2} g t ; \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133.)$$

the latter of these two sets coinciding with (127.), and the former set conducting to (128.).

26. Returning now from this simpler motion to the more complex motion first mentioned, and denoting by S_2 the *disturbing part* or function which must be added to S_1 in order to make up the whole principal function S of that more complex motion ; we have, by applying our general method, the following rigorous expression for this disturbing function,

$$S_2 = -\int_0^t H_2\, d t + \int_0^t \frac{1}{2} \left\{ \left(\frac{\delta S_2}{\delta \eta_1}\right)^2 + \left(\frac{\delta S_2}{\delta \eta_2}\right)^2 + \left(\frac{\delta S_2}{\delta \eta_3}\right)^2 \right\} d t, \quad . \quad (134.)$$

in which we may, approximately, neglect the second definite integral, and calculate the first by the help of the equations of undisturbed motion. In this manner we find, approximately, by (125.) (127.),

$$- H_2 = - \frac{\mu^2}{2} \left\{ (e_1 + p_1 t)^2 + (e_2 + p_2 t)^2 \right\} - \frac{\nu^2}{2} (e_3 + p_3 t - \tfrac{1}{2} g t^2)^2, \quad (135.)$$

and therefore, by integration,

$$\left.\begin{aligned} S_2 = &- \frac{1}{2} \{\mu^2 (e_1^2 + e_2^2) + \nu^2 e_3^2\} t - \frac{1}{2} \{\mu^2 (e_1 p_1 + e_2 p_2) + \nu^2 e_3 p_3\} t^2 \\ &- \frac{1}{6} \{\mu^2 (p_1^2 + p_2^2) + \nu^2 (p_3^2 - g e_3)\} t^3 + \frac{1}{8} \nu^2 g p_3 t^4 - \frac{1}{40} \nu^2 g^2 t^5, \end{aligned}\right\} (136.)$$

or, by (133.),

$$\left.\begin{aligned} S_2 = &- \frac{\mu^2 t}{6} (\eta_1^2 + e_1 \eta_1 + e_1^2 + \eta_2^2 + e_2 \eta_2 + e_2^2) \\ &- \frac{\nu^2 t}{6} \left\{ \eta_3^2 + e_3 \eta_3 + e_3^2 + \frac{1}{4} g (\eta_3 + e_3) t^2 + \frac{1}{40} g^2 t^4 \right\} : \end{aligned}\right\} \quad . \quad . \quad (137.)$$

the error being of the fourth order, with respect to the small quantities μ, ν. And neglecting this small error, we can deduce, by our general method, approximate forms for the integrals of the equations of disturbed motion, from the corrected function $S_1 + S_2$, as follows:

$$\left.\begin{aligned} \varpi_1 &= \frac{\delta S_1}{\delta \eta_1} + \frac{\delta S_2}{\delta \eta_1} = \frac{\eta_1 - e_1}{t} - \frac{\mu^2 t}{3} \left(\eta_1 + \frac{1}{2} e_1\right), \\ \varpi_2 &= \frac{\delta S_1}{\delta \eta_2} + \frac{\delta S_2}{\delta \eta_2} = \frac{\eta_2 - e_2}{t} - \frac{\mu^2 t}{3} \left(\eta_2 + \frac{1}{2} e_2\right), \\ \varpi_3 &= \frac{\delta S_1}{\delta \eta_3} + \frac{\delta S_2}{\delta \eta_3} = \frac{\eta_3 - e_3}{t} - \frac{1}{2} g t - \frac{\nu^2 t}{3} \left(\eta_3 + \frac{1}{2} e_3 + \frac{1}{8} g t^2\right); \end{aligned}\right\} . \quad (138.)$$

and

$$\left.\begin{aligned}
p_1 &= -\frac{\delta S_1}{\delta e_1} - \frac{\delta S_2}{\delta e_1} = \frac{\eta_1 - e_1}{t} + \frac{\mu^2 t}{3}\left(e_1 + \frac{1}{2}\eta_1\right), \\
p_2 &= -\frac{\delta S_1}{\delta e_2} - \frac{\delta S_2}{\delta e_2} = \frac{\eta_2 - e_2}{t} + \frac{\mu^2 t}{3}\left(e_2 + \frac{1}{2}\eta_2\right), \\
p_3 &= -\frac{\delta S_1}{\delta e_3} - \frac{\delta S_2}{\delta e_3} = \frac{\eta_3 - e_3}{t} + \frac{1}{2} g t + \frac{\nu^2 t}{3}\left(e_3 + \frac{1}{2}\eta_3 + \frac{1}{8} g t^2\right);
\end{aligned}\right\} \quad (139.)$$

or, in the same order of approximation,

$$\left.\begin{aligned}
\eta_1 &= e_1 + p_1 t - \frac{1}{2}\mu^2 t^2 \left(e_1 + \frac{1}{3} p_1 t\right), \\
\eta_2 &= e_2 + p_2 t - \frac{1}{2}\mu^2 t^2 \left(e_2 + \frac{1}{3} p_2 t\right), \\
\eta_3 &= e_3 + p_3 t - \frac{1}{2} g t^2 - \frac{1}{2}\nu^2 t^2 \left(e_3 + \frac{1}{3} p_3 t - \frac{1}{12} g t^2\right),
\end{aligned}\right\} \quad . \quad . \quad (140.)$$

and

$$\left.\begin{aligned}
\varpi_1 &= p_1 - \mu^2 t \left(e_1 + \frac{1}{2} p_1 t\right), \\
\varpi_2 &= p_2 - \mu^2 t \left(e_2 + \frac{1}{2} p_2 t\right), \\
\varpi_3 &= p_3 - g t - \nu^2 t \left(e_3 + \frac{1}{2} p_3 t - \frac{1}{6} g t^2\right).
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (141.)$$

Accordingly, if we develope the rigorous integrals of disturbed motion, (113.) and (114.), as far as the squares (inclusive) of the small quantities μ and ν, we are conducted to these approximate integrals; and if we develope the rigorous expression (120.) for the principal function of such motion, to the same degree of accuracy, we obtain the sum of the two expressions (130.) and (137.).

27. To illustrate still further, in the present example, our general method of successive approximation, let S_3 denote the small unknown correction of the approximate expression (137.), so that we shall now have, rigorously, for the present disturbed motion,

$$S = S_1 + S_2 + S_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (142.)$$

S_1 and S_2 being here determined rigorously by (130.) and (137.). Then, substituting $S_1 + S_2$ for S_1 in the general transformation (87.), we find, rigorously, in the present question,

$$\left.\begin{aligned}
S_3 = &-\int_0^t \frac{1}{2}\left\{\left(\frac{\delta S_2}{\delta \eta_1}\right)^2 + \left(\frac{\delta S_2}{\delta \eta_2}\right)^2 + \left(\frac{\delta S_2}{\delta \eta_3}\right)^2\right\} dt \\
&+\int_0^t \frac{1}{2}\left\{\left(\frac{\delta S_3}{\delta \eta_1}\right)^2 + \left(\frac{\delta S_3}{\delta \eta_2}\right)^2 + \left(\frac{\delta S_3}{\delta \eta_3}\right)^2\right\} dt:
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (143.)$$

and if we neglect only terms of the eighth and higher dimensions with respect to the small quantities μ, ν, we may confine ourselves to the first of these two definite integrals, and may employ, in calculating it, the approximate expressions (140.) for

the coordinates of disturbed motion. In this manner we obtain the very approximate expression,

$$\left.\begin{aligned} S_3 = & -\frac{\mu^4}{18}\int_0^t t^2 \left\{ \left(\eta_1 + \frac{1}{2} e_1\right)^2 + \left(\eta_2 + \frac{1}{2} e_2\right)^2 \right\} dt \\ & -\frac{\nu^4}{18}\int_0^t t^2 \left(\eta_3 + \frac{1}{2} e_3 + \frac{1}{8} g\, t^2\right)^2 dt \\ = & -\frac{\mu^4 t^3}{360} (4\,\eta_1^2 + 7\,\eta_1 e_1 + 4\, e_1^2 + 4\,\eta_2^2 + 7\,\eta_2 e_2 + 4\, e_2^2) \\ & -\frac{\nu^4 t^3}{360} (4\,\eta_3^2 + 7\,\eta_3 e_3 + 4\, e_3^2) - \frac{\nu^4 g\, t^5}{240} (\eta_3 + e_3) - \frac{17\,\nu^4 g^2 t^7}{40320} \\ & -\frac{\mu^6 t^5}{945} \left(\eta_1^2 + \frac{31}{16} \eta_1 e_1 + e_1^2 + \eta_2^2 + \frac{31}{16} \eta_2 e_2 + e_2^2\right) \\ & -\frac{\nu^6 t^5}{945} \left(\eta_3^2 + \frac{31}{16} \eta_3 e_3 + e_3^2\right) - \frac{17\,\nu^6 g\, t^7 (\eta_3 + e_3)}{40320} - \frac{31\,\nu^6 g^2 t^9}{725760}; \end{aligned}\right\} \quad (144.)$$

which is accordingly the sum of the terms of the fourth and sixth dimensions in the development of the rigorous expression (120.), and gives, by our general method, correspondingly approximate expressions for the integrals of disturbed motion, under the forms

$$\left.\begin{aligned} \varpi_1 &= \frac{\delta S_1}{\delta \eta_1} + \frac{\delta S_2}{\delta \eta_1} + \frac{\delta S_3}{\delta \eta_1}, \\ \varpi_2 &= \frac{\delta S_1}{\delta \eta_2} + \frac{\delta S_2}{\delta \eta_2} + \frac{\delta S_3}{\delta \eta_2}, \\ \varpi_3 &= \frac{\delta S_1}{\delta \eta_3} + \frac{\delta S_2}{\delta \eta_3} + \frac{\delta S_3}{\delta \eta_3}, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (145.)$$

and

$$\left.\begin{aligned} p_1 &= -\frac{\delta S_1}{\delta e_1} - \frac{\delta S_2}{\delta e_1} - \frac{\delta S_3}{\delta e_1}, \\ p_2 &= -\frac{\delta S_1}{\delta e_2} - \frac{\delta S_2}{\delta e_2} - \frac{\delta S_3}{\delta e_2}, \\ p_3 &= -\frac{\delta S_1}{\delta e_3} - \frac{\delta S_2}{\delta e_3} - \frac{\delta S_3}{\delta e_3}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (146.)$$

28. To illustrate by the same example the theory of gradually varying elements, let us establish the following definitions, for the present disturbed motion,

$$\left.\begin{aligned} \varkappa_1 &= \eta_1 - \varpi_1 t, & \varkappa_2 &= \eta_2 - \varpi_2 t, & \varkappa_3 &= \eta_3 - \varpi_3 t - \frac{1}{2} g\, t^2, \\ \lambda_1 &= \varpi_1, & \lambda_2 &= \varpi_2, & \lambda_3 &= \varpi_3 + g\, t, \end{aligned}\right\} \quad \ldots \quad (147.)$$

and let us call these six quantities $\varkappa_1\, \varkappa_2\, \varkappa_3\, \lambda_1\, \lambda_2\, \lambda_3$ the *varying elements* of that motion, by analogy to the six constant quantities $e_1\, e_2\, e_3\, p_1\, p_2\, p_3$, which may, for the undisturbed motion, be represented in a similar way, namely, by (127.) and (128.),

$$\left.\begin{array}{lll} e_1 = \eta_1 - \varpi_1\, t, & e_2 = \eta_2 - \varpi_2\, t, & e_3 = \eta_3 - \varpi_3\, t - \frac{1}{2}\, g\, t^2, \\ p_1 = \varpi_1, & p_2 = \varpi_2, & p_3 = \varpi_3 + g\, t. \end{array}\right\} \quad . \quad . \quad . \quad . \quad (148.)$$

We shall then have rigorously, for the six disturbed variables $\eta_1\, \eta_2\, \eta_3\, \varpi_1\, \varpi_2\, \varpi_3$, expressions of the same forms as in the integrals (127.) and (128.) of undisturbed motion, but with variable instead of constant elements, namely, the following:

$$\left.\begin{array}{lll} \eta_1 = \varkappa_1 + \lambda_1\, t, & \eta_2 = \varkappa_2 + \lambda_2\, t, & \eta_3 = \varkappa_3 + \lambda_3\, t - \frac{1}{2}\, g\, t^2, \\ \varpi_1 = \lambda_1, & \varpi_2 = \lambda_2, & \varpi_3 = \lambda_3 - g\, t; \end{array}\right\} \quad . \quad . \quad . \quad . \quad (149.)$$

and the rigorous determination of the six varying elements $\varkappa_1\, \varkappa_2\, \varkappa_3\, \lambda_1\, \lambda_2\, \lambda_3$, as functions of the time and of their own initial values $e_1\, e_2\, e_3\, p_1\, p_2\, p_3$, depends on the integration of the 6 following equations, in ordinary differentials of the first order, of the forms (105.):

$$\left.\begin{array}{l} \frac{d\, \varkappa_1}{d\, t} = \frac{\delta\, \mathrm{H}_2}{\delta\, \lambda_1} = \mu^2\, t\, (\varkappa_1 + \lambda_1\, t), \\ \frac{d\, \varkappa_2}{d\, t} = \frac{\delta\, \mathrm{H}_2}{\delta\, \lambda_2} = \mu^2\, t\, (\varkappa_2 + \lambda_2\, t), \\ \frac{d\, \varkappa_3}{d\, t} = \frac{\delta\, \mathrm{H}_2}{\delta\, \lambda_3} = \nu^2\, t\, \left(\varkappa_3 + \lambda_3\, t - \frac{1}{2}\, g\, t^2\right), \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (150.)$$

and

$$\left.\begin{array}{l} \frac{d\, \lambda_1}{d\, t} = -\frac{\delta\, \mathrm{H}_2}{\delta\, \varkappa_1} = -\mu^2\, (\varkappa_1 + \lambda_1\, t), \\ \frac{d\, \lambda_2}{d\, t} = -\frac{\delta\, \mathrm{H}_2}{\delta\, \varkappa_2} = -\mu^2\, (\varkappa_2 + \lambda_2\, t), \\ \frac{d\, \lambda_3}{d\, t} = -\frac{\delta\, \mathrm{H}_2}{\delta\, \varkappa_3} = -\nu^2\, \left(\varkappa_3 + \lambda_3\, t - \frac{1}{2}\, g\, t^2\right), \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (151.)$$

H_2 being here the expression

$$\mathrm{H}_2 = \frac{\mu^2}{2}\, \left\{(\varkappa_1 + \lambda_1\, t)^2 + (\varkappa_2 + \lambda_2\, t)^2\right\} + \frac{\nu^2}{2}\, \left(\varkappa_3 + \lambda_3\, t - \frac{1}{2}\, g\, t^2\right)^2, \quad . \quad (152.)$$

which is obtained from (125.) by substituting for the disturbed coordinates $\eta_1\, \eta_2\, \eta_3$ their values (149.), as functions of the varying elements and of the time. It is not difficult to integrate rigorously this system of equations (150.) and (151.); and we shall soon have occasion to state their complete and accurate integrals: but we shall continue for a while to treat these rigorous integrals as unknown, that we may take this opportunity to exemplify our general method of indefinite approximation, for all such dynamical questions, founded on the properties of the *functions of elements* C and E. Of these two functions either may be employed, and we shall use here the function C.

29. This function, by (109.) and (152.), may rigorously be expressed as follows:

$$\left.\begin{aligned} C = \frac{\mu^2}{2}\int_0^t (\lambda_1^2 t^2 - \varkappa_1^2 + \lambda_2^2 t^2 - \varkappa_2^2)\, d t \\ + \frac{\nu^2}{2}\int_0^t \left\{\left(\lambda_3 t - \frac{1}{2} g t^2\right)^2 - \varkappa_3^2\right\} d t ; \end{aligned}\right\} \quad . \quad . \quad . \quad (153.)$$

and has therefore the following for a first approximate value, obtained by treating the elements $\varkappa_1\ \varkappa_2\ \varkappa_3\ \lambda_1\ \lambda_2\ \lambda_3$ as constant and equal to their initial values $e_1\ e_2\ e_3\ p_1\ p_2\ p_3$,

$$\left.\begin{aligned} C = -\frac{t}{2}\left\{\mu^2 (e_1^2 + e_2^2) + \nu^2 e_3^2\right\} + \frac{t^3}{6}\left\{\mu^2 (p_1^2 + p_2^2) + \nu^2 p_3^2\right\} \\ - \frac{t^4}{8}\nu^2 g\, p_3 + \frac{t^5}{40}\nu^2 g^2. \end{aligned}\right\} \quad (154.)$$

In like manner we have, as first approximations, of the kind expressed by the general formula (Z^1), the following results deduced from the equations (151.),

$$\left.\begin{aligned} \lambda_1 &= p_1 - \mu^2\left(e_1 t + \frac{1}{2} p_1 t^2\right), \\ \lambda_2 &= p_2 - \mu^2\left(e_2 t + \frac{1}{2} p_2 t^2\right), \\ \lambda_3 &= p_3 - \nu^2\left(e_3 t + \frac{1}{2} p_3 t^2 - \frac{1}{6} g t^3\right), \end{aligned}\right\} \quad . \quad . \quad . \quad (155.)$$

and therefore, as approximations of the same kind,

$$\left.\begin{aligned} e_1 &= -\frac{1}{2} p_1 t - \frac{\lambda_1 - p_1}{\mu^2 t}, \\ e_2 &= -\frac{1}{2} p_2 t - \frac{\lambda_2 - p_2}{\mu^2 t}, \\ e_3 &= -\frac{1}{2} p_3 t + \frac{1}{6} g t^2 - \frac{\lambda_3 - p_3}{\nu^2 t}. \end{aligned}\right\} \quad . \quad . \quad . \quad (156.)$$

Substituting these values for the initial constants $e_1\ e_2\ e_3$ in the approximate value (154.) for the function of elements C, we obtain the following approximate expression C_1 for that function, of the form supposed by our theory:

$$\left.\begin{aligned} C_1 = -\frac{1}{2t}\left\{\frac{(\lambda_1 - p_1)^2 + (\lambda_2 - p_2)^2}{\mu^2} + \frac{(\lambda_3 - p_3)^2}{\nu^2}\right\} \\ - \frac{t}{2}\left\{(\lambda_1 - p_1) p_1 + (\lambda_2 - p_2) p_2 + (\lambda_3 - p_3)\left(p_3 - \frac{1}{3} g t\right)\right\} \\ + \frac{t^3}{24}\left\{\mu^2 (p_1^2 + p_2^2) + \nu^2 p_3^2\right\} - \frac{t^4}{24}\nu^2 g\, p_3 + \frac{t^5}{90}\nu^2 g^2. \end{aligned}\right\} \quad (157.)$$

The rigorous function C must satisfy, in the present question, by the principles of the eighteenth number, the partial differential equation,

$$\frac{\delta C}{\delta t} = \frac{\mu^2}{2}\left\{\left(\frac{\delta C}{\delta \lambda_1} + \lambda_1 t\right)^2 + \left(\frac{\delta C}{\delta \lambda_2} + \lambda_2 t\right)^2\right\} + \frac{\nu^2}{2}\left(\frac{\delta C}{\delta \lambda_3} + \lambda_3 t - \frac{1}{2} g t^2\right)^2 ; \quad (158.)$$

and if it be put under the form (U^1.),

$$C = C_1 + C_2,$$

C_1 being a first approximation, supposed to vanish with the time, then the correction C_2 must satisfy rigorously the condition

$$\left.\begin{aligned}C_2=\int_0^t\Big\{-\frac{\delta C_1}{\delta t}+\frac{\mu^2}{2}\Big(\frac{\delta C_1}{\delta\lambda_1}+\lambda_1 t\Big)^2+\frac{\mu^2}{2}\Big(\frac{\delta C_1}{\delta\lambda_2}+\lambda_2 t\Big)^2+\frac{\nu^2}{2}\Big(\frac{\delta C_1}{\delta\lambda_3}+\lambda_3 t-\frac{1}{2}g t^2\Big)^2\Big\}dt\\-\frac{1}{2}\int_0^t\Big\{\mu^2\Big(\frac{\delta C_2}{\delta\lambda_1}\Big)^2+\mu^2\Big(\frac{\delta C_2}{\delta\lambda_2}\Big)^2+\nu^2\Big(\frac{\delta C_2}{\delta\lambda_3}\Big)^2\Big\}dt.\end{aligned}\right\}\tag{159.}$$

In passing to a second approximation we may neglect the second definite integral, and may calculate the first by the help of the approximate equations (155.); which give, in this manner,

$$\left.\begin{aligned}C_2&=-\int_0^t\Big\{(\lambda_1-p_1)^2+(\lambda_2-p_2)^2+(\lambda_3-p_3)^2\Big\}dt\\&\quad+\frac{\mu^2}{2}\int_0^t\Big\{\lambda_1(\lambda_1-p_1)+\lambda_2(\lambda_2-p_2)\Big\}dt\\&\quad+\frac{\nu^2}{2}\int_0^t\Big(\lambda_3-\frac{2}{3}gt\Big)(\lambda_3-p_3)\,t^2\,dt\\&=-\frac{t}{3}\{(\lambda_1-p_1)^2+(\lambda_2-p_2)^2+(\lambda_3-p_3)^2\}\\&\quad+\frac{t^3}{24}\{\mu^2p_1(\lambda_1-p_1)+\mu^2p_2(\lambda_2-p_2)+\nu^2p_3(\lambda_3-p_3)\}\\&\quad-\frac{t^4}{45}\nu^2g(\lambda_3-p_3)+\frac{t^5}{240}(\mu^4p_1^2+\mu^4p_2^2+\nu^4p_3^2)\\&\quad-\frac{t^6}{240}\nu^4gp_3+\frac{t^7}{945}\nu^4g^2.\end{aligned}\right\}\tag{160.}$$

We might improve this second approximation in like manner, by calculating a new definite integral C_3, with the help of the following more approximate forms for the relations between the varying elements $\lambda_1\ \lambda_2\ \lambda_3$ and the initial constants, deduced by our general method:

$$\left.\begin{aligned}e_1&=-\frac{\delta C_1}{\delta p_1}-\frac{\delta C_2}{\delta p_1}=-\frac{\lambda_1-p_1}{\mu^2 t}\Big(1+\frac{\mu^2t^2}{6}+\frac{\mu^4t^4}{24}\Big)-\frac{tp_1}{2}\Big(1+\frac{\mu^2t^2}{12}+\frac{\mu^4t^4}{60}\Big),\\e_2&=-\frac{\delta C_1}{\delta p_2}-\frac{\delta C_2}{\delta p_2}=-\frac{\lambda_2-p_2}{\mu^2 t}\Big(1+\frac{\mu^2t^2}{6}+\frac{\mu^4t^4}{24}\Big)-\frac{tp_2}{2}\Big(1+\frac{\mu^2t^2}{12}+\frac{\mu^4t^4}{60}\Big),\\e_3&=-\frac{\delta C_1}{\delta p_3}-\frac{\delta C_2}{\delta p_3}=-\frac{\lambda_3-p_3}{\nu^2 t}\Big(1+\frac{\nu^2t^2}{6}+\frac{\nu^4t^4}{24}\Big)-\frac{tp_3}{2}\Big(1+\frac{\nu^2t^2}{12}+\frac{\nu^4t^4}{60}\Big)\\&\quad+\frac{gt^2}{6}\Big(1+\frac{7\nu^2t^2}{60}+\frac{\nu^4t^4}{40}\Big);\end{aligned}\right\}\tag{161.}$$

in which we can only depend on the terms as far as the second order, but which acquire a correctness of the fourth order when cleared of the small divisors, and give then

$$\left.\begin{aligned}\lambda_1&=p_1-\mu^2t\Big(e_1+\frac{1}{2}p_1t\Big)+\frac{1}{6}\mu^4t^3\Big(e_1+\frac{1}{4}p_1t\Big),\\\lambda_2&=p_2-\mu^2t\Big(e_2+\frac{1}{2}p_2t\Big)+\frac{1}{6}\mu^4t^3\Big(e_2+\frac{1}{4}p_2t\Big),\\\lambda_3&=p_3-\nu^2t\Big(e_3+\frac{1}{2}p_3t-\frac{1}{6}gt^2\Big)+\frac{1}{6}\nu^4t^3\Big(e_3+\frac{1}{4}p_3t-\frac{1}{20}gt^2\Big).\end{aligned}\right\}\tag{162.}$$

But a little attention to the nature of this process shows that all the successive corrections to which it conducts can be only rational and integer and homogeneous functions, of the second dimension, of the quantities $\lambda_1 \lambda_2 \lambda_3 p_1 p_2 p_3 g$, and that they may all be put under the following form, which is therefore the form of their sum, or of the whole sought function C;

$$\left.\begin{aligned} \mathrm{C} = {} & \mu^{-2} a_\mu (\lambda_1 - p_1)^2 + b_\mu p_1 (\lambda_1 - p_1) + \mu^2 c_\mu p_1^2 \\ & + \mu^{-2} a_\mu (\lambda_2 - p_2)^2 + b_\mu p_2 (\lambda_2 - p_2) + \mu^2 c_\mu p_2^2 \\ & + \nu^{-2} a_\nu (\lambda_3 - p_3)^2 + b_\nu p_3 (\lambda_3 - p_3) + \nu^2 c_\nu p_3^2 \\ & + f_\nu g (\lambda_3 - p_3) + \nu^2 h_\nu g p_3 + \nu^2 i_\nu g^2, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (163.)$$

the coefficients a_μ a_ν &c. being functions of the small quantities μ, ν, and also of the time, of which it remains to discover the forms. Denoting therefore their differentials, taken with respect to the time, as follows,

$$d\, a_\mu = a'_\mu\, d\, t, \; d\, a_\nu = a'_\nu\, d\, t, \; \&\text{c.}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (164.)$$

and substituting the expression (163.) in the rigorous partial differential equation (158.), we are conducted to the six following equations in ordinary differentials of the first order:

$$\left.\begin{aligned} & 2\, a'_\nu = (2\, a_\nu + \nu^2 t)^2; \; b'_\nu = (2\, a_\nu + \nu^2 t)\,(b_\nu + t); \; c'_\nu = \tfrac{1}{2}(b_\nu + t)^2; \\ & f'_\nu = (2\, a_\nu + \nu^2 t)\,(f_\nu - \tfrac{1}{2} t^2); \; h'_\nu = (b_\nu + t)\,(f_\nu - \tfrac{1}{2} t^2); \; i'_\nu = \tfrac{1}{2}(f_\nu - \tfrac{1}{2} t^2)^2; \end{aligned}\right\} (165.)$$

along with the 6 following conditions, to determine the 6 arbitrary constants introduced by integration,

$$a_0 = -\frac{1}{2\,t}; \; b_0 = -\frac{t}{2}; \; f_0 = \frac{t^2}{6}; \; c_0 = \frac{t^3}{24}; \; h_0 = -\frac{t^4}{24}; \; i_0 = \frac{t^5}{90}. \quad . \quad (166.)$$

In this manner we find, without difficulty, observing that a_μ b_μ c_μ may be formed from a_ν b_ν c_ν by changing ν to μ,

$$\left.\begin{aligned} & a_\nu = -\tfrac{1}{2}\nu^2 t - \tfrac{1}{2}\nu \operatorname{cotan} \nu t, \quad a_\mu = -\tfrac{1}{2}\mu^2 t - \tfrac{1}{2}\mu \operatorname{cotan} \mu t, \\ & b_\nu = -t + \frac{1}{\nu}\tan\frac{\nu t}{2}, \qquad b_\mu = -t + \frac{1}{\mu}\tan\frac{\mu t}{2}, \\ & c_\nu = -\frac{t}{2\,\nu^2} + \frac{1}{\nu^3}\tan\frac{\nu t}{2}, \qquad c_\mu = -\frac{t}{2\,\mu^2} + \frac{1}{\mu^3}\tan\frac{\mu t}{2}, \\ & f_\nu = \tfrac{1}{2} t^2 - \frac{1}{\nu^2} + \frac{t}{\nu}\operatorname{cotan} \nu t, \\ & h_\nu = \frac{t^2}{2\,\nu^2} - \frac{t}{\nu^3}\tan\frac{\nu t}{2}, \\ & i_\nu = \frac{t}{2\,\nu^4} - \frac{t^3}{6\,\nu^2} - \frac{t^2}{2\,\nu^3}\operatorname{cotan} \nu t. \end{aligned}\right\} \quad . \quad (167.)$$

The form of the function C is therefore entirely known, and we have for this *function of elements* the following rigorous expression,

$$\left.\begin{aligned}
C = &- \frac{(\lambda_1 - p_1)^2 + (\lambda_2 - p_2)^2}{2\mu\tan\mu t} - \frac{(\lambda_3 - p_3)^2}{2\nu\tan\nu t} \\
&- \frac{t}{2}\{(\lambda_1 - p_1)^2 + (\lambda_2 - p_2)^2 + (\lambda_3 - p_3)^2\} \\
&- t\{p_1(\lambda_1 - p_1) + p_2(\lambda_2 - p_2) + p_3(\lambda_3 - p_3)\} \\
&+ \frac{1}{\mu}\{p_1(\lambda_1 - p_1) + p_2(\lambda_2 - p_2)\}\tan\frac{\mu t}{2} + \frac{1}{\nu}p_3(\lambda_3 - p_3)\tan\frac{\nu t}{2} \\
&- \frac{t}{2}(p_1^2 + p_2^2 + p_3^2) + \frac{1}{\mu}(p_1^2 + p_2^2)\tan\frac{\mu t}{2} + \frac{1}{\nu}p_3^2\tan\frac{\nu t}{2} \\
&+ \left(\frac{t^2}{2} - \frac{1}{\nu^2} + \frac{t}{\nu}\operatorname{cotan}\nu t\right)g(\lambda_3 - p_3) + \left(\frac{t^2}{2} - \frac{t}{\nu}\tan\frac{\nu t}{2}\right)g p_3 \\
&+ \left(\frac{t}{2\nu^2} - \frac{t^3}{6} - \frac{t^2}{2\nu}\operatorname{cotan}\nu t\right)g^2,
\end{aligned}\right\} \quad (168.)$$

which may be variously transformed, and gives by our general method the following systems of rigorous integrals of the differential equations of varying elements, (150.), (151.):

$$\left.\begin{aligned}
e_1 &= -\frac{\delta C}{\delta p_1} = -\frac{\lambda_1 - p_1}{\mu\sin\mu t} - \frac{p_1}{\mu}\tan\frac{\mu t}{2}, \\
e_2 &= -\frac{\delta C}{\delta p_2} = -\frac{\lambda_2 - p_2}{\mu\sin\mu t} - \frac{p_2}{\mu}\tan\frac{\mu t}{2}, \\
e_3 &= -\frac{\delta C}{\delta p_3} = -\frac{\lambda_3 - p_3}{\nu\sin\nu t} - \frac{p_3}{\nu}\tan\frac{\nu t}{2} + \frac{g}{\nu}\left(\frac{t}{\sin\nu t} - \frac{1}{\nu}\right),
\end{aligned}\right\} \quad . \quad . \quad (169.)$$

and

$$\left.\begin{aligned}
x_1 &= \frac{\delta C}{\delta\lambda_1} = -(\lambda_1 - p_1)\left(t + \frac{1}{\mu}\operatorname{cotan}\mu t\right) + p_1\left(-t + \frac{1}{\mu}\tan\frac{\mu t}{2}\right), \\
x_2 &= \frac{\delta C}{\delta\lambda_2} = -(\lambda_2 - p_2)\left(t + \frac{1}{\mu}\operatorname{cotan}\mu t\right) + p_2\left(-t + \frac{1}{\mu}\tan\frac{\mu t}{2}\right), \\
x_3 &= \frac{\delta C}{\delta\lambda_3} = -(\lambda_3 - p_3)\left(t + \frac{1}{\nu}\operatorname{cotan}\nu t\right) + p_3\left(-t + \frac{1}{\nu}\tan\frac{\nu t}{2}\right) \\
&\qquad + g\left(\frac{t^2}{2} - \frac{1}{\nu^2} + \frac{t}{\nu}\operatorname{cotan}\nu t\right);
\end{aligned}\right\} (170.)$$

that is,

$$\left.\begin{aligned}
\lambda_1 &= p_1\cos\mu t - e_1\mu\sin\mu t, \\
\lambda_2 &= p_2\cos\mu t - e_2\mu\sin\mu t, \\
\lambda_3 &= p_3\cos\nu t - e_3\nu\sin\nu t + g\left(t - \frac{1}{\nu}\sin\nu t\right),
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (171.)$$

and

$$\left.\begin{aligned}
x_1 &= e_1(\cos\mu t + \mu t\sin\mu t) + p_1\left(\frac{1}{\mu}\sin\mu t - t\cos\mu t\right), \\
x_2 &= e_2(\cos\mu t + \mu t\sin\mu t) + p_2\left(\frac{1}{\mu}\sin\mu t - t\cos\mu t\right), \\
x_3 &= e_3(\cos\nu t + \nu t\sin\nu t) + p_3\left(\frac{1}{\nu}\sin\nu t - t\cos\nu t\right) \\
&\qquad - g\left(\frac{\operatorname{vers}\nu t}{\nu^2} - \frac{t}{\nu}\sin\nu t + \frac{t^2}{2}\right).
\end{aligned}\right\} \quad . \quad (172.)$$

Accordingly, these rigorous expressions for the 6 varying elements, in the present dynamical question, agree with the results obtained by the ordinary methods of integration from the 6 ordinary differential equations (150.) and (151.), and with those obtained by elimination from the equations (113.) (114.) (147.).

Remarks on the foregoing Example.

30. The example which has occupied us in the last six numbers is not altogether ideal, but is realised to some extent by the motion of a projectile in a void. For if we consider the earth as a sphere, of radius R, and suppose the accelerating force of gravity to vary inversely as the square of the distance r from its centre, and to be $= g$ at the surface, this force will be represented generally by $\frac{g\,R^2}{r^2}$; and to adapt the differential equations (78.) to the motion of a projectile in a void, it will be sufficient to make

$$U = g\,R^2\left(\frac{1}{r} - \frac{1}{R}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (173.)$$

If we place the origin of rectangular coordinates at the earth's surface, and suppose the semiaxis of $+ z$ to be directed vertically upwards, we shall have

$$r = \sqrt{(R + z)^2 + x^2 + y^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (174.)$$

and

$$U = - g\,z + \frac{g\,z^2}{R} - \frac{g\,(x^2 + y^2)}{2\,R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (175.)$$

neglecting only those very small terms which have the square of the earth's radius for a divisor: neglecting therefore such terms, the force-function U in this question is of that form (110.) on which all the reasonings of the example have been founded; the small constants μ, ν, being the real and imaginary quantities $\sqrt{\frac{g}{R}}$, $\sqrt{\frac{-2g}{R}}$, respectively. We may therefore apply the results of the recent numbers to the motions of projectiles in a void, by substituting these values for the constants, and altering, where necessary, trigonometrical to exponential functions. But besides the theoretical facility and the little practical importance of researches respecting such projectiles, the results would only be accurate as far as the first negative power (inclusive) of the earth's radius, because the expression (110.) for the force-function U is only accurate so far; and therefore the rigorous and approximate investigations of the six preceding numbers, founded on that expression, are offered only as mathematical illustrations of a general *method*, extending to all problems of dynamics, at least to all those to which the law of living forces applies.

Attracting Systems resumed: Differential Equations of internal or Relative Motion; Integration by the Principal Function.

31. Returning now from this digression on the motion of a single point, to the more important study of an attracting or repelling system, let us resume the differential equations (A.), which may be thus summed up:

$$d\,t\,\delta\,\mathrm{H} = \Sigma\,(d\,\eta\,\delta\,\varpi - d\,\varpi\,\delta\,\eta)\,; \quad \ldots \quad (\mathrm{A}^2.)$$

and in order to separate the absolute motion of the whole system in space from the motions of its points among themselves, let us choose the following marks of position:

$$x_{\prime\prime} = \frac{\Sigma\,.\,m\,x}{\Sigma\,m},\ y_{\prime\prime} = \frac{\Sigma\,.\,m\,y}{\Sigma\,m},\ z_{\prime\prime} = \frac{\Sigma\,.\,m\,z}{\Sigma\,m}, \quad \ldots \quad (176.)$$

and

$$\xi_i = x_i - x_n,\ \eta_i = y_i - y_n,\ \zeta_i = z_i - z_n\,; \quad \ldots \quad (177.)$$

that is, the 3 rectangular coordinates of the centre of gravity of the system, referred to an origin fixed in space, and the $3\,n-3$ rectangular coordinates of the $n-1$ masses $m_1\ m_2 \ldots m_{n-1}$, referred to the nth mass m_n, as an internal and moveable origin, but to axes parallel to the former. We then find, as in the former Essay,

$$\left.\begin{aligned} \mathrm{T} &= \tfrac{1}{2}\,(x'^{2}_{\prime\prime} + y'^{2}_{\prime\prime} + z'^{2}_{\prime\prime})\,\Sigma\,m \\ &+ \tfrac{1}{2}\,\Sigma_{\prime}\,.\,m\,(\xi'^2 + \eta'^2 + \zeta'^2) - \frac{1}{2\,\Sigma\,m}\{(\Sigma_{\prime}\,.\,m\,\xi')^2 + (\Sigma_{\prime}\,.\,m\,\eta')^2 + (\Sigma_{\prime}\,.\,m\,\zeta')^2\}, \end{aligned}\right\} \quad (178.)$$

the sign of summation $\Sigma_{\prime}$ referring to the first $n-1$ masses only; and therefore,

$$\left.\begin{aligned} \mathrm{T} &= \frac{1}{2\,\Sigma\,m}\left\{\left(\frac{\delta\,\mathrm{T}}{\delta\,x'_{\prime\prime}}\right)^2 + \left(\frac{\delta\,\mathrm{T}}{\delta\,y'_{\prime\prime}}\right)^2 + \left(\frac{\delta\,\mathrm{T}}{\delta\,z'_{\prime\prime}}\right)^2\right\} \\ &+ \tfrac{1}{2}\,\Sigma_{\prime}\,.\,\frac{1}{m}\left\{\left(\frac{\delta\,\mathrm{T}}{\delta\,\xi'}\right)^2 + \left(\frac{\delta\,\mathrm{T}}{\delta\,\eta'}\right)^2 + \left(\frac{\delta\,\mathrm{T}}{\delta\,\zeta'}\right)^2\right\} \\ &+ \frac{1}{2\,m_n}\left\{\left(\Sigma_{\prime}\,\frac{\delta\,\mathrm{T}}{\delta\,\xi'}\right)^2 + \left(\Sigma_{\prime}\,\frac{\delta\,\mathrm{T}}{\delta\,\eta'}\right)^2 + \left(\Sigma_{\prime}\,\frac{\delta\,\mathrm{T}}{\delta\,\zeta'}\right)^2\right\}. \end{aligned}\right\} \quad \ldots \quad (179.)$$

If then we put for abridgement,

$$\left.\begin{aligned} x'_{\prime} &= \frac{1}{m}\,\frac{\delta\,\mathrm{T}}{\delta\,\xi'} = \xi' - \frac{\Sigma_{\prime}\,.\,m\,\xi'}{\Sigma\,m}, \\ y'_{\prime} &= \frac{1}{m}\,\frac{\delta\,\mathrm{T}}{\delta\,\eta'} = \eta' - \frac{\Sigma_{\prime}\,.\,m\,\eta'}{\Sigma\,m}, \\ z'_{\prime} &= \frac{1}{m}\,\frac{\delta\,\mathrm{T}}{\delta\,\zeta'} = \zeta' - \frac{\Sigma_{\prime}\,.\,m\,\zeta'}{\Sigma\,m}, \end{aligned}\right\} \quad \ldots \quad (180.)$$

we shall have the expression

$$\left.\begin{aligned} \mathrm{H} &= \tfrac{1}{2}\,(x'^{2}_{\prime\prime} + y'^{2}_{\prime\prime} + z'^{2}_{\prime\prime})\,\Sigma\,m + \tfrac{1}{2}\,\Sigma_{\prime}\,.\,m\,(x'^{2}_{\prime} + y'^{2}_{\prime} + z'^{2}_{\prime}) \\ &+ \frac{1}{2\,m_n}\{(\Sigma_{\prime}\,.\,m\,x'_{\prime})^2 + (\Sigma_{\prime}\,.\,m\,y'_{\prime})^2 + (\Sigma_{\prime}\,.\,m\,z'_{\prime})^2\} - \mathrm{U}, \end{aligned}\right\} \quad \ldots \quad (\mathrm{B}^2.)$$

of which the variation is to be compared with the following form of (A².),

$$\left.\begin{aligned} dt\delta\mathrm{H} &= (dx_{\prime\prime}\delta x'_{\prime\prime} - dx'_{\prime\prime}\delta x_{\prime\prime} + dy_{\prime\prime}\delta y'_{\prime\prime} - dy'_{\prime\prime}\delta y_{\prime\prime} + dz_{\prime\prime}\delta z'_{\prime\prime} - dz'_{\prime\prime}\delta z_{\prime\prime})\,\Sigma\,m \\ &+ \Sigma_{\prime}.m(d\xi\delta x'_{\prime} - dx'_{\prime}\delta\xi + d\eta\delta y'_{\prime} - dy'_{\prime}\delta\eta + d\zeta\delta z'_{\prime} - dz'_{\prime}\delta\zeta), \end{aligned}\right\} \quad (\mathrm{C}^2.)$$

in order to form, by our general process, $6\,n$ differential equations of motion of the first order, between the $6\,n$ quantities $x_{\prime\prime}\,y_{\prime\prime}\,z_{\prime\prime}\,x'_{\prime\prime}\,y'_{\prime\prime}\,z'_{\prime\prime}\,\xi\,\eta\,\zeta\,x'_{\prime}\,y'_{\prime}\,z'_{\prime}$ and the time t. In thus taking the variation of H, we are to remember that the force-function U depends only on the $3\,n-3$ internal coordinates $\xi\,\eta\,\zeta$, being of the form

$$\left.\begin{array}{l} U = m_n\,(m_1 f_1 + m_2 f_2 + \ldots + m_{n-1} f_{n-1}) \\ \quad + m_1 m_2 f_{1,2} + m_1 m_3 f_{1,3} + \ldots + m_{n-2} m_{n-1} f_{n-2,n-1}, \end{array}\right\} \quad . \quad (D^2.)$$

in which f_i is a function of the distance of m_i from m_n, and $f_{i,k}$ is a function of the distance of m_i from m_k, such that their derived functions or first differential coefficients, taken with respect to the distances, express the laws of mutual repulsion, being negative in the case of attraction; and then we obtain, as we desired, two separate groups of equations, for the motion of the whole system of points in space, and for the motions of those points among themselves; namely, first, the group

$$\left.\begin{array}{l} d\,x_{\prime\prime} = x'_{\prime\prime}\,d\,t,\ d\,x'_{\prime\prime} = 0, \\ d\,y_{\prime\prime} = y'_{\prime\prime}\,d\,t,\ d\,y'_{\prime\prime} = 0, \\ d\,z_{\prime\prime} = z'_{\prime\prime}\,d\,t,\ d\,z'_{\prime\prime} = 0, \end{array}\right\} \quad \ldots \ldots \quad (181.)$$

and secondly the group

$$\left.\begin{array}{l} d\,\xi = \left(x'_{\prime} + \frac{1}{m_n}\,\Sigma_{\prime}\,.\,m\,x'_{\prime}\right) d\,t,\ d\,x'_{\prime} = \frac{1}{m}\,\frac{\delta\,U}{\delta\,\xi}\,d\,t, \\ d\,\eta = \left(y'_{\prime} + \frac{1}{m_n}\,\Sigma_{\prime}\,.\,m\,y'_{\prime}\right) d\,t,\ d\,y'_{\prime} = \frac{1}{m}\,\frac{\delta\,U}{\delta\,\eta}\,d\,t, \\ d\,\zeta = \left(z'_{\prime} + \frac{1}{m_n}\,\Sigma_{\prime}\,.\,m\,z'_{\prime}\right) d\,t,\ d\,z'_{\prime} = \frac{1}{m}\,\frac{\delta\,U}{\delta\,\zeta}\,d\,t. \end{array}\right\} \quad \ldots \quad (182.)$$

The six differential equations of the first order, (181.), between $x_{\prime\prime}\,y_{\prime\prime}\,z_{\prime\prime}\,x'_{\prime\prime}\,y'_{\prime\prime}\,z'_{\prime\prime}$ and t, contain the law of rectilinear and uniform motion of the centre of gravity of the system; and the $6\,n - 6$ equations of the same order, (182.), between the $6\,n - 6$ variables $\xi\,\eta\,\zeta\,x'_{\prime},y'_{\prime},z'_{\prime}$ and the time, are forms for the differential equations of internal or relative motion. We might eliminate the $3\,n-3$ auxiliary variables $x'_{\prime}, y'_{\prime}, z'_{\prime}$ between these last equations, and so obtain the following other group of $3\,n - 3$ equations of the second order, involving only the relative coordinates and the time,

$$\left.\begin{array}{l} \xi'' = \frac{1}{m}\,\frac{\delta\,U}{\delta\,\xi} + \frac{1}{m_n}\,\Sigma_{\prime}\,\frac{\delta\,U}{\delta\,\xi}, \\ \eta'' = \frac{1}{m}\,\frac{\delta\,U}{\delta\,\eta} + \frac{1}{m_n}\,\Sigma_{\prime}\,\frac{\delta\,U}{\delta\,\eta}, \\ \zeta'' = \frac{1}{m}\,\frac{\delta\,U}{\delta\,\zeta} + \frac{1}{m_n}\,\Sigma_{\prime}\,\frac{\delta\,U}{\delta\,\zeta}; \end{array}\right\} \quad \ldots \ldots \quad (183.)$$

but it is better for many purposes to retain them under the forms (182.), omitting, however, for simplicity, the lower accents of the auxiliary variables $x'_{\prime}, y'_{\prime}, z'_{\prime}$, because it is easy to prove that these auxiliary variables (180.) are the components of centrobaric velocity, and because, in investigating the properties of internal or relative motion, we are at liberty to suppose that the centre of gravity of the system is fixed in space, at the origin of $x\,y\,z$. We may also, for simplicity, omit the lower accent of $\Sigma_{\prime}$, understanding that the summations are to fall only on the first $n - 1$ masses, and denoting for greater distinctness the nth mass by a separate symbol M; and then we

may comprise the differential equations of relative motion in the following simplified formula,

$$d t \delta \mathrm{H} = \Sigma \,.\, m \,(d \xi \delta x' - d x' \delta \xi + d \eta \delta y' - d y' \delta \eta + d \zeta \delta z' - d z' \delta \zeta), \quad . \quad (\mathrm{E}^2.)$$

in which

$$\mathrm{H} = \tfrac{1}{2} \Sigma \,.\, m \,(x'^2 + y'^2 + z'^2) + \frac{1}{2\mathrm{M}} \{(\Sigma \,.\, m x')^2 + (\Sigma \,.\, m y')^2 + (\Sigma \,.\, m z')^2\} - \mathrm{U}. \quad (\mathrm{F}^2.)$$

And the integrals of these equations of relative motion are contained (by our general method) in the formula

$$\delta \mathrm{S} = \Sigma \,.\, m \,(x' \delta \xi - a' \delta \alpha + y' \delta \eta - b' \delta \beta + z' \delta \zeta - c' \delta \gamma), \quad . \quad . \quad (\mathrm{G}^2.)$$

in which $\alpha\ \beta\ \gamma\ a'\ b'\ c'$ denote the initial values of $\xi\ \eta\ \zeta\ x'\ y'\ z'$, and S is the *principal function of relative motion* of the system; that is, the former function S, simplified by the omission of the part which vanishes when the centre of gravity is fixed, and which gives in general the laws of motion of that centre, or the integrals of the equations (181.).

Second Example: Case of a Ternary or Multiple System with one Predominant Mass; Equations of the undisturbed motions of the other masses about this, in their several Binary Systems; Differentials of all their Elements, expressed by the coefficients of one Disturbing Function.

32. Let us now suppose that the $n - 1$ masses m are small in comparison with the nth mass M; and let us separate the expression (F².) for H into the two following parts,

$$\left.\begin{aligned} \mathrm{H}_1 &= \Sigma \,.\, \frac{m}{2}\left(1 + \frac{m}{\mathrm{M}}\right)(x'^2 + y'^2 + z'^2) - \mathrm{M}\,\Sigma \,.\, m f, \\ \mathrm{H}_2 &= \frac{m_1\, m_2}{\mathrm{M}} (x'_1 x'_2 + y'_1 y'_2 + z'_1 z'_2 - \mathrm{M} f_{1,2}) + \dots \\ &+ \frac{m_i\, m_k}{\mathrm{M}} (x'_i x'_k + y'_i y'_k + z'_i z'_k - \mathrm{M} f_{i,k}) + \dots, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (\mathrm{H}^2.)$$

of which the latter is small in comparison with the former, and may be neglected in a first approximation. Suppressing it accordingly, we are conducted to the following $6n - 6$ differential equations of the 1st order, belonging to a simpler motion, which may be called the *undisturbed*:

$$\left.\begin{aligned} \frac{d \xi}{d t} &= \frac{1}{m} \frac{\delta \mathrm{H}_1}{\delta x'} = \left(1 + \frac{m}{\mathrm{M}}\right) x'; \quad \frac{d x'}{d t} = -\frac{1}{m} \frac{\delta \mathrm{H}_1}{\delta \xi} = \mathrm{M} \frac{\delta f}{\delta \xi}; \\ \frac{d \eta}{d t} &= \frac{1}{m} \frac{\delta \mathrm{H}_1}{\delta y'} = \left(1 + \frac{m}{\mathrm{M}}\right) y'; \quad \frac{d y'}{d t} = -\frac{1}{m} \frac{\delta \mathrm{H}_1}{\delta \eta} = \mathrm{M} \frac{\delta f}{\delta \eta}; \\ \frac{d \zeta}{d t} &= \frac{1}{m} \frac{\delta \mathrm{H}_1}{\delta z'} = \left(1 + \frac{m}{\mathrm{M}}\right) z'; \quad \frac{d z'}{d t} = -\frac{1}{m} \frac{\delta \mathrm{H}_1}{\delta \zeta} = \mathrm{M} \frac{\delta f}{\delta \zeta}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (\mathrm{I}^2.)$$

These equations arrange themselves in $n - 1$ groups, corresponding to the $n - 1$ binary systems (m, M); and it is easy to integrate the equations of each group separately. We may suppose, then, these integrals found, under the forms,

$$\left.\begin{aligned} \varkappa &= \chi^{(1)}(t, \xi, \eta, \zeta, x', y', z'), & \nu &= \chi^{(4)}(t, \xi, \eta, \zeta, x', y', z'), \\ \lambda &= \chi^{(2)}(t, \xi, \eta, \zeta, x', y', z'), & \tau &= \chi^{(5)}(t, \xi, \eta, \zeta, x', y', z'), \\ \mu &= \chi^{(3)}(t, \xi, \eta, \zeta, x', y', z'), & \omega &= \chi^{(6)}(t, \xi, \eta, \zeta, x', y', z'), \end{aligned}\right\} \quad . \quad . \quad (\mathrm{K}^2.)$$

the six quantities $\varkappa\ \lambda\ \mu\ \nu\ \tau\ \omega$ being constant for the undisturbed motion of any one binary system; and therefore the six functions $\chi^{(1)}, \chi^{(2)}, \chi^{(3)}, \chi^{(4)}, \chi^{(5)}, \chi^{(6)}$, or $\varkappa, \lambda, \mu, \nu, \tau, \omega$, being such as to satisfy *identically* the following equation,

$$0 = m \frac{\delta \varkappa}{\delta t} + \frac{\delta \varkappa}{\delta \xi} \frac{\delta \mathrm{H}_1}{\delta x'} - \frac{\delta \varkappa}{\delta x'} \frac{\delta \mathrm{H}_1}{\delta \xi} + \frac{\delta \varkappa}{\delta \eta} \frac{\delta \mathrm{H}_1}{\delta y'} - \frac{\delta \varkappa}{\delta y'} \frac{\delta \mathrm{H}_1}{\delta \eta} + \frac{\delta \varkappa}{\delta \zeta} \frac{\delta \mathrm{H}_1}{\delta z'} - \frac{\delta \varkappa}{\delta z'} \frac{\delta \mathrm{H}_1}{\delta \zeta}, \quad . \quad (\mathrm{L}^2.)$$

with five other equations analogous, for the five other clements $\lambda, \mu, \nu, \tau, \omega$, in any one binary system (m, M).

33. Returning now to the original multiple system, we may retain as definitions the equations (K^2.), but then we can no longer consider the elements $\varkappa_i\ \lambda_i\ \mu_i\ \nu_i\ \tau_i\ \omega_i$ of the binary system (m_i, M) as constant, because this system is now disturbed by the other masses m_k; however, the $6n - 6$ equations of disturbed relative motion, when put under the forms

$$\left.\begin{aligned}
m \frac{d\xi}{dt} &= \frac{\delta \mathrm{H}_1}{\delta x'} + \frac{\delta \mathrm{H}_2}{\delta x'}, & m \frac{dx'}{dt} &= - \frac{\delta \mathrm{H}_1}{\delta \xi} - \frac{\delta \mathrm{H}_2}{\delta \xi}, \\
m \frac{d\eta}{dt} &= \frac{\delta \mathrm{H}_1}{\delta y'} + \frac{\delta \mathrm{H}_2}{\delta y'}, & m \frac{dy'}{dt} &= - \frac{\delta \mathrm{H}_1}{\delta \eta} - \frac{\delta \mathrm{H}_2}{\delta \eta}, \\
m \frac{d\zeta}{dt} &= \frac{\delta \mathrm{H}_1}{\delta z'} + \frac{\delta \mathrm{H}_2}{\delta z'}, & m \frac{dz'}{dt} &= - \frac{\delta \mathrm{H}_1}{\delta \zeta} - \frac{\delta \mathrm{H}_2}{\delta \zeta},
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (\mathrm{M}^2.)$$

and combined with the identical equations of the kind (L^2.), give the following simple expression for the differential of the element $\varkappa$, in its disturbed and variable state,

$$m \frac{d\varkappa}{dt} = \frac{\delta \varkappa}{\delta \xi} \frac{\delta \mathrm{H}_2}{\delta x'} - \frac{\delta \varkappa}{\delta x'} \frac{\delta \mathrm{H}_2}{\delta \xi} + \frac{\delta \varkappa}{\delta \eta} \frac{\delta \mathrm{H}_2}{\delta y'} - \frac{\delta \varkappa}{\delta y'} \frac{\delta \mathrm{H}_2}{\delta \eta} + \frac{\delta \varkappa}{\delta \zeta} \frac{\delta \mathrm{H}_2}{\delta z'} - \frac{\delta \varkappa}{\delta z'} \frac{\delta \mathrm{H}_2}{\delta \zeta}, \quad . \quad (\mathrm{N}^2.)$$

together with analogous expressions for the differentials of the other elements. And if we express $\xi\ \eta\ \zeta\ x'\ y'\ z'$, and therefore H_2 itself, as depending on the time and on these varying elements, we may transform the $6n - 6$ differential equations of the 1st order, (M^2.), between $\xi\ \eta\ \zeta\ x'\ y'\ z'\ t$, into the same number of equations of the same order between the varying elements and the time; which will be of the forms

$$\left.\begin{aligned}
m \frac{d\varkappa}{dt} &= \{\varkappa, \lambda\} \frac{\delta \mathrm{H}_2}{\delta \lambda} + \{\varkappa, \mu\} \frac{\delta \mathrm{H}_2}{\delta \mu} + \{\varkappa, \nu\} \frac{\delta \mathrm{H}_2}{\delta \nu} + \{\varkappa, \tau\} \frac{\delta \mathrm{H}_2}{\delta \tau} + \{\varkappa, \omega\} \frac{\delta \mathrm{H}_2}{\delta \omega}, \\
m \frac{d\lambda}{dt} &= \{\lambda, \varkappa\} \frac{\delta \mathrm{H}_2}{\delta \varkappa} + \{\lambda, \mu\} \frac{\delta \mathrm{H}_2}{\delta \mu} + \{\lambda, \nu\} \frac{\delta \mathrm{H}_2}{\delta \nu} + \{\lambda, \tau\} \frac{\delta \mathrm{H}_2}{\delta \tau} + \{\lambda, \omega\} \frac{\delta \mathrm{H}_2}{\delta \omega}, \\
m \frac{d\mu}{dt} &= \{\mu, \varkappa\} \frac{\delta \mathrm{H}_2}{\delta \varkappa} + \{\mu, \lambda\} \frac{\delta \mathrm{H}_2}{\delta \lambda} + \{\mu, \nu\} \frac{\delta \mathrm{H}_2}{\delta \nu} + \{\mu, \tau\} \frac{\delta \mathrm{H}_2}{\delta \tau} + \{\mu, \omega\} \frac{\delta \mathrm{H}_2}{\delta \omega}, \\
m \frac{d\nu}{dt} &= \{\nu, \varkappa\} \frac{\delta \mathrm{H}_2}{\delta \varkappa} + \{\nu, \lambda\} \frac{\delta \mathrm{H}_2}{\delta \lambda} + \{\nu, \mu\} \frac{\delta \mathrm{H}_2}{\delta \mu} + \{\nu, \tau\} \frac{\delta \mathrm{H}_2}{\delta \tau} + \{\nu, \omega\} \frac{\delta \mathrm{H}_2}{\delta \omega}, \\
m \frac{d\tau}{dt} &= \{\tau, \varkappa\} \frac{\delta \mathrm{H}_2}{\delta \varkappa} + \{\tau, \lambda\} \frac{\delta \mathrm{H}_2}{\delta \lambda} + \{\tau, \mu\} \frac{\delta \mathrm{H}_2}{\delta \mu} + \{\tau, \nu\} \frac{\delta \mathrm{H}_2}{\delta \nu} + \{\tau, \omega\} \frac{\delta \mathrm{H}_2}{\delta \omega}, \\
m \frac{d\omega}{dt} &= \{\omega, \varkappa\} \frac{\delta \mathrm{H}_2}{\delta \varkappa} + \{\omega, \lambda\} \frac{\delta \mathrm{H}_2}{\delta \lambda} + \{\omega, \mu\} \frac{\delta \mathrm{H}_2}{\delta \mu} + \{\omega, \nu\} \frac{\delta \mathrm{H}_2}{\delta \nu} + \{\omega, \tau\} \frac{\delta \mathrm{H}_1}{\delta \tau},
\end{aligned}\right\} (\mathrm{O}^2.)$$

if we put, for abridgement,

$$\{\varkappa, \lambda\} = \frac{\delta \varkappa}{\delta \xi}\frac{\delta \lambda}{\delta x'} - \frac{\delta \varkappa}{\delta x'}\frac{\delta \lambda}{\delta \xi} + \frac{\delta \varkappa}{\delta \eta}\frac{\delta \lambda}{\delta y'} - \frac{\delta \varkappa}{\delta y'}\frac{\delta \lambda}{\delta \eta} + \frac{\delta \varkappa}{\delta \zeta}\frac{\delta \lambda}{\delta z'} - \frac{\delta \varkappa}{\delta z'}\frac{\delta \lambda}{\delta \zeta}, \quad . \quad . \quad (\mathrm{P}^2.)$$

and form the other symbols $\{\varkappa, \mu\}$, $\{\lambda, \varkappa\}$, &c., from this, by interchanging the letters. It is evident that these symbols have the properties,

$$\{\lambda, \varkappa\} = - \{\varkappa, \lambda\}, \ \{\varkappa, \varkappa\} = 0 ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (184.)$$

and it results from the principles of the 15th number, that these combinations $\{\varkappa, \lambda\}$, &c., when expressed as functions of the elements, do not contain the time explicitly. There are in general, by (184.), only 15 such distinct combinations for each of the $n - 1$ binary systems; but there would thus be, in all, $15\,n - 15$, if they admitted of no further reductions: however, it results from the principles of the 16th number, that $12\,n - 12$ of these combinations may be made to vanish by a suitable choice of the elements. The following is another way of effecting as great a simplification, at least for that extensive class of cases in which the undisturbed distance between the two points of each binary system (m, M) admits of a minimum value.

Simplification of the Differential Expressions by a suitable choice of the Elements.

34. When the undisturbed distance r of m from M admits of such a minimum q, corresponding to a time τ, and satisfying at that time the conditions

$$r' = 0, \ r'' > 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (185.)$$

then the integrals of the group (I^2.), or the known rules of the undisturbed motion of m about M, may be presented in the following manner:

$$\left.\begin{aligned}
\varkappa &= \sqrt{\{(\xi y' - \eta x')^2 + (\eta z' - \zeta y')^2 + (\zeta x' - \xi z')^2\}} ;\\
\lambda &= \varkappa - \xi y' + \eta x' ;\\
\mu &= \frac{\mathrm{M} + m}{2\,\mathrm{M}} (x'^2 + y'^2 + z'^2) - \mathrm{M} f(r) ;\\
\nu &= \tan^{-1} . \frac{\eta z' - \zeta y'}{\xi z' - \zeta x'} ;\\
\tau &= t - \int_q^r \frac{\sqrt{\frac{\mathrm{M}}{\mathrm{M}+m}} . \frac{d\,r}{\sqrt{d\,r^2}} . d\,r}{\sqrt{\left\{2\,\mu + 2\,\mathrm{M} f(r) - \left(1 + \frac{m}{\mathrm{M}}\right) \frac{\varkappa^2}{r^2}\right\}}} ;\\
\omega &= \nu + \sin^{-1} . \frac{\varkappa\,\zeta\,r^{-1}}{\sqrt{2\,\lambda\,\varkappa - \lambda^2}} - \int_q^r \frac{\sqrt{\frac{\mathrm{M}+m}{\mathrm{M}}} . \frac{d\,r}{\sqrt{d\,r^2}} . \frac{\varkappa}{r^2} . d\,r}{\sqrt{\left\{2\,\mu + 2\,\mathrm{M} f(r) - \left(1 + \frac{m}{\mathrm{M}}\right) \frac{\varkappa^2}{r^2}\right\}}} ;
\end{aligned}\right\} . \ (\mathrm{Q}^2.)$$

the minimum distance q being a function of the two elements $\varkappa$, μ, which must satisfy the conditions

$$2\,\mu + 2\,\mathrm{M} f(q) - \left(1 + \frac{m}{\mathrm{M}}\right) \frac{\varkappa^2}{q^2} = 0, \ \mathrm{M} f'(q) + \left(1 + \frac{m}{\mathrm{M}}\right) \frac{\varkappa^2}{q^3} > 0 ; \quad . \quad (186.)$$

and $\sin^{-1} s$, $\tan^{-1} t$, being used (according to Sir JOHN HERSCHEL's notation) to ex-

press, *not* the cosecant and cotangent, but the *inverse functions* corresponding to sine and cosine, or the arcs which are more commonly called arc ($\sin = s$), arc ($\tan = t$). It must also be observed that the factor $\frac{d\,r}{\sqrt{d\,r^2}}$, which we have introduced under the signs of integration, is not superfluous, but is designed to be taken as equal to positive or negative unity, according as $d\,r$ is positive or negative; that is, according as r is increasing or diminishing, so as to make the element under each integral sign constantly positive. In general, it appears to be a useful rule, though not always followed by analysts, to employ the real radical symbol $\sqrt{\mathrm{R}}$ only for positive quantities, unless the negative sign be expressly prefixed; and then $\frac{r}{\sqrt{r^2}}$ will denote positive or negative unity, according as r is positive or negative. The arc given by its sine, in the expression of the element ω, is supposed to be so chosen as to increase continually with the time.

35. After these remarks on the notation, let us apply the formula (P^2.) to calculate the values of the 15 combinations such as $\{\varkappa, \lambda\}$, of the 6 constants or elements (Q^2.). Since

$$r = \sqrt{(\xi^2 + \eta^2 + \zeta^2)}, \quad \ldots \quad (187.)$$

it is easy to perceive that the six combinations of the 4 first elements are as follows:

$$\{\varkappa, \lambda\} = 0,\ \{\varkappa, \mu\} = 0,\ \{\varkappa, \nu\} = 0,\ \{\lambda, \mu\} = 0,\ \{\lambda, \nu\} = 1,\ \{\mu, \nu\} = 0. \quad (188.)$$

To form the 4 combinations of these 4 first elements with τ, we may observe, that this 5th element τ, as expressed in (Q^2.), involves explicitly (besides the time) the distance r, and the two elements $\varkappa, \mu$; but the combinations already determined show that these two elements may be treated as constant in forming the four combinations now sought; we need only attend, therefore, to the variation of r, and if we interpret by the rule (P^2.) the symbols $\{\varkappa, r\}$ $\{\lambda, r\}$ $\{\mu, r\}$ $\{\nu, r\}$, and attend to the equations (I^2.), we see that

$$\{\varkappa, r\} = 0,\ \{\lambda, r\} = 0,\ \{\mu, r\} = -\frac{dr}{dt},\ \{\nu, r\} = 0, \quad \ldots \quad (189.)$$

$\frac{dr}{dt}$ being the total differential coefficient of r in the undisturbed motion, as determined by the equations (I^2.); and, therefore, that

$$\{\varkappa, \tau\} = 0,\ \{\lambda, \tau\} = 0,\ \{\nu, \tau\} = 0, \quad \ldots \quad (190.)$$

and

$$\{\mu, \tau\} = -\frac{\delta \tau}{\delta r}\frac{dr}{dt} = +\frac{dt}{dr}\frac{dr}{dt} = 1: \quad \ldots \quad (191.)$$

observing that in differentiating the expressions of the elements (Q^2.), we may treat those elements as constant, if we change the differentials of $\xi\,\eta\,\zeta\,x'\,y'\,z'$ to their undisturbed values. It remains to calculate the 5 combinations of these 5 elements with the last element ω; which is given by (Q^2.) as a function of the distance r, the coordinate ζ, and the 4 elements $\varkappa, \lambda, \mu, \nu$; so that we may employ this formula,

$$\{e, \omega\} = \frac{\delta \omega}{\delta r}\{e, r\} + \frac{\delta \omega}{\delta \zeta}\{e, \zeta\} + \frac{\delta \omega}{\delta \varkappa}\{e, \varkappa\} + \frac{\delta \omega}{\delta \lambda}\{e, \lambda\} + \frac{\delta \omega}{\delta \mu}\{e, \mu\} + \frac{\delta \omega}{\delta \nu}\{e, \nu\}, \ (192.)$$

in which, if e be any of the first five elements, or the distance r,

$$\{e,r\} = -\frac{1}{r}\left(\xi\frac{\delta e}{\delta x'} + \eta\frac{\delta e}{\delta y'} + \zeta\frac{\delta e}{\delta z'}\right), \{e,\zeta\} = -\frac{\delta e}{\delta z'}, \{e,\varkappa\} = 0, \quad . \quad (193.)$$

and

$$\frac{\delta\omega}{\delta\zeta} = \left(\frac{\delta\varkappa}{\delta z'}\right)^{-1}, \quad \frac{\delta\omega}{\delta r} = -\frac{d\zeta}{dr}\frac{\delta\omega}{\delta\zeta}, \quad \frac{\delta\omega}{\delta\nu} = 1; \quad \ldots \quad (194.)$$

the formula (192.) may therefore be thus written:

$$\left.\begin{array}{l}\{e,\omega\} = \left\{\dfrac{z'\left(\xi\frac{\delta e}{\delta x'} + \eta\frac{\delta e}{\delta y'} + \zeta\frac{\delta e}{\delta z'}\right)}{\xi x' + \eta y' + \zeta z'} - \dfrac{\delta e}{\delta z'}\right\}\left(\dfrac{\delta\varkappa}{\delta z'}\right)^{-1} \\ \qquad + \{e,\nu\} + \dfrac{\delta\omega}{\delta\lambda}\{e,\lambda\} + \dfrac{\delta\omega}{\delta\mu}\{e,\mu\}.\end{array}\right\} \quad \ldots \quad (195.)$$

We easily find, by this formula, that

$$\{\varkappa,\omega\} = -1; \ \{\lambda,\omega\} = 0; \ \{\mu,\omega\} = 0; \ \{r,\omega\} = \frac{dr}{dt}\frac{\delta\omega}{\delta\mu}; \quad \ldots \quad (196.)$$

and

$$\{\nu,\omega\} = -\frac{\delta\nu}{\delta z'}\frac{\delta\omega}{\delta\zeta} - \frac{\delta\omega}{\delta\lambda} = 0. \quad \ldots \quad (197.)$$

The formula (195.) extends to the combination $\{\tau,\omega\}$ also; but in calculating this last combination we are to remember that τ is given by (Q^2.) as a function of $\varkappa, \mu, r$, such that

$$\frac{\delta\tau}{\delta r} = -\frac{dt}{dr}; \quad \ldots \quad (198.)$$

and thus we see, with the help of the combinations (196.) already determined, that

$$\{\tau,\omega\} = -\frac{\delta\tau}{\delta\varkappa} - \frac{\delta\omega}{\delta\mu} = \frac{\delta}{\delta\varkappa}\int_q^r \Theta_r\, dr + \frac{\delta}{\delta\mu}\int_q^r \Omega_r\, dr, \quad \ldots \quad (199.)$$

if we represent for abridgement by Θ_r and Ω_r the coefficients of dr under the integral signs in (Q^2.), namely,

$$\left.\begin{array}{l}\Theta_r = \sqrt{\dfrac{M}{M+m}}\dfrac{dr}{\sqrt{dr^2}}\left\{2\mu + 2Mf(r) - \dfrac{M+m}{M}\cdot\dfrac{\varkappa^2}{r^2}\right\}^{-\frac{1}{2}} \\ \Omega_r = \dfrac{\varkappa}{r^2}\sqrt{\dfrac{M+m}{M}}\dfrac{dr}{\sqrt{dr^2}}\left\{2\mu + 2Mf(r) - \dfrac{M+m}{M}\cdot\dfrac{\varkappa^2}{r^2}\right\}^{-\frac{1}{2}}.\end{array}\right\} \quad . \quad . \quad (200.)$$

These coefficients are evidently connected by the relation

$$\frac{\delta\Theta_r}{\delta\varkappa} + \frac{\delta\Omega_r}{\delta\mu} = 0, \quad \ldots \quad (201.)$$

which gives

$$\frac{\delta}{\delta\varkappa}\int_{r_\prime}^r \Theta_r\, dr + \frac{\delta}{\delta\mu}\int_{r_\prime}^r \Omega_r\, dr = 0, \quad \ldots \quad (202.)$$

$r_\prime$ being any quantity which does not vary with the elements $\varkappa$ and μ; we might therefore at once conclude by (199.) that the combination $\{\tau,\omega\}$ vanishes, if a diffi-

culty were not occasioned by the necessity of varying the lower limit q, which depends on those two elements, and by the circumstance that at this lower limit the coefficients Θ_r Ω_r become infinite. However, the relatión (202.) shows that we may express this combination $\{\tau, \omega\}$ as follows:

$$\{\tau, \omega\} = \frac{\delta}{\delta \varkappa} \int_q^{r_i} \Theta_r \, d\, r + \frac{\delta}{\delta \mu} \int_q^{r_i} \Omega_r \, d\, r, \quad \ldots \ldots \ldots \quad (203.)$$

r_i being an auxiliary and arbitrary quantity, which cannot really affect the result, but may be made to facilitate the calculation; or in other words, we may assign to the distance r any arbitrary value, not varying for infinitesimal variations of $\varkappa$, μ, which may assist in calculating the value of the expression (199.). We may therefore suppose that the increase of distance $r - q$ is small, and corresponds to a small positive interval of time $t - \tau$, during which the distance r and its differential coefficient r' are constantly increasing; and then after the first moment τ, the quantity

$$\Theta_r = \frac{1}{r'} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (204.)$$

will be constantly finite, positive, and decreasing, during the same interval, so that its integral must be greater than if it had constantly its final value; that is,

$$t - \tau = \int_q^r \Theta_r \, d\, r > (r - q)\, \Theta_r. \quad \ldots \ldots \ldots \ldots \quad (205.)$$

Hence, although Θ_r tends to infinity, yet $(r - q)\, \Theta_r$ tends to zero, when by diminishing the interval we make r tend to q; and therefore the following difference

$$\int_q^r \Omega_r \, d\, r - \frac{\mathrm{M}+m}{\mathrm{M}} \frac{\varkappa}{q^2} \int_q^r \Theta_r \, d\, r = \frac{\mathrm{M}+m}{\mathrm{M}} \int_q^r \left(\frac{\varkappa}{r^2} - \frac{\varkappa}{q^2}\right) \Theta_r \, d\, r, \quad . \; . \quad (206.)$$

will also tend to 0, and so will also its partial differential coefficient of the first order, taken with respect to μ. We find therefore the following formula for $\{\tau, \omega\}$, (remembering that this combination has been shown to be independent of r,)

$$\{\tau, \omega\} = \overset{\Lambda}{r = q} \left\{ \frac{\delta}{\delta \varkappa} \int_q^r \Theta_r \, d\, r + \frac{\mathrm{M}+m}{\mathrm{M}} \frac{\varkappa}{q^2} \frac{\delta}{\delta \mu} \int_q^r \Theta_r \, d\, r \right\}; \quad . \; . \; . \quad (207.)$$

the sign $\overset{\Lambda}{r = q}$ implying that the limit is to be taken to which the expression tends when r tends to q. In this last formula, as in (199.), the integral $\int_q^r \Theta_r \, d\, r$ may be considered as a known function of $r, q, \varkappa, \mu$, or simply of $r, q, \varkappa$, if μ be eliminated by the first condition (186.); and since it vanishes independently of $\varkappa$ when $r = q$, it may be thus denoted:

$$\int_q^r \Theta_r \, d\, r = \varphi\, (r, q, \varkappa) - \varphi\, (q, q, \varkappa), \quad \ldots \ldots \ldots \ldots \quad (208.)$$

the form of the function φ depending on the law of attraction or repulsion. This integral therefore, when considered as depending on $\varkappa$ and μ, by depending on $\varkappa$ and q, need not be varied with respect to $\varkappa$, in calculating $\{\tau, \omega\}$ by (207.), because

its partial differential coefficient $\left(\frac{\delta}{\delta \varkappa}\int_q^r \Theta_r\, d r\right)$, obtained by treating q as constant, vanishes at the limit $r = q$; nor need it be varied with respect to q, because, by (186.),

$$\frac{\delta q}{\delta \varkappa} + \frac{\mathrm{M} + m}{\mathrm{M}} \frac{\varkappa}{q^2} \frac{\delta q}{\delta \mu} = 0: \quad \ldots \quad (209.)$$

it may therefore be treated as constant, and we find at last

$$\{\tau, \omega\} = 0, \quad \ldots \quad (210.)$$

the two terms (199.) or (203.) both tending to infinity when r tends to q, but always destroying each other.

36. Collecting now our results, and presenting for greater clearness each combination under the two forms in which it occurs when the order of the elements is changed, we have, for each binary system, the following thirty expressions:

$$\left.\begin{array}{l}
\{\varkappa, \lambda\} = 0,\ \{\varkappa, \mu\} = 0,\ \{\varkappa, \nu\} = 0,\ \{\varkappa, \tau\} = 0,\ \{\varkappa, \omega\} = -1, \\
\{\lambda, \varkappa\} = 0,\ \{\lambda, \mu\} = 0,\ \{\lambda, \nu\} = 1,\ \{\lambda, \tau\} = 0,\ \{\lambda, \omega\} = 0, \\
\{\mu, \varkappa\} = 0,\ \{\mu, \lambda\} = 0,\ \{\mu, \nu\} = 0,\ \{\mu, \tau\} = 1,\ \{\mu, \omega\} = 0, \\
\{\nu, \varkappa\} = 0,\ \{\nu, \lambda\} = -1,\ \{\nu, \mu\} = 0,\ \{\nu, \tau\} = 0,\ \{\nu, \omega\} = 0, \\
\{\tau, \varkappa\} = 0,\ \{\tau, \lambda\} = 0,\ \{\tau, \mu\} = -1,\ \{\tau, \nu\} = 0,\ \{\tau, \omega\} = 0, \\
\{\omega, \varkappa\} = 1,\ \{\omega, \lambda\} = 0,\ \{\omega, \mu\} = 0,\ \{\omega, \nu\} = 0,\ \{\omega, \tau\} = 0;
\end{array}\right\} \quad . \quad . \quad (\mathrm{R}^2.)$$

so that the three combinations

$$\{\mu, \tau\}\ \{\omega, \varkappa\}\ \{\lambda, \nu\}$$

are each equal to positive unity; the three inverse combinations

$$\{\tau, \mu\}\ \{\varkappa, \omega\}\ \{\nu, \lambda\}$$

are each equal to negative unity; and all the others vanish. The six differential equations of the first order, for the 6 varying elements of any one binary system (m, M), are therefore, by (O^2.),

$$\left.\begin{array}{ll}
m \frac{d \mu}{d t} = \frac{\delta \mathrm{H}_2}{\delta \tau}, & m \frac{d \tau}{d t} = -\frac{\delta \mathrm{H}_2}{\delta \mu}, \\
m \frac{d \omega}{d t} = \frac{\delta \mathrm{H}_2}{\delta \varkappa}, & m \frac{d \varkappa}{d t} = -\frac{\delta \mathrm{H}_2}{\delta \omega}, \\
m \frac{d \lambda}{d t} = \frac{\delta \mathrm{H}_2}{\delta \nu}, & m \frac{d \nu}{d t} = -\frac{\delta \mathrm{H}_2}{\delta \lambda};
\end{array}\right\} \quad \ldots \quad (\mathrm{S}^2.)$$

and, if we still omit the variation of t, they may all be summed up in this form for the variation of H_2,

$$\delta \mathrm{H}_2 = \Sigma \,.\, m\,(\mu' \delta \tau - \tau' \delta \mu + \omega' \delta \varkappa - \varkappa' \delta \omega + \lambda' \delta \nu - \nu' \delta \lambda), \quad . \quad . \quad (\mathrm{T}^2.)$$

which single formula enables us to derive all the $6\,n - 6$ differential equations of the first order, for all the varying elements of all the binary systems, from the variation or from the partial differential coefficients of a single quantity H_2, expressed as a function of those elements.

If we choose to introduce into the expression (T^2.), for δH_2, the variation of the time t, we have only to change $\delta \tau$ to $\delta \tau - \delta t$, because, by (Q^2.), δt enters only so accompanied; that is, t enters only under the form $t - \tau_i$, in the expressions of $\xi_i\, \eta_i\, \zeta_i\, x'_i\, y'_i\, z'_i$ as functions of the time and of the elements; we have, therefore,

$$\frac{\delta H_2}{\delta t} = - \Sigma \frac{\delta H_2}{\delta \tau} = - \Sigma \,.\, m \mu' ; \quad \ldots \quad (211.)$$

and since, by (H^2.), (Q^2.),

$$H_1 = \Sigma \,.\, m \mu, \quad \ldots \quad (212.)$$

we find finally,

$$\frac{d H_1}{dt} = - \frac{\delta H_2}{\delta t}. \quad \ldots \quad (U^2.)$$

This remarkable form for the differential of H_1, considered as a varying element, is general for all problems of dynamics. It may be deduced by the general method from the formulæ of the 13th and 14th numbers, which give

$$\left.\begin{aligned} \frac{d H_1}{d t} &= \frac{\delta H_2}{\delta \varkappa_1} \Sigma \left(\frac{\delta H_1}{\delta \eta} \frac{\delta \varkappa_1}{\delta \varpi} - \frac{\delta H_1}{\delta \varpi} \frac{\delta \varkappa_1}{\delta \eta} \right) + \ldots + \frac{\delta H_2}{\delta \varkappa_{6n}} \Sigma \left(\frac{\delta H_1}{\delta \eta} \frac{\delta \varkappa_{6n}}{\delta \varpi} - \frac{\delta H_1}{\delta \varpi} \frac{\delta \varkappa_{6n}}{\delta \eta} \right) \\ &= \frac{\delta H_2}{\delta \varkappa_1} \frac{\delta \varkappa_1}{\delta t} + \frac{\delta H_2}{\delta \varkappa_2} \frac{\delta \varkappa_2}{\delta t} + \ldots + \frac{\delta H_2}{\delta \varkappa_{6n}} \frac{\delta \varkappa_{6n}}{\delta t} = - \frac{\delta H_2}{\delta t}, \end{aligned}\right\} \quad (213.)$$

$\varkappa_1\, \varkappa_2 \ldots \varkappa_{6n}$ being any $6n$ elements of a system expressed as functions of the time and of the quantities $\eta\, \varpi$; or more concisely by this special consideration, that $H_1 + H_2$ is constant in the disturbed motion, and that in taking the first total differential coefficient of H_2 with respect to the time, the elements may by (F^1.) be treated as constant. It is also a remarkable corollary of the general principles just referred to, but one not difficult to verify, that the first partial differential coefficient $\frac{\delta \varkappa_s}{\delta t}$ of any element $\varkappa_s$, taken with respect to the time, may be expressed as a function of the elements alone, not involving the time explicitly.

On the essential distinction between the Systems of Varying Elements considered in this Essay and those hitherto employed by mathematicians.

37. When we shall have integrated the differential equations of varying elements (S^2.), we can then calculate the varying relative coordinates $\xi\, \eta\, \zeta$, for any binary system (m, M), by the rules of undisturbed motion, as expressed by the equations (I^2.), (Q^2.), or by the following connected formulæ:

$$\left.\begin{aligned} \xi &= r \left(\cos \theta + \frac{\lambda}{\varkappa} \sin (\theta - \nu) \sin \nu \right), \\ \eta &= r \left(\sin \theta - \frac{\lambda}{\varkappa} \sin (\theta - \nu) \cos \nu \right), \\ \zeta &= \frac{r}{\varkappa} \sqrt{2 \lambda \varkappa - \lambda^2} \sin (\theta - \nu) : \end{aligned}\right| \quad \ldots \quad (V^2.)$$

in which the distance r is determined as a function of the time t and of the elements τ, $\varkappa$, μ, by the 5th equation (Q².); and in which

$$\theta = \omega + \int_q^r \frac{\sqrt{\frac{\mathrm{M}+m}{\mathrm{M}}} \cdot \frac{d\,r}{\sqrt{d\,r^2}} \cdot \frac{\varkappa}{r^2}\,d\,r}{\sqrt{\left\{2\,\mu + 2\,\mathrm{M}\,f(r) - \frac{\mathrm{M}+m}{\mathrm{M}} \cdot \frac{\varkappa^2}{r^2}\right\}}}, \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{W}^2)$$

q being still the minimum of r, when the orbit is treated as constant, and being still connected with the elements $\varkappa$, μ, by the first equation of condition (186.). In astronomical language, M is the sun, m a planet, $\xi\ \eta\ \zeta$ are the heliocentric rectangular coordinates, r is the radius vector, θ the longitude in the orbit, ω the longitude of the perihelion, ν of the node, $\theta - \omega$ is the true anomaly, $\theta - \nu$ the argument of latitude, μ the constant part of the half square of undisturbed heliocentric velocity, diminished in the ratio of the sun's mass (M) to the sum ($\mathrm{M} + m$) of masses of sun and planet, $\varkappa$ is the double of the areal velocity diminished in the same ratio, $\frac{\lambda}{\varkappa}$ is the versed sine of the inclination of the orbit, q the perihelion distance, and τ the time of perihelion passage. The law of attraction or repulsion is here left undetermined; for NEWTON's law, μ is the sun's mass divided by the axis major of the orbit taken negatively, and $\varkappa$ is the square root of the semiparameter, multiplied by the sun's mass, and divided by the square root of the sum of the masses of sun and planet. But the varying ellipse or other orbit, which the foregoing formulæ require, differs essentially (though little) from that hitherto employed by astronomers: because it gives correctly the heliocentric coordinates, but *not* the heliocentric components of velocity, without differentiating the elements in the calculation; and therefore does *not touch*, but *cuts*, (though under a very small angle,) the actual heliocentric orbit, described under the influence of all the disturbing forces.

38. For it results from the foregoing theory, that if we differentiate the expressions (V².) for the heliocentric coordinates, without differentiating the elements, and then assign to those new varying elements their values as functions of the time, obtained from the equations (S².), and deduce the centrobaric components of velocity by the formulæ (I².), or by the following:

$$x' = \frac{\mathrm{M}\,\xi'}{\mathrm{M}+m}, \quad y' = \frac{\mathrm{M}\,\eta'}{\mathrm{M}+m}, \quad z' = \frac{\mathrm{M}\,\zeta'}{\mathrm{M}+m}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (214.)$$

then these centrobaric components will be the same functions of the time and of the new varying elements which might be otherwise deduced by elimination from the integrals (Q².), and will represent rigorously (by the extension given in the theory to those last-mentioned integrals) the components of velocity of the disturbed planet m, relatively to the centre of gravity of the whole solar system. We chose, as more suitable to the general course of our method, that these centrobaric components of velocity should be the auxiliary variables to be combined with the heliocentric coordinates, and to have their disturbed values rigorously expressed by the formulæ

of undisturbed motion; but in making this choice it became necessary to modify these latter formulæ, and to determine a varying orbit essentially distinct in theory (though little differing in practice) from that conceived so beautifully by LAGRANGE. The orbit which he imagined was more simply connected with the heliocentric motion of a *single planet*, since it gave, for such heliocentric motion, the velocity as well as the position; the orbit which we have chosen is perhaps more closely combined with the conception of a *multiple system*, moving about its common centre of gravity, and influenced in every part by the actions of all the rest. Whichever orbit shall be hereafter adopted by astronomers, they will remember that both are equally fit to represent the celestial appearances, if the numeric elements of either set be suitably determined by observation, and the elements of the other set of orbits be deduced from these by calculation. Meantime mathematicians will judge, whether in sacrificing a part of the simplicity of that geometrical conception on which the theories of LAGRANGE and POISSON are founded, a simplicity of another kind has not been introduced, which was wanting in those admirable theories; by our having succeeded in expressing rigorously the differentials of *all* our own new varying elements through the coefficients of a *single* function: whereas it has seemed necessary hitherto to employ one function for the Earth disturbed by Venus, and another function for Venus disturbed by the Earth.

Integration of the Simplified Equations, which determine the new varying Elements.

39. The simplified differential equations of varying elements, (S^2.), are of the same form as the equations (A.), and may be integrated in a similar manner. If we put, for abridgement,

$$(\tau, \varkappa, \nu) = \int_0^t \left\{ \Sigma \left(\tau \frac{\delta H_2}{\delta \tau} + \varkappa \frac{\delta H_2}{\delta \varkappa} + \nu \frac{\delta H_2}{\delta \nu} \right) - H_2 \right\} dt, \quad . \quad . \quad (X^2.)$$

and interpret similarly the symbols (μ, ω, λ), &c., we can easily assign the variations of the following 8 combinations, $(\tau, \varkappa, \nu)$ (μ, ω, λ) $(\mu, \varkappa, \nu)$ (τ, ω, λ) (τ, ω, ν) $(\mu, \varkappa, \lambda)$ $(\tau, \varkappa, \lambda)$ (μ, ω, ν); namely,

$$\left.\begin{aligned}
\delta(\tau,\varkappa,\nu) &= \Sigma \,.\, m\,(\tau\,\delta\mu - \tau_0\,\delta\mu_0 + \varkappa\,\delta\omega - \varkappa_0\,\delta\omega_0 + \nu\,\delta\lambda - \nu_0\,\delta\lambda_0) - H_2\,\delta t,\\
\delta(\mu,\omega,\lambda) &= \Sigma \,.\, m\,(\mu_0\,\delta\tau_0 - \mu\,\delta\tau + \omega_0\,\delta\varkappa_0 - \omega\,\delta\varkappa + \lambda_0\,\delta\nu_0 - \lambda\,\delta\nu) - H_2\,\delta t,\\
\delta(\mu,\varkappa,\nu) &= \Sigma \,.\, m\,(\mu_0\,\delta\tau_0 - \mu\,\delta\tau + \varkappa\,\delta\omega - \varkappa_0\,\delta\omega_0 + \nu\,\delta\lambda - \nu_0\,\delta\lambda_0) - H_2\,\delta t,\\
\delta(\tau,\omega,\lambda) &= \Sigma \,.\, m\,(\tau\,\delta\mu - \tau_0\,\delta\mu_0 + \omega_0\,\delta\varkappa_0 - \omega\,\delta\varkappa + \lambda_0\,\delta\nu_0 - \lambda\,\delta\nu) - H_2\,\delta t,\\
\delta(\tau,\omega,\nu) &= \Sigma \,.\, m\,(\tau\,\delta\mu - \tau_0\,\delta\mu_0 + \omega_0\,\delta\varkappa_0 - \omega\,\delta\varkappa + \nu\,\delta\lambda - \nu_0\,\delta\lambda_0) - H_2\,\delta t,\\
\delta(\mu,\varkappa,\lambda) &= \Sigma \,.\, m\,(\mu_0\,\delta\tau_0 - \mu\,\delta\tau + \varkappa\,\delta\omega - \varkappa_0\,\delta\omega_0 + \lambda_0\,\delta\nu_0 - \lambda\,\delta\nu) - H_2\,\delta t,\\
\delta(\tau,\varkappa,\lambda) &= \Sigma \,.\, m\,(\tau\,\delta\mu - \tau_0\,\delta\mu_0 + \varkappa\,\delta\omega - \varkappa_0\,\delta\omega_0 + \lambda_0\,\delta\nu_0 - \lambda\,\delta\nu) - H_2\,\delta t,\\
\delta(\mu,\omega,\nu) &= \Sigma \,.\, m\,(\mu_0\,\delta\tau_0 - \mu\,\delta\tau + \omega_0\,\delta\varkappa_0 - \omega\,\delta\varkappa + \nu\,\delta\lambda - \nu_0\,\delta\lambda_0) - H_2\,\delta t,
\end{aligned}\right\} (Y^2.)$$

$\varkappa_0\ \lambda_0\ \mu_0\ \nu_0\ \tau_0\ \omega_0$ being the initial values of the varying elements $\varkappa\ \lambda\ \mu\ \nu\ \tau\ \omega$. If, then, we consider, for example, the first of these 8 combinations $(\tau, \varkappa, \nu)$, as a function of

all the $3n-3$ elements $\mu_i\ \omega_i\ \lambda_i$, and of their initial values $\mu_{0,i}\ \omega_{0,i}\ \lambda_{0,i}$, involving also in general the time explicitly, we shall have the following forms for the $6n-6$ rigorous integrals of the $6n-6$ equations (S^2.):

$$\left.\begin{array}{l} m_i\tau_i = \frac{\delta}{\delta\mu_i}(\tau, \varkappa, \nu)\,;\ m_i\tau_{0,i} = -\frac{\delta}{\delta\mu_{0,i}}(\tau, \varkappa, \nu)\,; \\ m_i\varkappa_i = \frac{\delta}{\delta\omega_i}(\tau, \varkappa, \nu)\,;\ m_i\varkappa_{0,i} = -\frac{\delta}{\delta\omega_{0,i}}(\tau, \varkappa, \nu)\,; \\ m_i\nu_i = \frac{\delta}{\delta\lambda_i}(\tau, \varkappa, \nu)\,;\ m_i\nu_{0,i} = -\frac{\delta}{\delta\lambda_{0,i}}(\tau, \varkappa, \nu)\,; \end{array}\right\} \quad \text{(Z}^2.)$$

and in like manner we can deduce forms for the same rigorous integrals, from any one of the eight combinations (Y^2.). The determination of all the varying elements would therefore be fully accomplished, if we could find the complete expression for any one of these 8 combinations.

40. A first approximate expression for any one of them can be found from the form under which we have supposed H_2 to be put, namely, as a function of the elements and of the time, which may be thus denoted:

$$H_2 = H_2(t, \varkappa_1, \lambda_1, \mu_1, \nu_1, \tau_1, \omega_1, \ldots \varkappa_{n-1}, \lambda_{n-1}, \mu_{n-1}, \nu_{n-1}, \tau_{n-1}, \omega_{n-1})\,; \quad \text{(A}^3.)$$

by changing in this function the varying elements to their initial values, and employing the following approximate integrals of the equations (S^2.),

$$\left.\begin{array}{l} \mu = \mu_0 + \frac{1}{m}\int_0^t \frac{\delta H_2}{\delta\tau_0}\,dt,\ \tau = \tau_0 - \frac{1}{m}\int_0^t \frac{\delta H_2}{\delta\mu_0}\,dt, \\ \omega = \omega_0 + \frac{1}{m}\int_0^t \frac{\delta H_2}{\delta\varkappa_0}\,dt,\ \varkappa = \varkappa_0 - \frac{1}{m}\int_0^t \frac{\delta H_2}{\delta\omega_0}\,dt, \\ \lambda = \lambda_0 + \frac{1}{m}\int_0^t \frac{\delta H_2}{\delta\nu_0}\,dt,\ \nu = \nu_0 - \frac{1}{m}\int_0^t \frac{\delta H_2}{\delta\lambda_0}\,dt. \end{array}\right\} \quad \text{(B}^3.)$$

For if we denote, for example, the first of the 8 combinations (Y^2.) by G, so that

$$G = \{\tau, \varkappa, \nu\}, \quad \text{(C}^3.)$$

we shall have, as a first approximate value,

$$G_1 = \int_0^t \left\{ \Sigma\left(\tau_0\frac{\delta H_2}{\delta\tau_0} + \varkappa_0\frac{\delta H_2}{\delta\varkappa_0} + \nu_0\frac{\delta H_2}{\delta\nu_0}\right) - H_2 \right\} dt\,; \quad \text{(D}^3.)$$

and after thus expressing G_1 as a function of the time, and of the initial elements, we can eliminate the initial quantities of the forms $\tau_0\ \varkappa_0\ \nu_0$, and introduce in their stead the final quantities $\mu\ \omega\ \lambda$, so as to obtain an expression for G_1 of the kind supposed in (Z^2.), namely, a function of the time t, the varying elements $\mu\ \omega\ \lambda$, and their initial values $\mu_0\ \omega_0\ \lambda_0$. An approximate expression thus found may be corrected by a process of that kind, which has often been employed in this Essay for other similar purposes. For the function G, or the combination $(\tau, \varkappa, \nu)$, must satisfy rigorously, by (Y^2.) (A^3.), the following partial differential equation:

$$0 = \frac{\delta G}{\delta t} + H_2\left(t, \frac{1}{m_1}\frac{\delta G}{\delta \varpi_1}, \lambda_1, \mu_1, \frac{1}{m_1}\frac{\delta G}{\delta \lambda_1}, \frac{1}{m_1}\frac{\delta G}{\delta \mu_1}, \varpi_1, \frac{1}{m_2}\frac{\delta G}{\delta \varpi_2}, \ldots \varpi_{n-1}\right); \quad . \quad . \quad (\mathrm{E}^3.)$$

and each of the other analogous functions or combinations (Y^2.) must satisfy an analogous equation: if then we change G to $G_1 + G_2$, and neglect the squares and products of the coefficients of the small correction G_2, G_1 being a first approximation such as that already found, we are conducted, as a second approximation, on principles already explained, to the following expression for this correction G_2:

$$G_2 = -\int_0^t \left\{\frac{\delta G_1}{\delta t} + H_2\left(t, \frac{1}{m_1}\frac{\delta G_1}{\delta \varpi_1}, \lambda_1, \mu_1, \frac{1}{m_1}\frac{\delta G_1}{\delta \lambda_1}, \frac{1}{m_1}\frac{\delta G_1}{\delta \mu_1}, \varpi_1, \ldots\right)\right\} dt: \quad (\mathrm{F}^3.)$$

which may be continually and indefinitely improved by a repetition of the same process of correction. We may therefore, theoretically, consider the problem as solved; but it must remain for future consideration, and perhaps for actual trial, to determine which of all these various processes of successive and indefinite approximation, deduced in the present Essay and in the former, as corollaries of one general Method, and as consequences of one central Idea, is best adapted for numeric application, and for the mathematical study of phenomena.

VIII. *Continuation of a former Paper on the Twenty-five Feet Zenith Telescope lately erected at the Royal Observatory.* By John Pond, *Esq. A.R. F.R.S.*

Received March 11,—Read March 12, 1835.

DURING the last summer I had the honour of submitting to this Society a short paper on the subject of the large zenith telescope lately erected at this Observatory.

It is now nearly twenty years since the erection of such an instrument was first suggested to the President and Council of this Society; at that time the Royal Observatory was in a very inefficient state compared to what it is at present. We had only one circle; and there existed doubts as to the excellence of this instrument, though not any were ever entertained by me. The erection of a second circle put this question at rest; it has been abundantly shown in various volumes of the Greenwich Observations, by a series of more rigorous investigations than any instrument was ever submitted to before, that both the circles may be considered as perfect, their errors being less than their respective makers themselves assigned.

This circumstance, though satisfactory to myself, a little diminished the importance of the new zenith telescope. It was hardly to be expected that any new instrument could throw light upon errors already reduced within such small limits; this, however, has been done, and the object of this paper is to explain the process I have employed for the purpose.

Whoever is acquainted with the method of constructing the Greenwich Catalogue, must have perceived that the places of those stars which are observed by reflection are, according to all probability, more exactly determined than those which have been observed only by direct vision. γ Draconis, a star which since the time of Bradley has been of first-rate importance in the Greenwich Observations, cannot be observed by reflection. The probability of error was therefore greater in the place of this star than in that of any other. The new instrument has shown that this error does not exceed a quarter of a second; a degree of accuracy scarcely credible, and no doubt requiring to be confirmed by future observations.

The nature of the question to be determined in this case has happily produced a competition for excellence among the observers with the different instruments, which gives me an opportunity of showing the present state of practical astronomy at Greenwich.

The new instrument has been employed during the last summer under very unfavourable circumstances, both the building and the instrument having been almost constantly under repair. It is not requisite on this occasion to enter into the details

of these difficulties; I only wish to explain the nature of the experiments, the results of which I am now about to lay before the Society.

We have now three distinct methods of determining the place of any star passing the meridian near the zenith. First, by means of the mural circles; secondly, by the zenith telescope used alternately east and west, as is usually done with similar instruments; and lastly, by means of a small subsidiary star, as described by me last year in a paper laid before this Society, and which I am inclined to think more exact than any other method. By the following computations it will be seen that the three methods give results nearly identical; and that when the observations with the two circles are numerous and made with sufficient care, a quarter of a second is the greatest error to be apprehended.

Royal Observatory, March 10, 1835.

Results of Observations on γ Draconis and Bode 170 *Draconis.*

Zenith distance of γ Draconis determined by three different methods.

	Zenith distance, 1834.
First,—Result by 324 observations with the Mural Circles reduced to the latitude of the Zenith Telescope room, the difference between which and the Circle room being 0″·65 North	2′ 1″·36 North.
By Zenith Telescope employed in the unsual manner by alternate observations East and West; 28 results	2 1·11
By means of the subsidiary angle as described in my former paper of last year, and which result I prefer to either of the others	2 1·09

Zenith distance of Bode 170 Draconis determined by three different methods.

	Zenith distance, 1834.
First,—Result by 132 observations with the Mural Circles reduced to the latitude of the Zenith Telescope room, the difference between which and the Circle room being 0″·65 North	1′ 0″·45 South.
By Zenith Telescope employed in the usual manner by alternate observations East and West; 14 results	1 0·61
By means of the subsidiary angle as described in my former paper of last year, and which result I prefer to either of the others	1 0·74

Table I.

Containing 60 successive observations of the small auxiliary star, Bode 170 Draconis, divided into series of 10 each.

1833.	Bode 170 Draconis, by Jones's Circle.			1834.	Bode 170 Draconis, by Jones's Circle.		
	N. P. D. Jan. 1, 1834.	Diff. of each Obs. from Mean of 60.	Diff. between the Mean of 10 and the Mean of 60.		N. P. D. Jan. 1, 1834.	Diff. of each Obs. from Mean of 60.	Diff. between the Mean of 10 and the Mean of 60.
	° ′ ″	″			° ′ ″	″	
July 22.	38 32 20·68	0·40		July 8.	38 32 21·40	0·32	
23.	21·62	0·54		9.	21·00	0·08	
25.	20·83	0·25		10.	20·68	0·40	
26.	21·70	0·62		11.	21·08	0·00	
27.	21·25	0·17		12.	21·39	0·31	
29.	21·87	0·79		14.	20·99	0·09	
31.	21·73	0·65		15.	20·48	0·60	
Aug. 1.	21·79	0·71		16.	20·97	0·11	
3.	22·17	1·09		17.	20·77	0·31	
4.	21·81	0·73		21.	20·39	0·69	
Mean of 10 obs.	38 32 21·55		0·47	Mean of 10 obs.	38 32 20·92		0·16
6.	21·70	0·62		22.	21·07	0·01	
9.	21·57	0·49		24.	21·84	0·76	
11.	21·09	0·01		25.	20·11	0·97	
13.	20·91	0·17		30.	20·92	0·16	
14.	20·50	0·58		Aug. 1.	20·42	0·66	
16.	20·65	0·43		2.	21·68	0·60	
23.	20·79	0·29		6.	21·24	0·16	
25.	21·20	0·12		11.	21·02	0·06	
26.	20·85	0·23		12.	20·14	0·94	
27.	20·43	0·65		16.	20·86	0·22	
Mean of 10 obs.	38 32 20·97		0·11	Mean of 10 obs.	38 32 20·93		0·15
28.	21·07	0·01		19.	20·63	0·45	
Sept. 1.	20·82	0·26		22.	21·57	0·49	
3.	20·86	0·22		23.	21·36	0·28	
4.	20·86	0·22		25.	20·76	0·32	
5.	21·13	0·05		27.	21·18	0·10	
6.	21·36	0·28		Sept. 4.	20·89	0·19	
12.	21·31	0·23		5.	21·19	0·11	
18.	21·07	0·01		12.	21·21	0·13	
20.	20·95	0·13		13.	21·38	0·30	
23.	20·92	0·16		15.	20·58	0·50	
Mean of 10 obs.	38 32 21·04		0·04	Mean of 10 obs.	38 32 21·08		0·00
Mean of 60 obs. . . . . =					38 32 21·08	0·357	0·155

From this it appears that the mean error of 10 observations = 0″·155, and that the mean error of 30 observations, as deduced from the next page, = 0″·067.

The zenith distance from this result	=	1′ 0″·078 South. (Assumed co-latitude = 38° 31′ 21″*.)
Difference of latitude for zenith telescope	=	+ 0·65
Zenith distance for the latitude of zenith telescope	=	1 0·728
By zenith telescope by means of the subsidiary angle from the preceding page . .	=	1 0·74, which two quantities are identical.

* The accuracy of this quantity is of no importance, as the circles, according to our present mode of employing them, give, in fact, zenith distances, which are afterwards converted into polar distances by the application of the above co-latitude, and as such are registered in the Greenwich Catalogues.

TABLE II.

The same observations of Bode 170 Draconis arranged alternately in two columns of 30 observations each.

	N. P. D. Jan. 1, 1834.		N. P. D. Jan. 1, 1834.
	° ′ ″		° ′ ″
1833. July 22.	38 32 20·68	1833. July 23.	38 32 21·62
25.	20·83	26.	21·70
27.	21·25	29.	21·87
31.	21·73	Aug. 1.	21·79
Aug. 3.	22·17	4.	21·81
6.	21·70	9.	21·57
11.	21·09	13.	20·91
14.	20·50	16.	20·65
23.	20·79	25.	21·20
26.	20·85	27.	20·43
Mean of 10 obs.	38 32 21·159	Mean of 10 obs.	38 32 21·355
Aug. 28.	38 32 21·07	Sept. 1.	38 32 20·82
Sept. 3.	20·86	4.	20·86
5.	21·13	6.	21·36
12.	21·31	18.	21·07
20.	20·95	23.	20·92
1834. July 8.	21·40	1834. July 9.	21·00
10.	20·68	11.	21·08
12.	21·39	14.	20·99
15.	20·48	16.	20·97
17.	20·77	21.	20·39
Mean of 10 obs.	38 32 21·004	Mean of 10 obs.	38 32 20·946
July 22.	38 32 21·07	July 24.	38 32 21·84
25.	20·11	30.	20·92
Aug. 1.	20·42	Aug. 2.	21·68
6.	21·24	11.	21·02
12.	20·14	16.	20·86
19.	20·63	22.	21·57
23.	21·36	25.	20·76
27.	21·18	Sept. 4.	20·89
Sept. 5.	21·19	12.	21·21
13.	22·38	15.	20·58
Mean of 10 obs.	38 32 20·872	Mean of 10 obs.	38 32 21·133
Mean of 30 = 38 32 21·012		Mean of 30 = 38 32 21·145	
Mean of 60 = 38° 32′ 21″·08.			

Table III.

Difference of North Polar Distance of γ Draconis and Bode 170 Draconis, 1833.

1833.	Observed Difference. Troughton.	Observed Difference. Jones.	Difference of Equations.	Difference, January 1, 1833. Troughton.	Difference, January 1, 1833. Jones.
	′ ″	′ ″	″	′ ″	′ ″
July 5.	3 6·2	3 5·4	−1·36	3 4·84	3 4·04
6.	5·3	6·7	−1·35	3·95	5·35
9.	6·0	4·9	−1·31	4·69	3·59
15.	5·5	5·5	−1·18	4·32	4·32
16.	5·9	5·7	−1·15	4·75	4·55
22.	4·8	5·3	−1·04	3·76	4·26
23.	4·5	5·9	−1·01	3·49	4·89
25.	5·1	5·4	−0·98	4·12	4·42
26.	6·0	5·6	−0·95	5·05	4·65
27.	5·7	5·6	−0·94	4·76	4·66
29.	4·9	5·9	−0·89	4·01	5·01
31.	4·6	4·0	−0·84	3·76	3·16
August 1.	3·8	4·7	−0·81	2·99	3·89
3.	4·8	4·9	−0·77	4·03	4·13
4.	6·0	5·2	−0·74	5·26	4·46
6.	4·8	4·0	−0·69	4·11	3·31
9.	5·1	4·8	−0·61	4·49	4·19
11.	2·9	4·7	−0·56	2·34	4·14
13.	3·6	5·4	−0·50	3·10	4·90
14.	4·1	4·6	−0·47	3·63	4·13
16.	3·0	4·3	−0·42	2·58	3·88
21.	3·7	4·6	−0·19	3·51	4·41
25.	3·6	3·9	−0·13	3·47	3·77
26.	3·9	4·2	−0·09	3·81	4·11
27.	4·5	4·4	−0·06	4·44	4·34
28.	4·7	4·7	−0·04	4·66	4·66
Sept. 1.	3·0	3·6	+0·10	3·10	3·70
3.	1·6	3·7	+0·16	1·76	3·86
4.	3·6	3·9	+0·19	3·79	4·09
5.	2·8	3·7	+0·23	3·03	3·93
6.	3·5	4·3	+0·26	3·76	4·56
12.	2·7	3·9	+0·43	3·13	4·33
18.	2·8	3·0	+0·67	3·47	3·67
20.	2·6	3·7	+0·75	3·35	4·45
23.	3·1	3·3	+0·86	3·96	4·16
			Mean of 35 obs. =	3 3·807	3 4·228
Mean of 70 obs. ...... =				3′ 4″·018	

Difference of North Polar Distance of γ Draconis and Bode 170 Draconis, 1834.

1834.	Observed Difference. Troughton.	Observed Difference. Jones.	Difference of Equations.	Difference, January 1, 1834. Troughton.	Difference, January 1, 1834. Jones.
	′ ″	′ ″	″	′ ″	′ ″
July 8.	3 1·4	3 3·6	−1·31	3 0·09	3 2·29
9.	3·6	4·2	−1·30	2·30	2·90
10.	3·5	3·3	−1·28	2·22	2·02
11.	3·7	3·0	−1·27	2·43	1·76
12.	3·1	3·0	−1·24	1·86	1·76
14.	2·7	2·4	−1·21	1·49	1·19
15.	3·3	3·0	−1·20	2·10	1·80
16.	1·9	3·8	−1·19	0·71	2·61
17.	3·8	2·4	−1·17	2·63	1·23
21.	2·7	2·1	−1·08	1·62	1·02
22.	4·3	3·0	−1·04	3·26	1·96
24.	3·3	3·6	−1·01	2·29	2·59
25.	3·2	2·6	−0·98	2·22	1·62
30.	3·2	3·1	−0·86	2·34	2·24
August 2.	2·3	3·4	−0·84	1·46	1·96
6.	2·3	1·8	−0·71	1·59	0·21
11.	3·6	2·6	−0·55	3·05	2·05
12.	2·4	1·3	−0·52	1·88	0·78
16.	2·5	1·0	−0·43	2·07	0·57
19.	3·1	1·6	−0·32	2·78	1·28
22.	2·0	2·3	−0·24	1·76	2·06
23.	2·0	3·0	−0·21	1·79	2·79
25.	1·4	2·0	−0·15	1·25	1·86
27.	2·3	2·1	−0·08	2·22	2·02
Sept. 5.	1·0	1·9	+0·24	1·24	2·14
12.	1·4	1·7	+0·45	1·85	2·15
13.	1·1	2·7	+0·49	1·59	3·19
15.	1·3	0·4	+0·57	1·87	1·97
16.	0·8	1·2	+0·60	1·40	1·80
			Mean of 29 obs. =	3 1·901	3 1·854
Mean of 58 obs. .... =				3′ 1″·877	

		′ ″
Mean of 70 obs. (= 35 × 2) with both circles for the epoch Jan. 1, 1833, as above	=	3 4·018
Sum of Annual Variations of both stars	= −	2·240
Difference of North Polar Distance reduced to Jan. 1, 1834, by 70 obs. in 1833 ..	=	3 1·778
Mean of 58 obs. (= 29 × 2) for Jan. 1, 1834, by 58 obs. in 1834	=	3 1·877
Mean of total 128 obs. Jan. 1, 1834	=	3 1·823

TABLE IV.

Fundamental determinations of the Zenith Distances of γ Draconis.

Epochs.	State of lunar nutation.	Observed zenith dist. reduced to the beginning of each year.	Side of zen.	Zenith distance deduced from M. BESSEL's Formula (TABULÆ REGIOMONTANÆ, p. 46.). Epochs. 1800.	1st term.	2nd term.	Resulting Z. D.	Difference of formula and observed zen. dist.	
1753.	$-6''{\cdot}87$	$3'\ 2''{\cdot}05$	N.	$2'\ 26''{\cdot}669$	$+33''{\cdot}555$	$+2''{\cdot}233$	$3'\ 2''{\cdot}457$	$+0''{\cdot}407$	By BRADLEY with Zen. Sector.
1768.	$+3{\cdot}83$	$2\ 50{\cdot}30$	..	$2\ 26{\cdot}669$	$+22{\cdot}899$	$+1{\cdot}035$	$2\ 50{\cdot}603$	$+0{\cdot}303$	MASKELYNE Ditto.
1802.	$+9{\cdot}52$	$2\ 25{\cdot}30$	..	$2\ 26{\cdot}669$	$-\ 1{\cdot}428$	$+0{\cdot}004$	$2\ 25{\cdot}245$	$-0{\cdot}055$	Ditto. Ditto.
1813.	$-7{\cdot}64$	$2\ 17{\cdot}40$	..	$2\ 26{\cdot}669$	$-\ 9{\cdot}281$	$+0{\cdot}171$	$2\ 17{\cdot}559$	$+0{\cdot}159$	POND Ditto.
1833.	$-4{\cdot}30$	$2\ \ 2{\cdot}47$	..	$2\ 26{\cdot}669$	$-23{\cdot}560$	$+1{\cdot}101$	$2\ \ 4{\cdot}210$	$+1{\cdot}740$	Ditto New Zen. Telescope.

The above results (column 3rd) are those that have been obtained with the greatest care during their respective periods; and having been deduced from observations with the zenith sector, they are quite independent of the latitude of the Observatory.

M. BESSEL's formula is deduced from the observations for the first sixty years, and therefore agree very well; but when we attempt to predict from the observations of these sixty years the place of the stars for twenty years to come, we find a difference of $1''{\cdot}74$ between the predicted and observed zenith distance, the observed place being this quantity south of it.

Explanation of the foregoing TABLES.

Table I. contains the results of 60 observations of the small star Bode 170 Draconis, made with JONES's circle, and is intended to show what degree of accuracy may be obtained by extreme care. The mean difference $0''{\cdot}357$, column 3, between the mean of the whole and each result, (and which is nearly the probable error of a single observation from this series,) demonstrates with what care they have been made. The same may be said with respect to the mean difference of column 4, namely, $0''{\cdot}155$, which is similarly obtained from the mean of the whole and the mean of each ten (a quantity which represents nearly the probable error of the mean of ten observations). However, it may be remarked, that the exact coincidence exhibited throughout this series does not prove the truth of the final north polar distance of the star here assigned, since some omissions or errors in the process of reduction would affect it. That no instrumental error exists is demonstrated by the identity of the result with that obtained with the new instrument.

Table II. contains the same observations arranged in a different manner.

This is the arrangement I have advantageously followed in investigating the difference of parallax; the object being to distinguish the effect arising from accidental error of observation from that which is due to any permanent astronomical cause.

This method should be employed when the object is to judge of the consistency of observations, without any reference to the astronomical result.

Table III. shows the manner in which the difference in zenith distance between the two stars is obtained by means of the circles; a quantity, as I have shown, of the highest importance in the investigation.

This quantity, having been determined by the microscopes of the respective circles, might be erroneous if the runs of the microscopes were not exact, although the error here must be very small, twelve microscopes being constantly used. But as they have lately been taken down, examined, and replaced, without any sensible alteration, it may be presumed that the error from this source is sufficiently corrected.

Table IV. This Table contains in a very compressed form the result of an immense number of observations of γ Draconis during a period of eighty years; and it will be seen that if from M. Bessel's formula*, deduced from the first sixty years of these observations, we attempt to predict or assign the place of the star for the present time or twenty years in advance, the star will be found $1''\cdot75$ south of its computed place.

* By this formula the zenith distance of the star north for 1800, $+\, t = 2'\, 26''\cdot669 - t\,.\,0''\cdot71394 + t^2\,.\,0''\cdot001011$. Where t is the number of years before or after 1800, if before, the sign of t is minus.

IX. *Some account of the Eruption of Vesuvius, which occurred in the month of August 1834, extracted from the Manuscript Notes of the Cavaliere* MONTICELLI, *Foreign Member of the Geological Society, and from other sources; together with a Statement of the Products of the Eruption, and of the condition of the Volcano subsequently to it. By* CHARLES DAUBENY, *M.D. F.R.S. F.G.S. &c., Professor of Chemistry and Botany in the University of Oxford.*

Received February 25,—Read March 19, 1835.

THE eruption of Vesuvius which occurred in the month of August of last year, excited on the spot an unusual share of interest, from the largeness of the volume of lava at the time discharged, and the extent of the damage it occasioned in its progress down the mountain; whilst in a scientific point of view it attracted the greater attention, since it was regarded by many as the concluding link in a series of volcanic operations, which had been going on up to that period with only occasional intermissions from the year 1831.

It was therefore natural, that on my arrival at Naples shortly after the mountain had subsided into a state of comparative repose, I should seize upon the opportunity which appeared to offer of increasing my acquaintance with volcanic phenomena; first, by collecting on the spot such information as could be best relied on, with respect to the leading features of the past eruption; and secondly, by ascertaining from personal examination the actual condition of the volcano, and the products resulting either from its late operations, or from those in actual progress.

With a view to the former object, I solicited and obtained from the Cavaliere MONTICELLI (one of the Foreign Members of the Geological Society) a written account of the eruption, from which he has permitted me to extract such particulars as I might deem likely to interest the Members of the Royal Society; whilst in the hope of accomplishing the latter object, a considerable portion of the time I spent at Naples was taken up in visiting the several parts of Vesuvius, and in collecting the solid as well as aeriform substances, ejected from its crater, and from the recently erupted lava.

In the former part, therefore, of the present communication, I can claim no further share, than as the compiler of facts observed and reported to me by others; and all that I conceive myself personally responsible for is the latter portion, in which I have stated the several products and actual condition of the volcano at the time I visited it.

It would appear that for a considerable time previous to the eruption in question, the crater of the volcano had continued to throw up stones and scoriæ, which falling down for the most part almost perpendicularly round the point of their emission, had by degrees accumulated into two conical masses, which rose up in the midst of the great crater. The largest of these cones is calculated to have been more than 200 feet in height, and possessed at one time a regular pyramidal form, with an appearance of stability.

It is stated, however, by MONTICELLI, that in May last, from the 20th of which month up to the 20th of July, the volcano had continued to throw up stones and ashes, and even to emit lava, both these conical hillocks were observed to be broken away, and to sink towards the south; whence, in a memoir read by him to the Academy of Sciences at Naples on the 5th of August, he predicted their speedy disappearance.

These anticipations were realized at no long period subsequently. On the 22nd of August, after the volcano had continued for a month in a state of apparent repose, volumes of black smoke began to show themselves on the summit of the more recent of the two hillocks above noticed; and after a smart shock of an earthquake, this was succeeded by ejections of red-hot stones and scoriæ, which continued to be shot forth all the night with fresh quakings and rumblings of the soil.

Early on the 23rd, a current of lava was seen to issue from the foot of the great cone which encompasses the crater on its western side, and this bending in the direction of the point called Crocelle, reached the flanks of the rising ground denominated Contaroni, whence, moving continually forwards at the rate of about six feet per minute, and reinforced by a second stream of lava which had burst forth from an adjacent point, it reached about nightfall the path generally taken from the Hermitage to the summit of the mountain, which it completely blocked up.

During the 24th, lava continued to flow from the same points, and to advance down the western declivity of the mountain; and during the night a violent shaking of the volcano, which agitated the whole adjacent country, was apparently coincident with the falling in of both the conical hillocks described as existing in the interior of the crater, no traces of which were visible in the morning. Thus we have here a decided instance of two considerable pyramidal masses of volcanic materials, not blown into the air, as some might suppose to be the case, but actually swallowed up within the cavities of the mountain in the course of a single night.

Up to this time the western side of the volcano had been the point that yielded to the internal pressure, and the inhabitants of Portici and Resina had imagined themselves to be chiefly menaced. But on the evening of the 24th a fresh vent was established on the eastern side of the mountain near the *Grotta del Mauro*, whence the lava of 1817 had issued; and after this had taken place, no more lava was observed to flow from the western side of the cone. On the other hand, the current from the eastern side was reinforced on the morning of the 25th by a second stream,

which issuing forth from the foot of the great cone on the spot called Coutrel, flowed over the preceding one.

On the morning of the 26th, an immense column of black and dense smoke served as the prelude to the bursting forth of a new current of lava from the same point as before, as well as from several others in the neighbourhood; and the whole of this molten mass poured down the mountain in a single narrow stream, circumscribed within the boundaries of a hollow way or water-course. Here, its progress being favoured by the rapid slope of the declivity, it very soon reached Mauro, and took possession of the road leading from Bosco-tre-case to Ottayano.

On the 27th it was augmented by two fresh currents emitted from points not far distant; but now, instead of flowing on in a single stream as before, it became divided into three. The largest of these currents, going straight in the direction of Mauro, spread over some lands belonging to the hamlet of Torcigno; the second covered the cultivated fields above Bosco-reale; and the third invaded the upper part of the village of Bosco-tre-case.

It was the first, however, of these currents which effected the greatest damage. Widening as it descended, it had acquired, by the time it reached the base of the mountain, a breadth of nearly half a mile, retaining even there a depth which averaged from fifteen to eighteen feet.

At Mauro, the Casino of the Prince of Ottayano formed its precise boundary to the north, and one wall of that mansion was swept away by it, whilst all the rest of the building stood uninjured. From this point the lava proceeded to the road which leads from Torre del Annunziata to Ottayano, which it completely blocked up, and moving still further to the eastward, swept away in its course several detached hamlets included in the Commune.

It is calculated, that 180 houses, the abode of about 800 persons, were destroyed by the current, and that 500 acres (moggie) of land were covered over and reduced to sterility by it.

Among the remains of the houses overthrown by the lava, which I was able to examine, no traces of fusion were visible, and the lava seemed to have acted merely as so much dead weight pressing upon them from without. These, however, it is to be remarked, were on the verge of the stream, where the lava was least hot; for in the interior of the current I was unable to discover any vestiges of the houses that had been destroyed.

At the time that the eruption occurred, the villages in the neighbourhood were covered, to the depth, it is said, of two inches, by a shower of capilli; and from one account which I have seen, it would appear that torrents of hot water were poured down from the crater on the 28th.

The flow of lava from the crater continued all the 29th; but subsequently to that date no further eruption was perceived, and the principal current already described, being no longer urged forwards or augmented by fresh streams from above, gradually

slackened in its progress, and stopped at a distance of about a quarter of a mile beyond the road from Torre to Ottayano.

The lava is said to have been accompanied throughout its progress by a cloud of black sand, which hovered over its path, and from this cloud emanated frequent flashings of very vivid lightning, sometimes, but not always, followed by thunder.

These flashings Monticelli refers to the particles of sand being in an opposite state of electricity to that of the air, and consequently, when diffused through it by the wind, producing a discharge of electrical light. The same phenomenon was remarked by him in the preceding month of May, at which time the volcano, as has been stated, emitted a cloud of light volcanic sand. This was diffused by the wind over the whole of the circumambient atmosphere, and from the edges of this cloud, where the lightest and finest particles only of the sand were present, frequent coruscations of lightning appeared to emanate, whilst in its denser and blacker portion none such were discernible.

Towards the close of this eruption there occurred a phenomenon, which may perhaps be attributable to the volcanic action going on under Vesuvius. In a pond belonging to a private individual at Puzzuoli, all the fish suddenly died. In the lake of Fusaro, at this time, from twelve to thirteen hundred weight of fish were calculated to have perished; and it was remarked, that the victims principally belonged to those species which congregated at the bottom of pools, such as eels. Thus, too, a vast number of oysters at the bottom of this lake were found dead, whereas those which had attached themselves to the stones or the reeds on its sides are said to have escaped. In the neighbouring lake of Licola, also, several of the same species of fish were found to have perished.

After the 29th of August no further signs of internal commotion were exhibited by the mountain during the past year, except that disengagement of aqueous and aëriform vapours from the crater which is scarcely ever entirely absent.

So tranquil a condition of the volcano, although to a general observer it might appear deficient in that lively interest which belonged to the state of things that had preceded it, was at least favourable to a detailed examination of the several parts of the mountain, and allowed of my descending twice into the interior of the crater, which, owing to the falling in of the two conical hillocks alluded to, presented at that time a comparatively level surface. There were, indeed, three depressions or pits of considerable depth in the midst of it, which, though without any visible communication with the interior, were so charged with the noxious vapours evolved from an infinity of minute and scarcely visible spiracles, that it was judged unsafe to venture down into them. The rest of the crater, however, was a concavity of no great depth, which was traversed by my guide and myself with comparative facility, after we had remained within its precincts time enough, to collect the various sublimations that lined its walls, and to condense some of the vapours still copiously exhaled from its crevices. The sides of the crater consisted of strata which might be traced for a considerable

way round its brim in a direction nearly horizontal, except in one part, where, from some shock or fracture, they had sunk abruptly downwards. These strata consisted of loose volcanic sand and rapilli, coated with saline incrustations of common salt, coloured red and yellow by peroxide of iron, and presenting a beautiful and brilliant appearance. I could perceive no dykes intersecting these strata, as at the Monte Somma.

In order to collect the vapours, I caused to be constructed an apparatus, consisting of the head of a large alembic, fitted on to a cylindrical vessel of tinned iron with riveted joints, which, being open at bottom, and introduced a little way into the ground, served to conduct the exhalations into the receiver connected with it above. By this contrivance I succeeded in the course of an hour or two in condensing a sufficient quantity of the vapour for chemical examination at Naples. In the liquid collected I could detect no saline ingredient, and there appeared only a slight trace of sulphurous or sulphuric acids. The principal body condensed along with the steam was muriatic acid, which was uncombined with any base.

Whether carbonic acid might be disengaged from the crater I could devise no unexceptionable method of determining; yet by comparing the quantity of carbonate of barytes precipitated, by exposing a given quantity of barytic water for five minutes in the vapour of one of the Fumaroles, with what was obtained from the same quantity in equal times exposed to the open air out of the Fumaroles, I am led to conclude that this gas was exhaled.

Of nitrogen, the air of the Fumarole appeared to contain the same proportion, as atmospheric air does in general.

No sulphuretted hydrogen was emitted from the crater, neither could I discover, either in the condensed vapour or in the sublimations lining its walls, any trace of muriate of ammonia.

Muriatic salts principally were detected among the latter, but sulphates of lime, alumina, and iron were likewise present.

The next point in the volcano which arrested my attention was the vent on the eastern side of the great cone, from which issued one of the principal streams of lava that burst from the mountain in August last.

The vapours here collected appeared to agree in composition entirely with those from the interior of the crater; and the sublimations were of the same nature, with the addition of much specular iron ore and some muriate of copper.

The lava, which had been emitted in August, continued, when I visited it in November, to give out throughout the whole of its course white vapours; and even after the copious rains which fell subsequently, many of the spiracles, so late as the end of December, continued to emit the same. The interior of the current appeared also at both these periods to retain a considerable proportion of its original temperature. After removing about six feet of loose scoriæ, I at length reached the upper surface of the bed of lava itself, into which it would have been impossible to penetrate without the

assistance of mining implements. The surface temperature of the lava was indeed not high enough to melt lead, but one of DANIELL's pyrometers, with an iron rod, left in contact with it for a few minutes, rose more than one degree. It is probable, however, that I had failed in this instance in obtaining the full temperature of the superficies; for nearly a month afterwards, that is, late in December, after much rain had fallen, I removed the scoriæ from another contiguous portion of the bed, and found that a thermometer placed upon it, and merely covered over with a little sand, rose to 390° of FAHRENHEIT. From the cracks and cavities of this lava much aqueous vapour was still exhaling, and this I succeeded in condensing by means of the same apparatus which I had employed within the crater.

The condensed steam on examination was found to be impregnated, not only with free muriatic acid, but also with muriate of ammonia; and as the vapours were collected at the very point of their escape from the lava, it can hardly be doubted, that the latter salt is actually present ready formed within the cavities of the stone, having been emitted from the volcano along with the lava itself. The scoriæ which cover the surface of the bed are in some places quite incrusted over with beautiful crystals of this sort, some of which are perfectly white, whilst others are of an orange-yellow colour. The latter appears to be owing to the presence of oxide of iron. The quantity of sal ammoniac was large enough to repay the trouble of collecting, and much of it was carried away by the peasants to Naples to be sold to the workers in brass and jewellery. Muriate of soda was also common amongst the substances incrusting the scoriæ, but none could be detected in the vapour emitted at the period of my examination.

The very same substances I found to be exhaled, during my stay at Naples, from the crater of the Solfatara of Puzzuoli, which differed however in one respect, namely, in that of emitting much sulphuretted hydrogen, from which the vapours of Vesuvius were entirely free. Hence the film of minute crystals of sulphur which forms on the surface of the rock of the Solfatara in the immediate neighbourhood in the Fumaroles; whilst from the Vesuvian lava no sulphur in any form was given out at the time of my visit, although amongst the sublimations produced at an earlier stage of the operations, crystals of this body were not uncommon.

The disengagement of such principles, as water, muriatic acid, and sal ammoniac from a semi-extinct volcano like the Solfatara, is much more intelligible, than its escape from the substance of a bed of lava which has already undergone consolidation.

In the latter instance, what is the condition in which we are to imagine such bodies to exist in the heart of the mass? Not certainly in a state of chemical union with its constituents, for we cannot conceive any affinity inherent in salts of ammonia or soda for the earthy ingredients of a bed of lava; neither, if in combination with them, would they be separated, as the latter parted with its heat.

It seems necessary to suppose, that these bodies, being thrown up at the time of the eruption from the interior of the volcano, became entangled within the interstices

of the lava at the same time disengaged; that a portion of what was originally ejected still continues in a compressed state within the cavities of the rock, especially in its interior; and that it is only by slow degrees that they find means of escape through chinks and crevices to the surface.

We know that many trap rocks contain a portion of water and of muriatic acid, and that the latter body has even been detected in the domite of Auvergne, a volcanic production, which, comparatively speaking, must be regarded as of extreme antiquity*; so that we may more readily conceive, in what manner lavas of recent origin retain larger quantities of the same volatile principles, and even certain saline substances, diffused through their pores and fissures.

Perhaps indeed, although chemical attraction in these cases is out of the question, a certain degree of *adhesive affinity* may have been exerted, between the substances exhaled, and the walls of the cavities that had contained them. Dr. Faraday, in the Sixth Series of his Researches on Electricity, published in our Transactions, has introduced some pertinent remarks on this kind of influence, referring to it, amongst other phenomena, the operation of platina in determining the union of oxygen and hydrogen in Döbereiner's experiment. Nor indeed does it seem improbable, that, as heat exercises a repulsive power, not only between the particles of bodies, but likewise between masses of them†, so likewise a species of affinity may exist between masses of matter even where their particles are not mutually attractive; and that this may retard the operation of heat upon bodies possessing intrinsically a considerable degree of volatility, and prevent their entire disengagement all at once from the cavities of the substance which entangled them.

Be that as it may, it seems certain from the above observations, that ammonia is one of the original products of volcanic action in the case of Vesuvius; and it would be easy to extend the same inference to other volcanos,—a fact, I am aware, by no means new, but still one, the circumstances of which seem to deserve investigation, especially, as from the readiness with which nascent hydrogen enters into combination with azote, it might be imagined, that the ammonia was somehow or other generated in the open air, owing to a disengagement of hydrogen from the lava.

I trust, that the having traced it to the vapour directly issuing from the mass effectually dispels such a suspicion, and will serve as an additional argument in support of an opinion I have long entertained, that atmospheric air and water both find their way to the seat of volcanic operations, and are alike deprived of their oxygen by certain principles there existing; whilst the residuary nitrogen and hydrogen are evolved, in in some cases separately, in others united, in the form of ammonia.

* I might likewise refer to the existence of carburetted hydrogen in a condensed state in cavities of rock-salt at Wielitzka, and that of sal ammoniac in that of the Tyrol, as facts of the same description. The latter might lead to some speculations with regard to the origin of sea-salt, to which I may perhaps on some future occasion recur.

† See Professor Powell's Paper in the Philosophical Transactions for 1834, Part II.

X. *On the Atmospheric Tides and Meteorology of Dukhun (Deccan), East Indies. By Lieutenant-Colonel* W. H. Sykes, *F.R.S. L.S. G.S. Z.S. Vice-President of the Statistical and Entomological Societies.*

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THE value of the following meteorological observations depending on the goodness of my instruments, on certain precautions in the use of them, and on the care with which atmospheric changes were recorded, I shall preface my notices on the meteorology of Dukhun with an account of the instruments I had in use, and of my method to insure correct results. In determining atmospheric pressure, for the first two years I was confined to two of Thomas Jones's barometers: they required to be filled when employed, and were destitute of an adjustment for the change of level of the mercury in their cisterns, unless the position of the cistern had been altered at each observation; a measure attended with insuperable inconvenience. At first I experienced a good deal of vexation in expelling the moisture from the tubes; but by previously rubbing the inside with a tuft of floss silk tied to the end of an iron wire, I dried them so effectually (unless in the monsoon months) as to excite powerful electricity: and I have frequently had shocks in my right thumb, running up to my shoulder, in pouring the mercury into the tube, accompanied with cracking noises, until the approach of the mercury to within two inches of my thumb, when the electricity was discharged as described. I experienced these shocks at Salseh, near Purranda, on the 3rd of February; at Pairgaon, on the Beema River, on the 14th of February; at Kundallah, in the hilly tracts, on the 14th of March, 1828; and at many other places. Jones's barometers were each provided with a thermometer let into one of the legs of the tripod on which the barometer was suspended. The scale of this thermometer was of thin ivory, and the tube excessively slender. During the heat of the day in the dry season, the scale was contracted, by parting with its moisture, into the segment of a circle, bending the tube of the thermometer. At night the ivory scale relaxed from its curvature, and at sunrise it had returned to a right line. This operation continued daily for more than three weeks; but on the 15th of February 1827, the contraction of the scale was too great for the flexibility of the glass, and the tube of thermometer No. 1. broke. The thermometer attached to barometer No. 2. subsequently shared the same fate, from a similar cause. Thomas Jones's barometers pack well, carry easily, and are certainly very useful as checks upon permanently filled barometers, which frequently give false indications, from the unknown escape of the mercury, or the admission of air, which could not be detected

without the aid of a second barometer: but they are very troublesome to fill; are destitute of a thermometer near the cistern, to determine the temperature of the mercury; and want the means of adjusting the lower level of the barometric column; the tubes are frequently breaking, from the pressure of the iron screw which fixes the cistern to the tube, (I have broken seven tubes from this cause,) and in case of not being tightly screwed on, the cistern falls off from the weight of the mercury in it, and the mercury is lost; and from the uncertainty of expelling air and moisture from the tubes, particularly in the moist months, the indications of the instrument can only be looked upon as approximations to the truth.

On the 12th of April 1827, I had the gratification to receive three barometers from England: they were made by CARY on the ENGLEFIELD construction, which admits of a most delicate adjustment of the lower level of the barometric column in the reservoir. They were beautifully finished, but unluckily had reservoirs of ivory; and I instantly foresaw the inconvenience to which such selection of material would subject me. In the dry weather the ivory contracted, and permitted the escape of the mercury by the screws (male and female) which joined the two portions of the reservoir. Subsequently the reservoirs cracked at the spots where the metallic screws attached the reservoir to the brass cylinder surrounding the tube of the barometer. I was finally compelled from these disasters, within a twelvemonth, to send two barometers back to England to have glass or iron reservoirs put to them. From the ease, accuracy, and delicacy with which the contrivance in these instruments permits the mercury to be adjusted at its lower level, they require only an iron cistern to render them quite efficient; and they are peculiarly suited to measure minute changes in the atmospheric tides. Mr. NEWMAN of Regent-street has acted upon my suggestion, and has constructed two ENGLEFIELD barometers with iron cisterns, to which he has applied an excellent improvement of his own to prevent the oscillation of the mercury in the tube *en route*.

Having broken the seventh and last tube belonging to JONES's barometers, to prevent my observations being confined to a solitary instrument I had recourse to one of the India Company's barometers made by GILBERT: it was very heavy, and clumsily constructed, had air in the tube, and I ascertained the mercury not to be of the specific gravity engraved on the reservoir. The instrument had a glass reservoir, and the manner of fixing it to the tube was sufficiently ingenious; but it wanted an accurate and efficient method of adjusting the lower level of the mercury. This operation was to be effected by looking through the glass reservoir and screwing up the mercury to a line marked on it; but the oxidation of the mercury usually dimmed the glass, and made it no easy task; even had it been readily practicable, the occurrence of the tube exactly in the centre of the convex surface of the mercury prevented its outline being fully seen, and the reading off could never be rigidly accurate. These causes combined to render unsatisfactory, observations taken with the instrument to fix the *exact time* of the flux and reflux of the diurnal and noc-

turnal atmospheric tides; but it answered sufficiently well as a check upon my other barometers. Several others by GILBERT, used by myself and my friends, were found to be similarly defective.

Auxiliary to the barometers, I had in use ADIE's sympiesometer. This instrument, so ingenious in its construction, I soon found to be utterly inadequate to measure the pressure of the atmosphere with the correction given to it for the expansion from heat of the hydrogen gas in the tube. An inspection of my meteorological register will show by a glance the inefficiency of the instrument as a substitute for the barometer within the range of my observations. In fact, it constantly sunk with increase of heat, and gradually rose with the return of cold. In very few instances in a whole year was it found to have stood higher at 9½ A.M., the period of the maximum atmospheric pressure, than at sunrise; and in these trifling approximations to truth it evidently deteriorated since the first record of its indications. A sympiesometer in possession of the Assay Master at Bombay was subject to the same defects. I nevertheless continued my observations with the instrument simultaneously with the barometers, to supply the inventor with the necessary elements for its correction, should he desire to make use of them. I was further induced to keep the instrument on my register, from the advantages I derived from the attached elegant and accurate thermometer, which had its degrees divided into fifths.

The temperature of the air was determined by two excellent DOLLOND's FAHRENHEIT thermometers, one of which had been in my possession twenty-two years. One had a brass scale, the other a stout ivory scale sufficiently robust to prevent the dry air warping it materially. These thermometers never differed from each other more than half a degree, and I had great confidence in their indications. No. 1. with the brass scale was used for several years to determine the temperature of boiling water at different levels. In this process small particles of mercury rose from the surface, and fixed themselves at the apex of the tube; but this was easily remedied by driving the mercury by heat up to the apex, and in retiring it always carried with it the particles which had risen.

For the determination of the moisture in the atmosphere, two of LESLIE's and one of DANIELL's hygrometers were sent to me from Calcutta. The former were kept in use from the 21st of March 1826 until the 7th of April 1827, when, finding them destitute of uniform indications with respect to each other and to DANIELL's hygrometer, I was induced to give up employing them further. DANIELL's hygrometer was continued in use from the 21st of March 1826 until the 30th of September 1827, when it was unfortunately broken. There not being an instrument of the kind for sale in India, Colonel GOODFELLOW, Chief Engineer at Bombay, was good enough to assist we with one, which was brought into use on the 25th of October following. This continued in use, with occasional interruptions from the want of æther, until the 28th of March 1828, when it shared the fate of the former. From this period until the 11th of June 1829, I was disabled from making hygrometric observa-

tions, when the arrival of other hygrometers from Europe permitted me to resume them.

Daniell's hygrometer I found to be an admirable instrument, ingenious in its construction, definite and uniform in its indications, simple in its use, and satisfactory in its results. But it is not without a drawback upon its utility. Independently of the demand for a constant supply of æther, there are periods of the year in Dukhun when the high temperature and extreme dryness render the dew-point only obtainable at such an expense of æther as to render it an object of pecuniary consideration; and with the very best æther I have never been able to reduce the temperature more than 61° of Fahrenheit's scale, that is to say, from 90° to 29°, on the 16th of February 1828, at 4 p.m.; and at that hour the attempt in the months of March and April 1827 proved fruitless, and I was obliged to give up a register of the dewing-point in the afternoon. In the month of January, at sunrise, on the 4th and 6th respectively, I got the dewing-point at three degrees below that of congelation of water, namely at 29°, the temperature of the air being 62°; and on the 3rd of February 1828, the dewing-point was at 28° Fahr., the air 56°, at sunrise. On the 17th of February the lowest dewing-point ever registered was obtained, namely 27° Fahr., temperature of air 57°·50 at sunrise. The objections, therefore, to this instrument are, the great expense of æther in the dry months, and the occasional inability to obtain the dewing-point when the temperature is very high and the day very dry. I never had any difficulty in Bombay or in the Konkun in obtaining the dewing-point, even at a temperature of the air of 91°·50 Fahr., nor will the efficiency of the instrument ever be doubtful within the tropics near to the sea shore. It is necessary to mention that the temperature of the air in Dukhun sometimes exceeds the boiling-point of good æther.

The measure of the quantity of rain which fell was taken with two instruments, one of which was sent to me from Calcutta under the apposite name of ombrometer, and the other was obtained from the medical stores at Bombay with the hybrid appellation of pluviometer attached to it. A hollow cylinder closed at one end had a metallic float with gage-rod, resting on the bottom. The rain was received into a round funnel fixed to the top of the cylinder: the diameter of the mouth of the funnel was in a certain ratio to that of the cylinder, and this ratio regulates the length of the inches on the gage-rod. The ombrometer was made of brass, neatly finished. The pluviometer, of lackered iron, large, rudely finished, and unwieldy; and it had further the disadvantage (unlike the ombrometer) of its funnel-shaped mouth not closing round the gage-rod, an improvement preventing the evaporation of the water that falls into the instrument. Both rain-gages stood more than three feet high, but their cylinders were of different diameters. In both, the inches on the gage-rod were so large as to admit of hundredths (and even thousandths if it were required) of an inch of water being read off with ease. They always worked very well together, the only discrepancy being in the larger instrument indicating two or three hun-

dredths of an inch of water less than the smaller in the first tenth of an inch of rain. Subsequently they coincided in their indications even to the hundredth of an inch.

With CARY's ENGLEFIELD barometers, I received three thermometrical barometers, for determining heights by the difference in temperature of boiling water at different levels. Owing to faultiness in their construction, they proved complete failures, and were sent back to England. I satisfied myself there was a good deal superfluous in the apparatus accompanying them, which made them moreover expensive, and I efficiently supplied the place of these barometrical thermometers by two good common thermometers with metallic scales and a tin shaving-pot with a slit in the lid, in which the thermometer was placed, being moveable in a collar of cork. Pure water and dry sticks were always found, an attendant carried a light, and my boiling-operation was concluded in a quarter of an hour without the aid of tallow, lamp, sulphuric acid, phosphoric matches, trimming-scissors, tweezers, hanging-screw to fix into trees, water-bottle, &c. &c., involving the outlay of several pounds. Accuracy in the indications of the instrument also was risked, owing to three fourths of the stem of the thermometer being exposed to the wind or cold air during the time of the immersion of the bulb in the boiling water, which checked the rise of the mercury.

Having for several years practised the barometrical and thermometrical methods to determine heights, I have no hesitation in expressing my opinion, that a good thermometer and a boiling-pot may efficiently supply the place of the expensive and delicate barometer where *great accuracy is not required.* In many instances I found the results by the two processes almost identical.

My electrometers consisted of two balls of pith suspended in small glass jars capped with brass, having an elevated point on the plane of one cap, and a wire projecting from the apex of the other, which was bell-shaped. They were in fact CAVALLO's pith ball bottle electrometers, with SAUSSURE's addition of pointed wires, but without a graduated scale on the bottle to measure the divergence of the balls. Owing to some peculiarity in the instruments, they feebly indicated the presence of electricity in the atmosphere, although at certain seasons it was so rife as to be painful to the feelings. When first received, they were sensible to artificial electricity, but latterly, without having been injured, they lost all susceptibility. I never could make a record of their indications. Even had they been available, the want of a scale rendered it impossible to give any positive idea of the extent of the electric state of the atmosphere at any time. A scale, in case it did not measure definite quantities, would nevertheless be highly useful to determine the electricity of any particular period relatively to that of any other period.

In placing the instruments for measuring the pressure and heat of the atmosphere, I was particularly careful to secure them from the operation of causes capable of producing partial and unsatisfactory results. They were always in the shade, and always guarded from direct or reflected heat, but with a free admission of the external air. Annually, from October until May inclusive, they stood, for the most part,

just within the inner doors of a field officer's tent, having a third canopy or extra fly to it; and during the hot months commonly pitched under the shade of lofty trees. For the remaining, or monsoon months of the year, the instruments were kept in a room at Hay Cottage, Poona, through which there was a constant draft of air by two windows opposite to each other in the line east and west. In using the hygrometers, they were always taken to the door of the tent or to an open window. Whether in determining ordinary pressure, atmospheric tides, temperature, moisture, or heights, by the barometer or boiling-water process, I have invariably deemed it necessary to guard my observations from error by the employment of instruments *in pairs*. I have been thus minute in the description of my instruments and my manner of using them, not less to supply the means for a just estimate of the value of my meteorological observations, than to enable meteorologists who may tread in my steps to benefit by my experience and disasters.

The barometrical means have been reduced to 32° FAHR. by Professor SCHUMACHER'S tables, with corrections for the expansion of the brass scale; and the monthly means for 1830 were obtained by the ingenious process recommended by Professor FORBES.

In regard to the following barometrical observations, I must premise, that my three best barometers, although precisely of the same construction and placed under precisely similar circumstances, would occasionally differ slightly from each other, not only in the amount of the oscillations, but in the period at which the several tides turned; and this fact is of some importance to those who may be disposed to rely too confidently upon the indications of a single instrument.

My erratic life necessarily disabled me from determining the mean absolute height of the barometer at any one place for a period exceeding five or six consecutive months, excepting for the year 1830. I cannot, therefore, state the *annual* range of the barometer for several years successively; but repeated returns to Poona in various months of the year would have supplied the materials for tolerably just estimates, even had I not one entire year's observations made at Poona. The *monthly* range is recorded for many complete months, and the *diurnal* range for six years with few omissions; but I propose to confine my deductions to my observations for the last *four years*, as the instruments I had in use at first (JONES's) did not admit of a delicate adjustment of the lower level of the mercury.

The great features in the barometrical indications are the diurnal and nocturnal tides, embracing two maxima and two minima in the twenty-four hours; the former occurring with occasional exceptions between 9 and 10 A.M. and 10 and 11 P.M., and the latter between 4 and 5 P.M. and 4 and 5 A.M. The same hours obtain at Calcutta and Madras at the level of the sea; at Kotgherry on the Neelgherry mountains at 6407 feet; in South America at 12,000 feet; and in London and Edinburgh, and other places in Europe where careful observations have been made. Hitherto little has been known respecting the nocturnal atmospheric tides, but the existence of the diurnal tides is now established beyond doubt in most parts of the world. How-

BOLDT, on the authority of HORSBURGH, has been led into the expression of a belief that they are masked or suspended on the western coast of India during the prevalence of the south-west monsoon. I am, however, enabled to state that this is quite unfounded with respect to Dukhun, as they were never interrupted, even for a single day, during six monsoons; and the same fact was observed at Kotgherry on the Neelgherry mountains at 6407 feet above the sea during part of the monsoon of 1828. Of their occurrence on *the coast* I am also enabled to offer some evidence from registers kept at the Engineer Institution in Bombay, and regularly transmitted to me; but the hours selected for observation, 9 noon and 3 P.M., were not exactly adapted to fix the full amount of the tide; but on the whole the fact of their occurrence during the monsoon of 1829 in Bombay is undeniable; and they were similarly remarked at Calcutta in 1822 by General HARDWICKE, and by Mr. PRINSEP in 1829, 1830, and 1831. This fact will relieve HUMBOLDT from some of his difficulties in his reasoning on the tides.

With respect to the tides in general, I have to state that in many thousand observations made by myself there was not a solitary instance in which the barometer was not *higher* at 9—10 A.M. than at sunrise, *lower* at 4—5 P.M. than at 9—10 A.M., *whatever the indication of the thermometer or hygrometer might be*: nor was there a solitary instance in the year 1830 in which the maximum *night tide* was not higher than the 4—5 o'clock day tide, although it rarely, if ever, rose so high as the 9—10 A.M. day tide. The nocturnal minimum tide occurring at 4—5 A.M., from three to four hours after my usual time of retiring to rest, my observations of it were very limited in number; nevertheless the accompanying Tables will furnish some direct, and ample indirect, testimony of its existence, since the fact of the rapid, constant, and considerable rise of the barometer from sunrise until 9—10 A.M. justifies the inference that there must have been a considerable previous fall to have admitted of such rise: the commencement of such fall was necessarily observed by me in my labours during 1830 to determine the limit hours of the tides, as I was obliged to continue observing in each case until the tide had turned. Moreover, at different periods I devoted forty-nine nights to the investigation of the minimum A.M. tide. Dr. WALKER at Mahabuleshwar, at 4500 feet above the sea, bestowed eight months' labour upon the tides; and Mr. DALMAHOY, on the Neelgherry mountains, was similarly employed for four or five months. HUMBOLDT in his narrative mentions the determination of the extent of the diurnal oscillations, the duration of the stationary state of the barometer at its maxima and minima, and the exact periods at which it becomes stationary and is in action again, as desiderata. I shall take these subjects in order as I proceed.

The extreme oscillation of the barometer in the same day never amounted to two tenths of an inch, in fact to ·1950, with a difference of the attached thermometer during this range of $+7°\cdot6$, and the hygrometer 15° from the point of saturation. Wind light and variable. This took place on the 19th of April 1830. There were

great masses of clouds, and distant thunder and lightning; a storm threatened, but did not take place. The same appearances continued daily until the 21st, on which day there was a hail storm, whilst the thermometer stood at 86°·3. On the 23rd there was another hail storm and thunder: this weather continued to the end of the month, and the daily oscillations were so great as to make the mean exceed that of any other month in the year. Here there could be little doubt of the oscillations being affected by the state of the weather. In 1827 the maximum oscillation of ·1892, (difference of thermometer attached +10°, dewing-point 37° from saturation, wind none,) took place on the 7th of March, and the weather was free from any of the indications before noticed; but on the 9th of March there was a little lightning and some drops of rain. In 1828 the maximum oscillation of ·1856, (difference of thermometer attached +10° 8′, no wind, and clear sky,) took place on the 2nd of January. In 1829 the maximum oscillation was on the 26th of February, and amounted only to ·1648, difference of thermometer +11° 5′, wind light east, and clear sky.

In 1827 the minimum oscillation of the year occurred on the 7th of August, between 9 A.M. and 4 P.M., amounting to ·0150; difference of attached thermometer —0°·8, light west wind, sky quite overcast, but no rain, although the dewing-point was only 3° from the point of saturation. A nearly similar oscillation, ·0153, thermometer +5°·2, took place on the 29th of the preceding May, with a violent west wind and clear sky, and *no dew-point* obtainable at 4 P.M. In 1828 the smallest diurnal oscillation of ·0155, thermometer +2°·3, took place on the 19th of October during a gentle rain and light S.W. wind. In 1829 the smallest oscillation was ·0281, thermometer +0°·9, on the 2nd of July, with a partially clouded sky and fresh W.S.W. wind, the hygrometer being 6° from the point of saturation. On the 21st of March the next smallest oscillation of the year took place, with a misty sky, light west wind, and air *very dry*. In 1830 the minimum of the year was also in July, amounting to ·0327, the thermometer being half a degree lower at the minimum than at the maximum hour; sky overcast, no rain, wind light west, hygrometer 8° from the point of saturation. On the 20th of March there is also a small oscillation of ·0493, thermometer +9°·9, sky clear, fresh west wind, and hygrometer 29° from the point of saturation. I have been particular in noticing the state of the weather and the winds, &c., at the periods of these extreme oscillations, as Mr. Snow Harris of Plymouth suggests that the atmospheric tides may be influenced by the force of the wind, whilst others refer them to hygrometric causes.

The mean of the diurnal oscillation of the barometer in Dukhun from 9—10 A.M. to 4—5 P.M. for 1827 was ·1025, mean range of attached thermometer between the two periods +5°·99. In 1828 it was ·1093, thermometer +6°·36. In 1829 it was ·0991, thermometer +3°·92. The smallness of the range both of barometer and thermometer in this year is attributable to three months' observations having been taken at an elevation of nearly 4000 feet above the sea. In 1830 the barometers were stationary for the whole time at Poona, and I look upon these observations as affording the

best types of the meteorological phenomena of Dukhun. The fall of the tide from 9—10 A.M. to 4—5 P.M. was ·1166, thermometer +4°·9. Comparing this tide with the same tide observed in other places, we find that at Madras, lat. 13° 5′, from observations taken at the Observatory every tenth day in 1823, the mean oscillation was ·079, mean range of attached thermometer +8°·5. At Calcutta, latitude 22° 35′, the means of the years 1829, 1830, and 1831, make the oscillation amount to ·110, thermometer range +12°·2. At Saharunpoor in Hindoostan, 1000 feet above the sea, latitude 31° N., by Mr. ROYLE's registers, the tide was ·120, mean range of thermometer +24°·2. At Ava, latitude 21° 51′, Major BURNEY's observations in 1830 make the tide amount to ·126, mean diurnal range of thermometer +10°·6. Agreeably to Mr. PRINSEP, at Benares, latitude 25° 30′, it is ·105, range of thermometer attached +16°·6. Professor FORBES in Edinburgh found the oscillation to be ·0114, mean range of thermometer attached for three years −0°·57. And Mr. HUDSON, at the Royal Society in London in 1831, determined the oscillation to be ·0289, therm. +1°·73.

HUMBOLDT and BONPLAND in equatorial America, at Cumana, La Guayra, Payta, Lima, and Rio Janeiro, found the mean extent of the oscillation at most from ·0945 to ·1181 *. At Lima, latitude 12° 26′, it was a little less (·0669 to ·0905 †) than nearer to the equator, where it was from ·1023 to ·1291 ‡. BOUSSINGAULT and RIVERO in 1823–4, at Santa Fé de Bogota (latitude 4° 35′ N.), height 8196 feet, found it to be ·0905, approaching my mean for 1829. At La Guayra (latitude 10° 36′ N.), at the level of the sea, it was ·0960; but as the preceding observations in America, with the exception of those at Bogota, were for a few days only, they are valueless as indicative of the mean diurnal oscillations, much less the monthly and annual means. The extent of the diurnal oscillation from 9 A.M. until 4 P.M. on the table land of Bogota was from ·0248 to ·1433 §. In Dukhun in 1827 it was from ·0150 to ·1892; in 1828 from ·0155 to ·1856; in 1829 from ·0281 to ·1648; and in 1830 from ·0327 to ·1950. The mean of the monthly variations at Bogota are from ·0580 to ·1062 ‖. Mine for 1827 were from ·0489 in July to ·1616 in December; in 1828 from ·0471 in July to ·1505 in February; in 1829 from ·0654 in July to ·1358 in January; and in 1830 from ·0750 in July to ·1430 in April. Considering that my observations were made on a level more than 6000 feet lower than that of Messrs. BOUSSINGAULT and RIVERO, the above data exhibit curious approximations, and prove that diurnal variations in the pressure of the atmosphere at great differences of level *may have* considerable uniformity. But to this we find an immediate exception, for Dr. WALKER at Mahabuleshwur, at 4500 feet above the sea, and a few miles south of the latitude of Poona, found the mean fall for ten months, from 9—10 A.M. to 4—5 P.M., to be ·0694, difference of thermometer attached +2°·61, which is infinitely less than M. BOUSSINGAULT's mean at 8000 feet.

The monthly means of the diurnal oscillations in consecutive years, although not uniform, have marked approximations. The five monsoon months in each year ex-

* 2mm·4 to 3mm·0. † 1mm·7 to 2mm·3. ‡ 2mm·6 to 3mm·3. § 0mm·63 to 3mm·64. ‖ 1mm·5 to 2mm·7.

hibit comparatively a low range, in fact the month of July has the lowest mean di urnal oscillation in each year; and it may be broadly stated, that with two or three exceptions the monthly mean diurnal oscillation increases from July to December or January, and decreases from these months to July. How far the monthly mean oscillations of the barometer and the monthly *mean range* of the thermometer coincide, will be seen by the following Table. The monthly mean temperature and the monthly mean diurnal oscillations of the barometer have little coincidence.

Mean range of the Thermometer attached to Englefield's Barometer between sunrise and 4—5 P.M. at Poona and Mahabuleshwur.

	Poona. 1827. 1823 feet. and between lat. 17° 25′ and 19° 27′ N.			Poona. 1828. and between lat. 17° 40′ and 19° 11′ N.			Poona. 1829. and between lat. 18° 10′ and 19° 23′ N.			Mahabuleshwur. 1828–29. 4500 feet.		
	Barom. Monthly mean osc.	Thermometer. Mean range.		Barom. Monthly mean osc.	Thermometer. Mean range.		Barom. Monthly mean osc.	Thermometer. Mean range.		Barom. Monthly mean osc.	Thermometer. Mean range.	
January ..	·1134	14	4	·1483	25	7	·1358	20	81	·0735	8	75
February..	·1257	23	1	·1505	27	1	·1083	21	88	·0666	15	40
March....	·1248	21	6	·1123	*17	68	·1024	†13	82	·0827	9	09
April	·0836	*10	1	·1334	17	89	·0981	†14	51	·0835	7	84
May	·0624	13	1	·0836	19	82	·0903	†12	96	·0757	4	89
June	·0902	6	35	·1007	7	34	·0734	4	29	·0528	..	80
July......	·0489	4	36	·0471	3	6	·0654	3	26	·0556	..	85
August ..	·0600	4	33	·0706	3	88	·0866	3	87	·0503	..	64
September	·0813	6	47	·0910	5	38	·0772	6	18			
October ..	·1147	7	21	·1106	5	58	·1116	9	57			
November	·1444	21	4	·1277	8	5	·1067	11	23	·0801	4	68
December	·1616	26	7	·1141	*16	28	·1338	‡14	46	·0738	6	70

In 1827 the greatest monthly mean diurnal oscillation and the maximum mean range of attached thermometer were coincident in December; but the next greatest mean oscillation was in November, whilst the greatest mean range was in February. November and March have nearly the same mean thermometric range, but differ 0·196 in mean barometric oscillation. In 1828 the greatest mean oscillation and range of thermometer are coincident in February. January accords in a similar manner; but May is quite anomalous, having a mean barometric oscillation of the monsoon month of September, range of thermometer $+5°{\cdot}88$, whilst its own range is $+19°{\cdot}82$. In 1829 the movements of the barometer and thermometer are not coincident; the maximum of the former being in January, that of the latter in February. In December we find the oscillation of the barometer nearly identical with that of January preceding, whilst the mean range of the thermometer is nearly seven degrees less. At Mahabuleshwur the monthly mean maximum oscillation of the barometer is in April, whilst the maximum mean range of the thermometer is in February, and nearly doubles the range of April.

* The observation for those months with the * were taken in Bombay, with the † at Hurreechundurghur, and with ‡ at Chamblee.

Of the rise of the barometer from sunrise to 9—10 A.M. I shall say only a few words, as the period embraces but four sixths of the time occupied by the flux of the atmospheric tide, and the figures in consequence are of little further value than as affording presumptive evidence that the rise, without the exception of a single day for six years, must have been preceded by a nocturnal ebb. Although the annual means, ·0473, difference of thermometer attached +7°·27, for 1827; ·0481, thermometer +6°·71, for 1828; and ·0382, thermometer +7°·48, for 1829, agree tolerably well, yet the monthly means for successive years do not manifest the same accordance; as in the tide just noticed the smallest mean oscillation is in the monsoon months, and it increases until December—January, and then decreases to June—July. In 1828, however, June is a remarkable exception, the oscillation being greater than in any month of the year excepting January, and nearly double that of June 1827. Dr. WALKER at Mahabuleshwur, at 4500 feet, found the rise from sunrise to 9—10 A.M. to be nearly identical (·0476, thermometer +4°·18) with my rise at less than 2000 feet. Mr. DALMAHOY at Kotagherry, at 6407 feet, found the rise from sunrise to noon to be ·0490, thermometer +10°·4; and had his observations been taken at the hour of the maximum diurnal tide (9—10 A.M.), the oscillation would no doubt have exceeded those recorded by Dr. WALKER and myself at infinitely lower levels. Mr. GOLDINGHAM at the Observatory at Madras, a little above the level of the sea, makes this tide amount to ·0470; so that, in fact, it is less at the level of the sea than at 6407 feet!

The nocturnal rising tide from 4—5 P.M. to 10—11 P.M. I observed with great care for eleven months continuously in 1830. It amounted to ·0884, thermometer −7°·2. The indications of monsoon influence in this tide are scarcely perceptible. Indeed, the smallest monthly mean oscillation occurs in December, ·0450, thermometer −6°·3; and the greatest in May, ·1140, thermometer −9°·0. Unlike the preceding tides, we cannot trace a maximum in the coldest months, and a minimum in the most rainy. Dr. WALKER found it to amount to ·0439, thermometer −5°·58; the monthly mean maximum oscillation, ·0632, thermometer −3°·21, being in November, and the minimum, ·0291, thermometer −6°·74, in January. Mr. DALMAHOY, at 6407 feet, found it to be ·0430, the minimum, ·0280, being in June, and the maximum, ·0560, in April; but as he did not determine the exact period of the time of the tide between 9—12 P.M., the real extent of the tide is unknown. In my own observations I watched for the turn of the tide. Mr. GOLDINGHAM, at Madras, makes the value of this oscillation ·0630; whilst M. DUPERREY, at Payta in America, latitude 5° 5′ S., makes it ·1259.

I now pass to the fourth tide, the fall between 10—11 P.M. and 4—5 A.M. Here the data are defective, as observers have only, for very short intervals of time, endeavoured to fix its limit hours; and I have no reliance whatever upon occasional observations as types of a whole year, or even a month or week. For myself, I am not an exception, as my observations between 4—5 A.M. are very limited in number. The maximum night tide, I have before stated, was observed by me for eleven months, and the A.M. tide at sunrise for several years. Dr. WALKER, at Mahabu-

leshwur, observed at both these periods for eight months; and Mr. DALMAHOY, at Kotagherry, between 9—12 P.M. and a little before sunrise, observed for five months. On the 30th of November 1828, at Poona, the A.M. minimum tide turned at 4^h 30^m A.M., and the maximum nocturnal tide at 10^h 30^m P.M.; the fall between these periods being ·0150, and the difference of attached thermometer −7°·6. The other tides of this day were a rise of ·0572, thermometer +9°·0, from 4^h 30^m A.M. to 9^h 30^m A.M.; fall from 9^h 30^m to 4 P.M. ·1330, thermometer +3°·4, and a rise of ·0908, thermometer −11°·0, from the last hour to 10^h 30^m P.M. The mean of eighteen days in September at Poona, in 1827, gave a rise of ·0753, thermometer −5°·1, from 4—5 P.M. to 10—11 P.M., and a fall of ·0254, thermometer −1°·37, from 10—11 P.M. to sunrise. The rise from the latter hour to 9—10 A.M. was ·0352, thermometer +3°·65, and the fall from 9—10 A.M. to 4—5 P.M. was ·0844, thermometer +2°·82. For twenty-one nights in October the fall from 10—11 P.M. to sunrise was only ·0010, thermometer −2°·39; the rise from 4—5 P.M. to 10—11 P.M. ·0745, thermometer −4°·76. But for nine nights in November the nocturnal A.M. tide occurred with a *contrary sign*, the barometer being ·0052 less, thermometer −10°·2, at sunrise than at 10—11 P.M. The maximum night tide, however, appears with the proper sign, the rise being ·0801, thermometer —11°·65. On the 3rd of November of the following year the A.M. minimum tide appears with the proper sign, the fall being ·0040, thermometer −4°·0; and the rise from 4—5 P.M. to 10—11 P.M. was ·0714, thermometer −6°·5. In the above, although we find great discrepancies in the fall from 10—11 P.M. to sunrise, we yet observe great uniformity in the nocturnal rise from 4—5 P.M. to 10—11 P.M. with a falling thermometer. Dr. WALKER found the mean nocturnal A.M. tide for eight months to be ·0180, thermometer −1°·68; and the rise from 4—5 P.M. to 10—11 P.M. to be ·0439, thermometer −5°·58. Mr. DALMAHOY, at Kotagherry, from 9—12 P.M. to a little before sunrise, for four months, found the mean fall to amount to ·0433, and the rise from sunset to 9—12 P.M to be ·0430. Mr. PRINSEP, F.R.S., in a voyage from Calcutta to Bombay, during thirty-two days found the barometer fall ·0220 from 10 P.M. to sunrise, and rise ·0440 from sunrise to 10 A.M.; fall ·102 from 10 A.M. to 4 P.M., and rise ·0800 from the last hour to 10 P.M. Observations taken hourly at the Madras Observatory, every tenth day and night, make the night tide from 10 P.M. to 4 A.M. to amount to ·035, and the 4 A.M. to 10 A.M. tide to be ·047; the other two tides being respectively ·079 and ·063. The smallness of this maximum diurnal tide appears very anomalous, considering that Madras is in a low latitude and at the level of the sea.

In opposition to the above facts, Dr. RUSSELL, at Boorhanpoor, gives a nocturnal minimum tide with a contrary sign, or a *rise* instead of a *fall* of ·0200, between 10 P.M. and 5 A.M.; and in observations of Dr. ROYLE, at Saharunpoor in India, at 1000 feet above the sea, and of FRAY JUAN, at Vera Cruz, the nocturnal tide appears in the monthly means so often with a *plus* instead of a *minus* sign, that the annual mean establishes this tide only by ·001 at Saharunpoor, and by ·002 at Vera Cruz.

Mr. Hudson however, at the Royal Society, in his careful hourly observations even in the high latitude of London, found it amount to ·0120; and I feel assured that further observations will establish its existence at those places rendered doubtful by the data just quoted.

With respect to the exact periods of the diurnal flux and reflux, and the duration of the quiescent state of the atmospheric tides, the subject has been wholly overlooked, as far as I can learn, in Western India; but even had it not escaped attention, there have not been, I believe, instruments in use sufficiently delicate in their construction to read off very small quantities. Dr. Walker at Mahabuleshwur, Captain Jervis in Bombay, at the Engineer Institution, and myself, are the only persons who have made observations on the tides. For myself, my multitudinous avocations deprived me of the necessary leisure for some years to enable me to enter systematically into the inquiry. Occasionally, at the admitted limit hours of the diurnal oscillations of the barometer, I made a few observations; but they were of little further value than to show that the maxima and minima, on consecutive days, did not occur at the same exact period of time. In 1830, however, with two of Englefield's barometers, which admitted of the adjustment of the lower level of the mercury to the 1000th of an inch, I made observations every quarter of an hour, and sometimes every five minutes, during the whole of the year at the limit hours of the *diurnal maximum* and *minimum* A.M. and P.M. and *maximum* P.M. tides. Messrs. Walker and Jervis had in use Gilbert's barometers, and did not observe for the exact limit hours.

Messrs. Boussingault and Rivero, in addressing Humboldt from South America, state that their labours had verified the fact established by Humboldt, that the mercury between the tropics attains its maximum between 8 and 10 A.M., then descends till near 4 P.M., and is at the minimum between 3 and 5 P.M.: then it ascends till 11 at night, without reaching however the same height at which it was at 9 in the morning, and finally re-descends till 4 in the morning. It will be seen how closely these limit hours hold good in Dukhun as well as in America; and Mr. Hudson has determined that they hold good in London: but I have on record instances of the barometer rising until $10^h\ 45^m$ A.M., falling until 6 P.M., and rising until 12 at night; but the instances are rare: and even the tremendous storms preceding and closing the monsoons in India only modify and do not interrupt the tides*. Humboldt observes, that in Macao, in 1814, there were frequent tempests, and twenty-six stormy days, and yet there was not a single instance of the tides being *interverted*. He says also, that in reviewing the whole of his observations made at different heights, and

* The variations appear to be independent of those of temperature and the seasons. If the mercury was descending from 2^h till 4^h, or rising from 4^h till 11^h, a violent storm, an earthquake, showers, and the most impetuous winds, would not alter its movement, which nothing appears to determine but the real time or the position of the sun.—Humboldt, Personal Narrative, vol. vi. part ii. p. 701.

Humboldt further remarks that the hurricanes (in the West Indies) are not in general accompanied by such an extraordinary lowering of the barometer as is imagined in Europe. Captain Don Thomas de Ugarte, on

in latitudes more or less near the equator, it seemed to him that the extent of the variations diminished very little with the elevation of the spot. Mr. COLEBROOKE remarks, that in the interior of India the periodicity of the tides is manifest, and *independent of the variations* of the temperature and the seasons of the year. My observations, on the whole, tend to strengthen the opinions of Messrs. HUMBOLDT and COLEBROOKE.

I found the *stationary period* of the tides to vary from *nil* to one hour and a half. With respect to the maximum diurnal tide, it appears by the accompanying Table that it never turned before 9 A.M. or after 10^h 20^m A.M. during the whole of the year 1830; the seasons therefore were inoperative; and this is confirmatory of Mr. GOLDINGHAM's observations at Madras in 1823, although no great reliance can be placed upon them, from their having been made *only* every tenth day: confirmatory also of MARGUÉ VICTOR's observations made for years at Toulouse. I found the maximum diurnal tide (indeed *all* the tides) to oscillate in its time of turning, and in its stationary period between the hours stated, without relation to any change in the attached thermometer. On the 5th of February the tide turned before 10 A.M.; on the 14th it turned at 10^h 20^m; on the 11th of March at 9^h 15^m; on the 19th not before 10 A.M.; April 11th at 9^h 30^m; April 17th at 9^h 45^m; on the 14th of June it turned at 9 o'clock; on the 10th of June at 10 o'clock; and similar anomalies occur in the following months. The *stationary periods* of the maximum A.M. tide range from 0 to 45 minutes, and the Table shows several instances of the latter. The fall of the barometer in equal periods of time after the turn of the tide presents irregularities. On the 11th of March the fall was ·010 in 30 minutes: on the 11th of April in 30 minutes it was only ·001.

The afternoon tide has the same irregularities as the preceding. It never turned before 4 P.M., and in a few instances only after 5 P.M. On the 5th of February the tide turned at 4 P.M.; on the 8th at 4^h 30^m; on the 20th of August at 5 P.M.; on the 4th of October at 4 P.M., &c.

The stationary period was from 0 to 45 minutes; but of the latter there is only one instance in the Table, although there may be more in the registers, as the extracts were taken at random. On the 9th of February there is a curious instance of the tide turning at 4^h 15^m P.M.; then rising ·004 to 4^h 30^m, continuing stationary until 5 P.M., and then *resuming* its rise. As in the morning tide, the movements of the barometer were not equal in equal times. On the 5th of February the tide rose only ·002 in

board ship in the terrible hurricane of the 27th and 28th of August 1794, found that the column of mercury fell only ·4448 ($11^{mm}\cdot3$).—Personal Narrative, vol. vi. part ii. page 794.

I am enabled to strengthen this assertion by the following extract from the log-book of the Duke of Buccleuch, Captain HENNING, from Calcutta to London, in January 1833, during a frightful tempest of two days' duration off the Isle of France:—

"21st. Lat. 24° 31′. Long. 61° 49′. Bar. max. 30°·00. Bar. min. 29°·60. Temp. 80½°.
22nd. Lat. 25° 39′. Long. 57° 32′. Bar. max. 29°·76. Bar. min. 28°·94. Temp. 82°."

The whole fall, therefore, amounted to no more than one inch and six hundredths in the two days.

ninety minutes; on the 6th it rose ·008 in seventy-five minutes; and on the 8th it rose ·005 in fifteen minutes; and on the 11th of April ·001 only in forty-five minutes.

In the maximum *nocturnal* tide (10—11 P.M.) there was rarely any difference in the thermometer during the oscillations of the mercury; nevertheless the turn of the tide ranged from 9^h 30^m to 11^h 30^m, and in two remarkable instances even beyond these hours. On the 12th of October it turned at 9 P.M., and on the 9th of June at 12 P.M.; both of these anomalies may have been produced by the state of the weather, there having been a heavy thunder storm from 7^h to 8^h 30^m on the 9th of June, and several thunder storms *round the horizon* on the 12th of October, although not immediately at Poona. The stationary period ranged from 0 to 60 minutes, but of the latter there is only one instance in the Table. As in the preceding tides, the movement of the mercury in equal periods of time manifested occasional irregularities, although the thermometer remained stationary, or nearly so. On the 6th of February the night fall was ·008 in 15 minutes, and on the 8th it was only ·001 in 15 minutes; in neither instance was there any movement of the thermometer, whilst on the 10th of June, between 10^h 45^m and 11 P.M., the barometric fall amounted to ·010. From the above facts, and they could be infinitely multiplied, it is clear there is not any positive uniformity in the oscillations of the mercurial column, nor in the duration of the stationary periods; nevertheless as the irregularities are bounded by comparatively narrow limits, the movements may be considered subject to a general law, the rationale of which remains to be explained.

Experiments have determined that the *diurnal* atmospheric tides (the nocturnal tides have been less attended to) extend from the equator to high parallels of latitude, but that the oscillation decreases as the latitude increases. It is further presumed that the oscillation gradually diminishes in ascending from the level of the sea to great heights. Professor JAMES FORBES of Edinburgh has laid down an assumed curve, in which the diurnal oscillation amounts to ·1190 at the equator, and *nil* at latitude 64° 8′ N.; and beyond that latitude the tide occurs with a contrary sign, the maximum hour becoming the minimum. More extended and careful observations in different parts of the earth will probably confirm the empirical law sought to be established by Professor FORBES, but our present meteorological data offer many exceptions to it. In the valuable table given by Professor FORBES in his paper*, there are exceptions to his law in the observations of RUSSELL at Boorhanpoor, and PRINSEP at Benares, each for three years. The mean diurnal oscillation, agreeably to the former, in latitude 24° 4′, being less (·0877), mean temperature 75°·2, than that at Benares (·1059), mean temperature 78°·8, in latitude 25° 30′. Mr. GOLDINGHAM in 1823, at the Madras Observatory, latitude 13° 5′, observing every tenth day, found the diurnal oscillation amount only to ·0790, mean temperature 81°·69. At Ava in the Birman empire, latitude 21° 51′, agreeably to Major BURNEY, the oscillation amounted to ·1260, mean temperature 78°·39, being greater than in any other series

* Transactions of the Royal Society of Edinburgh, vol. xii. Part I. p. 170.

of observations made in India. My own observations are also exceptions to the law. In latitude 18° 30′, at 1823 feet, with a mean temperature of 78°, the mean diurnal oscillation for one year, at the limit hour of the tide, was ·1166, whilst in Calcutta, latitude 22° 33′, mean temperature 78°·13, the mean of three years' oscillations (1829, 1830, and 1831,) give only ·1100; and as the observer was Mr. PRINSEP, his name is a guarantee for his accuracy.

But the exceptions are not confined to the tropics, for we find that the mean of five years' observations at Marseilles, latitude 43° 16′, mean temperature 60°·8, gives a less oscillation (·0326*) at a few toises above the sea than at Berne, latitude 46° 57′, mean temperature 53°·6, at 532 toises above the sea, where the mean of ten years' observations gives an oscillation of ·0354†. Here, therefore, the oscillation unquestionably should have been less, because the latitude is higher, the temperature lower, and the height above the sea greater. But these discrepancies may be attributed to the observations not having been taken at the exact limit hours of the tides, and do not therefore give the true oscillation; nor will satisfactory light be thrown upon the irregularities of the tides until hourly observations are made for lengthened periods in various parts of the earth.

On the subject of decrement in oscillation, consequent on elevation above the sea, I have collected such data as were available, and have thrown them into the form of a table. HUMBOLDT found that at the Caraccas, at 936 toises above the sea, the oscillation was greater (·1063‡, mean temperature 69°·8,) than at Cumana at 10 toises above the sea, where it was ·1004, mean temperature 78°·8. My own careful observations at Poona furnish a similar anomaly. At 1823 feet above the sea the mean oscillation for a year was greater (·1166) than at Bombay, where for nine months the mean was ·0765 at the Engineer Institution; and in my occasional visits I found it respectively ·0836 in April 1827, ·1123 in March 1828, and ·1141 in December 1828. At Madras, in a lower latitude than Poona, at the level of the sea, I have shown it to be only ·0790; whilst at Calcutta, in a higher latitude than Poona, the means of three years make it ·1100. Proceeding to higher levels, however, we find a marked diminution in the extent of the diurnal tide. At Mahabuleshwur, at 4500 feet, the means of eight months reduce the oscillation to ·0694; at Hurreechundurghur, at 3900 feet, the oscillation for the three hottest months was ·0969; whilst at Kotagherry, at 6407 feet, it was for five months from *noon to sunset* only ·0498. The oscillation at Mahabuleshwur, at 4500 feet, was in fact *less* than HUMBOLDT's oscillation at Mexico of ·0708 at nearly 7000 feet.

When we pass to the other tides we find the same puzzling anomalies. The mean rise from sunrise to 9—10 A.M., whether at Hurreechundurghur, at Mahabuleshwur, or Kotagherry, instead of being less than at Poona, is in fact greater. The mean of three years on the level of Poona gives ·0445, whilst the first place gives ·0488, the second place ·0476, and the last ·0490. The maximum night tide, on the contrary,

* $0^{mm}\cdot83$. † $0^{mm}\cdot90$. ‡ $2^{mm}\cdot70$.

is infinitely greater at Poona than at Mahabuleshwur or Kotagherry (it was not determined at Hurreechundurghur), being ·0884 at Poona, ·0439 at Mahabuleshwur, and ·0430 at Kotagherry. The fourth or *minimum* nocturnal tide occurring in the dead of night, has been rarely observed at the exact A.M. limit hour; but the observations have been taken at sunrise, which is from one and a half to two hours after the turn of the tide. I have previously shown that at different times I found this tide to amount to −·0150, −·0254, −·0010, and +·0053, and −·0040; taking the mean of these, after deducting the plus sign, we have ·0134 as an approximation to the amount of the oscillation in this tide; and this corrected for a presumed proportional increase from 4 A.M. to sunrise, would make its value ·0181. During eight months at Mahabuleshwur Dr. Walker found the mean fall from 10—11 P.M. to sunrise to be ·0180, thermometer −1°·68; corrected to 4 A.M. it would be about ·0240. Mr. Dalmahoy at Kotagherry, at 6407 feet, found the fall from 9—12 P.M. to a little before sunrise, amount to ·0350; and as it is probable he took his observations as often after the tide had turned at 10—11 P.M. as he took them after the limit hours of the 4—5 A.M. tide, the errors may be considered as compensating each other, and the oscillation may be left uncorrected.

Mr. Prinsep, in a voyage of thirty two days from Calcutta to Bombay, found the fall of the barometer from 10 P.M. to sunrise, amount to ·022, which corrected to 4 A.M. would be about ·0293. Correcting the rise of the tide from sunrise to 9—10 A.M. in the same rough way, the following will be the amount of the mean oscillation of the barometer in the different tides.

	Nocturnal *falling* minimum tide from 10—11 P.M. to 4—5 A.M.	Diurnal *rising* tide from 4—5 A.M. to 9—10 A.M.	Diurnal maximum *falling* tide from 9—10 A.M. to 4—5 P.M.	Nocturnal maximum *rising* tide from 4—5 P.M. to 10—11 P.M.
Mr. Prinsep, 32 days, level of the sea	−·0293	+·0587	−·1020	+·0800
M. Duperrey, Ship Coquille, Payta, lat. 5° 5′ S., two days	−·0669	+·0629	−·1417	+·1259*
Mr. Goldingham, Madras Observatory, every tenth day	−·0350	+·0470	−·0790	+·0630
Mr. Hudson, Royal Society, London	−·0162	+·0185	−·0289	+·0272
Colonel Sykes, 1800 to 2000 feet, Poona, one year	−·0181	+·0445	−·1166	+·0884
Dr. Walker, 4500 feet, Mahabuleshwur, ten months	−·0240	+·0636	−·0694	+·0439
Mr. Dalmahoy, 6407 feet, Kotagherry, five months	−·0433	+·0490	−·0498	+·0430

The observations of Duperrey, Goldingham, and Hudson were made during the limit hours of the several tides, and have not in consequence any correction applied by myself. Prinsep's, Dr. Walker's and my own observations are corrected from sunrise back to 4 A.M.; but the other tides are as they were observed. Mr. Dalma-

* Humboldt, Personal Narrative, vol. vi. part ii. page 703.

HOY's hours of observations have been previously noticed. M. DUPERREY's observations were made every fifteen minutes, but continued only for two days; and it is remarkable, although the first day gave a diurnal maximum falling tide of ·1417, the next day gave only ·0984: any deductions, therefore, from observations for *short periods* of time even in the tropics must be fallacious. The above data unquestionably prove the existence of *nocturnal* tides, of which doubts exist, or did exist, in Europe; although HUMBOLDT says they were observed in Dutch Guiana as far back as 1722 by a naturalist whose name is unknown.

Unhappily, during 1830, whilst observing the exact time of the turn of *three* tides, I was so harassed by public duties, that I omitted to record the barometer at sunrise, and therefore want the data to assist in determining for any *lengthened* continuous period the amount of the tide between 10—11 P.M. and 4—5 A.M. or sunrise. But as there is a remarkable accordance in the absolute height of the barometer at Poona during the monsoons of 1829 and 1830, if I were to adopt the mean height of the barometer at sunrise in 1829 for 1830, and then deduct this amount from the mean height at 10—11 P.M. in 1830, we shall have ·0332 as the value of the oscillations, which, corrected to 4 A.M., will give ·0442 as the value of the falling tide between 10—11 P.M. and 4—5 A.M.; and this amount I have little doubt would be infinitely nearer the truth than my forty or fifty nights' observations taken at different periods.

From the above short table it will appear that in my observations for four years, the maximum oscillation was between 9—10 A.M. and 4—5 P.M.; the next greatest was that between 4—5 P.M. and 10—11 P.M., amounting to a little more than 75¾ per cent. of the preceding fall: then follows the rising tide between 4—5 A.M. and 9—10 A.M., amounting to nearly 40 per cent. of the diurnal tide; and finally comes the falling tide between 10—11 P.M. and 4—5 A.M., which by the few direct observations I made would not be more than 15½ per cent. of the great tide, but which by the process above noticed, I suppose would be about 38 per cent.; and this would accord tolerably well with Mr. PRINSEP's proportion, which is nearly 28¾ per cent. I shall not remark upon the discrepancies between the ratio thus eliminated, and that deducible from the observations of the other gentlemen quoted in the Table. One fact, however, appears established by all the observers, that the greatest oscillation is during the day, the least during the night; the second greatest from 4—5 to 10—11 P.M., and the second least from 4—5 to 9—10 A.M.; and that all the irregularities occur within comparatively narrow limit hours.

My barometrical observations were taken on various levels, excepting for the yearly residence of five or six months at Poona during the monsoon, and for the entire year 1830: the statements of the absolute height of the barometer and the annual and monthly changes of pressure of the atmosphere will therefore be comparatively limited; and it may be as well to confine my remarks almost entirely to the year 1830. The maximum height of the barometer, and the mean monthly maximum in that year, both occurred in January, the former being 28·242 inches, thermometer 73°·6, and the

latter 28·087 inches, thermometer 75°·4. The minimum height in the year and the mean monthly minimum, in like manner, both occur in the same month, July, the former being 27·570 inches, thermometer 75°, and the latter 27·7666, thermometer 76°·95. The annual range of the barometer, therefore, amounted only to ·6720; and the difference of the thermometer at the extreme periods was 1°·4; the greatest monthly range, ·3710, was in November; the difference of the attached thermometer at the extreme periods was 10°·2; the smallest monthly range of ·2170 was in August; the difference of the attached thermometer at the extreme periods being 0°·5. In 1827 the barometer ranged during six months whilst I was stationary, only ·5103. In seven months in 1828 it was ·5656, and for seven months in 1829 it was ·4867; and in no instance did a range of eight tenths of an inch come under my observation, even in comparing the maximum of one year with the minimum of another. Whilst in England, at Edmonton and Cheltenham, in 1827, the extreme range of the barometer was respectively 1·88 inch and 1·75 inch. In 1828, at Edmonton, Cheltenham, and Weycomb, the range was 1·44 inch, 1·41 inch, and 1·61 inch respectively. An inspection of my tables will show that in four years, in the five *monsoon months*, from the maximum height 28·1343 inches, thermometer 76°·4 in October 1827, to the minimum height of 27·570, thermometer 75° in July 1830, the range amounted only to ·5643, difference of thermometer attached 1°·4. In looking over Mr. Goldingham's tables for twenty-one years at Madras, the greatest annual range (with a solitary exception of 1·430 inch in a terrific hurricane in May 1820,) amounted to ·9640 in 1818, and the greatest monthly range was in October of the same year ·7940; the smallest annual range was ·4620 in 1814; in fact, the annual range very rarely exceeded six tenths of an inch.

I found the mean monthly pressure of the atmosphere at its maximum in the coldest months, December and January; it gradually diminished until July or August, the most damp months: and gradually increased again until the cold months. Mr. Goldingham's means of twenty-one years give nearly the same results; the maximum pressure 30·085 inches, thermometer 75°·168, being in December or January; it then diminishes until May, June, and July, the mean height of the barometer, 29·860, thermometer 86°·907, being nearly the same in those months. But it is to be remarked, that two of these months, which at Poona are the most damp, at Madras are the hottest of the year: the minimum pressure, therefore, was as independent of moisture at Madras, as it was independent of extreme heat at Poona. From July the pressure gradually increases as at Poona, until December or January. Three years' observations at Calcutta indicate the same alternations. The barometer is highest in January, 30·0225 inches, and lowest in June, 29·5155 inches. At the Havannah the mean of three years gives a maximum pressure in January and a minimum in September. Opposed to these indications of uniformity of atmospheric action over a wide range of latitude and longitude, M. Boussingault found the maximum height of the barometer at Bogota for one year greatest in June and July, and least in Decem-

ber and January *. The means of four years' observations, from 1827 to 1830 inclusive, made by Mr. HUDSON at the Royal Society, give two maxima and two minima in the year, the former occurring in February and October, and the latter in April and September. Professor FORBES's observations in the same years at Edinburgh give a mean maximum in the winter months, December, January, and February, of 29·442 inches, and a mean minimum in spring, March, April, and May, of 29·0359 inches.

The annual mean height of the barometer at Poona was 27·9254 inches; at Madras for twenty-one years it was 29·958 inches; at Calcutta the means of three years make it 29·764; M. ARAGO, at Paris, by nine years' observations, reduced to the level of the sea, makes the mean height 29·9546 inches, almost identical with the mean height at Madras.

The climate of Dukhun is subject to very considerable variations of temperature, more, however, in the diurnal than in the monthly or annual ranges; indeed, less so in the last particular than in Europe. In 1827, the extreme range of the thermometer at Edmonton was 75° FAHR. (83° highest, 8° lowest); at Cheltenham it was 64°·5 (80°·5 highest, 16° lowest); in 1828 at Edmonton it was 61° (83° highest, 22° lowest). These extremes are even exceeded on the continent of Europe. In St. Petersburgh the thermometer has been as low as 35°·7 *below zero*, and as high as 91°·4, the range therefore 127°·1. At Berne in Switzerland the range has been from 24° below zero to 95°·25 FAHR. The extreme range of my thermometer in 1826 was from 93°·9 to 40°·50 or 53°·4; the former occurring on the 12th of March at 4 P.M., and the latter on the 15th of January at sunrise. In 1827 the extreme range was from 96°·8 to 48°, exhibiting a difference of 48°·8, the maximum being on the 28th of March at 4 P.M., and the minimum on the 12th of December at sunrise. In 1828 the maximum occurred on the 7th of May at 4 P.M., being 101°, and the minimum 56° on the 16th of February and 4th of December at sunrise, the range not exceeding 45°. I have to remark, however, that for a short time on the 7th of May, the thermometer rose to 105° (this was at the source of the Beema river, at a height of 3090 feet above the sea), the highest record of the instrument I have ever had in Dukhun, in the shade, in very many years' observations. These occasional manifestations of extreme heat would appear not to be confined to the equatorial regions, there being many similar instances in the temperate zones. At Montpelier in France, in 1823, the thermometer stood for some days at 100° FAHR. In Paris, in 1793, it was at 99°·6; and HUMBOLDT, in his Personal Narrative, mentions, on the authority of ARAGO, it being even 101°·12 FAHR. at Paris. The range of the thermometer in Paris, between 1793 and 1795 inclusive, was from 8°·6 below the freezing-point of FAHR. to 99°·6 or 81°.

The monthly means do not differ much from each other in Dukhun. In 1826 the difference between the *means* of the hottest month, May (83°·28), and the coldest, Ja-

* January, 0^{mm}·56045; Temperature 15°·7. June, 0^{mm}·56124; Temperature 15°·1.
December, 0^{mm}·56013; Temperature 15°·0. July, 0^{mm}·56134; Temperature 14°·2.
HUMBOLDT, Personal Narrative, vol. vi. part ii. page 748.

nuary (65°·90), was only 17°·38. In 1827 January was the coldest month, and the hottest was April, their mean difference being 14°·06. In 1828 the coldest month was December and the hottest May, their difference 15°·41. In 1829 March was the hottest and November the coldest, their difference 13°·66. The greatest diurnal range in 1826 was 37°·30, on the 5th of March, from 50°·5 to 87°·8. In 1827 it was 39°·5, on the 12th of December, from 49°·5 to 89°. In 1828 it was 34°·8, on the 16th of February, from 56° to 90°·8. In 1829 the maximum diurnal range was 37°·5, in December. The least diurnal range in 1826 was on the 22nd of August, amounting only to 0·60. In 1827 it also occurred in August (9th), being only 0°·40. In 1828 the minimum range was on the 18th of October, amounting to 0°·40; an unprecedented circumstance in that month. In 1829 the minimum range was 0°·60, in August. In 1830 it was 0°·5, in July.

With respect to the greatest diurnal and the greatest monthly range of the thermometer, the winter months have a range nearly in a quadruple ratio to the monsoon months, June, July, August, and September. The latter have mostly their temperature very equable, the difference of the monthly means rarely exceeding 3°, and the greatest diurnal range in five years only once amounted to 13°·6. The latter end of March, and April and May are the hottest months in the year, from the position of a nearly vertical sun, the intensity of whose influence is but slightly modified by the occasionally cloudy weather in May preceding the monsoon. The temperature falls in June, and continues nearly stationary until the end of September; it then rises in October, but falls at the end of the month until its annual minimum in December or January. It is low the early part of March, but rises *suddenly* after the middle of the month, occasioning a difference of 6° or 8° between the means of February and March, which is more than double that of other consecutive months in the year. The rise in October is also sudden, but does not occasion so great a difference of means as between February and March. It will thus be remarked that the temperature does not follow the sun's declination, owing to the interference of the monsoon.

My thermometrical observations in Dukhun were made upon levels ranging from 1400 feet above the sea to 4500. At the latter height, however, they were very limited in number, and beyond the levels of 1600 and 2200 feet they may be considered to have scarcely any sensible influence upon a mean temperature struck for tracts traversed between 1900 and 2000 feet. For instance, the mean temperature of Ahmednuggur in 1828 (1900 feet), Dr. Walker determined to be 78° Fahr., and my mean temperature for the country I traversed in that year was 77°·93. In 1827 it was 77·25; and in 1826, when my researches were a good deal confined to the hilly tracts, the mean temperature was 76°·46; and in 1829 the mean temperature was reduced to 74°·8, three months' observations of the year having been taken at 3943 feet above the sea, and one month's observations at 2416 feet. One fact is very remarkable; the observed mean temperature of places on the table land of India is much higher than the calculated mean temperature of the same places agreeably to

Mayer's formula. Ahmednuggur is 1900 feet above the sea with a mean temperature of 78°: the calculated mean temperature is 72°·27. Mhow in Malwa, at 2000 feet, observed mean temperature 74°; calculated 69°·86. A spring in the hill fort of Hurreechundurghur I found to be 69°·5: the calculated mean temperature for the latitude of that fort, at an elevation of 3900 feet, is 65°·45. The calculated mean temperature of Poona is 72°·78; the observed 77°·7. But I purpose enlarging on this subject in a future paper on the mensuration of heights in Dukhun, determined barometrically and thermometrically.

An inspection of my tables of temperature will show that the mean temperature of $9^h\ 30^m$ A.M. is almost identical with the annual mean temperature deduced from the maxima and the minima. Professor Forbes observes that the same holds good at Edinburgh. To show the importance of position in placing instruments for observations of temperature, in November 1828, I put thermometer No. 2 under a grass roof adjoining the eastern wall of my house, but within twelve feet of thermometer No. 1, which remained in its usual place. The instrument was secure from direct or reflected heat. At sunrise the mean for the month of No. 2 was 7°·42 *lower* than the mean of No. 1; at $9^h\ 30^m$ it was 1°·76 *higher*; and at 4 P.M. it was 2°·71 *higher*; but its mean for the whole month was 2°·35 less than the mean of the thermometer kept in the house near the open window.

To ascertain the numerical cooling effect of shutting out the external diurnal air from acting upon the thermometer in the hot months, I hung thermometer No. 2, in the month of April 1827, in my drawing-room, communicating by double doors with a large dining-room surrounded by an inclosed and glazed verandah. I had all the external windows and doors carefully shut at 7 A.M. daily, and opened again at sunset. Thermometer No. 1 was in its usual place in my library, with a free circulation of air. Thermometer No. 2 was 1°·73 *higher* than No. 1 at sunrise; at $9^h\ 30^m$ A.M. it was 0°·63 *lower*; and at 4 P.M. it was 5°·5 *lower*; and the difference of the monthly means was 3°·62 minus in favour of thermometer No. 2. There cannot be a doubt, therefore, of the advantage of closing a room in the tropics during the heat of the day.

My hygrometric observations with Daniell's hygrometer for forty-three months, from April 1826 until March 1828, and from June 1829 until January 1831, were very complete and satisfactory. The first great feature was the annual mean dewing-point being higher at 9½ A.M. than at sunrise or 4 P.M., excepting in 1829—1830; but it did not uniformly hold good in each month of the year. In 1826 the mean dewing-points in Dukhun at sunrise and 9½ A.M. were respectively 66°·58 and 67°·56; temperature of air 73°·66 and 77°·53, containing 7·473 and 7·634 grains of water in a cubic foot of air; but in the monthly means, October had a higher dewing-point at sunrise than at 9½ A.M.: October, however, was the only month in which this occurred. In the mean for 4 P.M., September had a higher dewing-point at 4 P.M. than at 9½ A.M. On the whole, it may be asserted that the mean dewing-points of the three periods of the day were tolerably uniform, although at 4 P.M. there was a much less absolute

weight of moisture in the air, allowing for the correction for increased temperature, than at sunrise or 9½ A.M. From June to December, inclusive, the mean dewing-point was 66°·75, mean temperature 77°·23, a cubic foot of air containing 7·455 grains of water.

The highest dewing-point recorded in Dukhun in 1826, occurred at 4 o'clock on the 21st of October, being 76°; temperature of air 84°·50; a cubic foot of air containing 9·945 grains of water. The lowest dewing-point occurred at sunrise on the 4th of December, being 44°; a cubic foot of air containing 3·673 grains of aqueous vapour at a temperature of air of 56°. But the lowest dewing-point did not indicate the driest state of the atmosphere, as a dewing-point of 45° in November, with a temperature of 87°, at 4 P.M. gave only 3·587 grains of water in a cubic foot of air. The most moist month was July, the mean weight of water in the atmosphere in a cubic foot of air being 8·775 grains, and the point of saturation 4°·85 from the dewing-point.

The greatest monthly range of the dewing-point was in October (30°), and the smallest range in July and August (7°). An inspection of the monthly ranges will show that they conform to a limited extent only with the ranges of the barometer and thermometer. From June to December inclusive, the extreme dewing-points differed 32°.

In 1827 my hygrometric observations are complete for the whole year. The following are the results. In the means for the months, as in 1826, with the exception of part of April and the months of May and October, the 9½ A.M. means give a greater quantity of moisture in the atmosphere than at sunrise; the mean for the year at half-past nine having a dewing-point of 60°·74, temperature of air 78°·50, the cubic foot of air containing 6·140 grains of moisture; and the yearly mean at sunrise having a dewing-point of 59°·26, temperature 71°·20, and the cubic foot of air containing 5·940 grains of aqueous vapour. In part of April and in the month of August only does the mean at 4 P.M. give a higher dewing-point and a greater quantity of vapour in the air than at 9½ A.M. and at sunrise. In August we find a cubic foot of air at 4 P.M. containing 8·692 grains of aqueous vapour.

The quantity, however, is only great in relation to the quantity contained in the air at other hours of observation in the same month, and it will not bear comparison with the mean quantity held suspended at other periods during the monsoon; for we see by the Table, that in June, at 4 P.M., a cubic foot of air held 8·883 grains of water, and the other hours of observation had still larger quantities: nevertheless the monthly mean indicates August being the most moist month in 1827; for although a cubic foot of air contained only 8·574 grains, and June held 8·931 grains of water in a cubic foot of air, yet the difference between the dewing-point and the temperature of the air in August was only 5°·18, while in June those points were 7°·51 from each other: the air in August, therefore, was nearest to saturation; but the remaining months of the monsoon differ very slightly from these results. The highest dewing-point in Dukhun, in 1827, occurred at 4 P.M., on the 13th of June, being 76°,

temperature 79°; a cubic foot of air containing 10·049 grains of aqueous vapour. This may be looked upon as great, the temperature of the air at Poona being rarely 76° FAHR. when it absolutely rains.

A very dry state of the atmosphere occurred in January, the dewing-point on the 4th of the month at sunrise being obtained three degrees below the congelation of water, temperature 62°. A cubic foot of air at this observation contained 2·146 grains of water; but this did not indicate the driest state of the atmosphere, the dewing-point from the point of saturation being 33°, while on the 5th of December it differed 46°, the dewing-point being 37°, and temperature of air 83°.

As in the preceding year, the smallest range of the hygrometer is found in July and August. From these months there is a rapid increase in the range until January, when the greatest monthly range occurs, namely 38°. December has also a very high range of 32°. The extreme range in the year amounts to 47°; that is to say, from a dewing-point of 29°, temperature 62°, in January, to 76°, temperature 79°, in June.

In 1827, as in the preceding year, there is a limited conformity in the range of the hygrometer to that of the thermometer. The monsoon months have the smallest range, the cold months the greatest, and the remaining months a range between those already noticed.

In 1828 my hygrometric observations in Dukhun extend through three months only. In these months, as in the preceding years, there was more aqueous vapour in the atmosphere at 9½ A.M. than at sunrise or at 4 P.M. In February of this year the lowest dewing-point ever recorded on the general level of the country took place, being 5° below the freezing-point, namely, 27° FAHR., the temperature of the air being 57°·50, and a cubic foot supporting 2·032 grains of aqueous vapour. Even this is not the lowest degree of absolute dryness remarked in the Dukhun, as on the hill fort of Loghur, on the 12th of March, the dewing-point, although 27° FAHR., took place when the temperature of the air was 67° FAHR.: consequently, a cubic foot of air contained only 1·995 grains of aqueous vapour instead of 2·032 grains. A yet further degree of dryness occurred on the 16th of February at 4 P.M., at Downd, near Pairgaon, on the Beema river, when the dewing-point was 61° from the point of saturation, the former being 29°, and the temperature of the air 90°. The highest dewing-point in the three months of winter occurred at 9½ A.M. in January, namely, 69° FAHR., the weight of moisture being 7·988 grains; a state of the atmosphere which may be looked upon as very unusual in that dry month. The range of the dewing-point in January (37°) approximates very closely to that of the same month in the preceding year. The same observation applies to the month of March; but there is a discrepancy with respect to February.

In 1829 my observations extend from June to December, inclusive: the mean of the three periods of the day is nearly identical for the monsoon months, viz. 69°·03, 69°·77 and 70°·06: the maxima, 77°, occurred in June and October at 4 P.M.; the minima all in October, 58° at sunrise, 50° at 9½ A.M., and 44° at 4 P.M. The mean

dewing-point for the monsoon was 69°·62, temperature of the air 75°·83, the cubic foot of air containing 8·191 grains of water; the maximum diurnal range 6° in September, and the maximum monthly range 8° in June and September. In October the mean dewing-point fell to 65°·83, temperature 78°·13. The maximum diurnal range increased to 26°, and the extreme monthly range was 33°. In 1830 the observations are only complete for 9—10 A.M.: the mean dewing-point was 61°·9, mean temperature 78°·4, and a cubic foot of air contained 6·351 grains of water: the extreme range of the hygrometer was 47°, and the lowest dewing-point 31°, temperature 50°, in December. An inspection of the tables Nos. 17—21 will show the gradual increase of moisture in a cubic foot of air from the most dry month, February, until June or July. Hence the moistness remains nearly stationary until the beginning of October, when it diminishes, somewhat rapidly and regularly, until February.

It might be supposed that the hottest months in the year, March, April, and May, would also be the driest; but such is not the fact. The powerful action of the sun on the ocean in the middle of March raises a large quantity of aqueous vapour, which continues to increase in the ratio of the sun's progress north. The westerly winds waft this aqueous vapour into Dukhun: much of it is arrested by the Ghàts and hilly tracts eastward of those hills; accounting for the sensible moistness of the air, the frequent night-fogs, and deposition of dew on this line in the end of March and in all April and May. The supply of moisture diminishes in proportion to the distance eastward from the sea to the limits of the Coromandel coast monsoon: we in consequence find the Ghàts, Poona, Ahmednuggur, and the Bala Ghàt, all with very different dewing-points in the hot months.

My visits to Bombay on public duty in successive years, in the hot and cold months, enabled me to determine, in the most satisfactory manner, with the aid of DANIELL's hygrometer, the usual surcharged state of the air of the coast with moisture, and its ample means of supplying the interior table land with aqueous vapour.

In April and May 1826, in Bombay, the monthly mean dewing-points were respectively 72°·84 and 75°·59, temperature 83°·48 and 84°·52, a cubic foot of air holding 8·988 grains, and 9·748 grains of water suspended; whilst July, the most rainy month during the monsoon at Poona, had only a mean of 8·775 grains of water suspended. In 1827 the mean of ten days' dewing-points in Bombay in April gave 10·243 grains. The greatest mean quantity at Poona during the monsoon in June was only 8·931 grains of water in a cubic foot of air. In 1828 I was enabled, in the month of March, to establish comparisons, derived from observations on consecutive days, between Bombay, the top of the Ghàts, the hill fort of Loghur, and Poona. At 4 P.M. in Bombay, on the 10th of March, a cubic foot of air held 11·205 grains of water. At Poona, at the same hour on the 14th of March, a cubic foot of air contained only 2·273 grains of water. At Bombay, on the 10th, at sunrise and at 9¼ A.M., the dewing-points were respectively 72° and 71°, temperature 75° and 81°·50; a cubic foot of air containing 8·873 grains at the former hour, and 8·487 grains of

water at the latter hour. The following morning, at Kundallah, on the top of the Ghàts, 1744 feet above the sea, at the same hours, the dewing-points were 36° and 40°, temperature 72° and 78°, equivalent only to 2·690 grains and 3·004 grains of water in a cubic foot of air. In the afternoon of the same day, at Karleh, 2015 feet above the sea, seven miles east of Kundallah, a cubic foot of air held 2·954 grains, and on the 12th, at 4 P.M., 2·611 grains of aqueous vapour. On the summit of Loghur, 3381 feet above the level of the sea, and 1366 feet above Karleh, the dewing-point at sunrise the next day was 5° Fahr. below the freezing-point, temperature of air 67°; and a cubic foot of air held only 1·995 grains of water in a state of vapour.

These facts fully establish the remarkable discrepancies between the hygrometric state of the air in Bombay and Dukhun, and that too within a difference of a few miles of latitude and longitude. A comparison of the absolute fall of rain in Bombay and in Poona for the years 1826, 1827, and 1828, shows an agreement (to a certain extent) in their ratio to the relative hygrometric state of the air at Poona and Bombay above noticed*. The occasional extreme dryness of the air in the months of December, January, February, and part of March, is productive of some inconvenience; new furniture cracks, planks separate from each other, doors shrink so much that the locks will not catch; the leaves of card-tables warp, and manifest a disposition to curl up, and are only kept level by the constant application of brackets; ink disappears as if by magic, and the nibs of pens, by their recession from each other, manifest a provoking mutual antipathy.

I will confine my observations on the fall of rain in Dukhun within a narrow compass, as a glance of the eye over the Tables, Nos. 23—28, will afford every information. The rains are light, uncertain, and in all years barely sufficient for the wants of the husbandman, and a slight failure occasions much distress. They usually commence at the end of May, with some heavy thunder showers from the E. to the S.E., the lightning being terrific, and frequently dangerous. They set in regularly within the first ten days in June, and continue until the end of September from the W. to the S.W., and break up with thunder storms from the E. to S.E. before the middle of October. During the remaining months of the year an accidental shower or two may fall from the Coromandel monsoon; and the further the distance eastward from Poona, the greater the chance of showers in the cold months. The monsoon temperature is equable and agreeable, and the rain occurs almost always in showers, rarely continuing uninterruptedly for a day or more, as is common on the coast and in the Konkun. There does not appear to be any uniformity in the

* The mean annual fall of rain in Bombay for those years was 93·62 inches, and the mean fall at Poona 26·926 inches, or 28¾ per cent. only of the fall in Bombay. The absolute weight of aqueous vapour at Poona in March 1828, was 41¼ per cent. of the quantity suspended in the air in Bombay in the same month. The comparison of the means of the annual fall of rain in Bombay for twelve years, from 1817 to 1828 inclusive, viz. 82·01 inches, and of the fall of rain at Poona, 23·43 inches, from 1826 to 1830 inclusive, gives the same result.

fall of rain in the same months in consecutive years. In 1826, July was the most rainy month, and August the driest. In 1827, June had the most rain, and July the least. In 1828, July was the most rainy, and, unlike the two preceding years, June the least so. In 1829 and 1830, June had the most rain, and September less than any monsoon month for many years previously. In five years' observations in Dukhun, the greatest quantity of rain fell in the months of June and July. October, the month in which the monsoon breaks up, is the next most rainy, but the rain falls in a few heavy squalls, and the greatest part of the month is quite fair and bright. September, August, and May follow in the order of their aggregate supply of water. In those five years no rain whatever fell in February, twice only in December, and only once in January, March, and April respectively. The mean annual fall was 23½ inches, while the mean fall for twelve years in Bombay, only 80 or 90 miles to the westward, was 82 inches. The clouds supplying the monsoon torrents would appear to have a low elevation, as I have frequently seen through breaks, as they were passing rapidly from the west to the east, a superior stratum, apparently stationary, or moving slowly in a contrary direction, and gilded by the sun's rays. The greatest fall of rain in any one day was 2·58 inches, on the 6th of July 1826; and in the whole five years there were only six other instances of the diurnal fall having exceeded 2 inches, namely, on the 15th of January, 2·17 inches; on the 29th of June, 2·57 inches; on the 26th of September 1827, 2·54 inches; on the 30th of August 1828, 2·24 inches; on the 24th of June 1830, 2·31 inches; and 25th of July, 2·41 inches.

At Hurnee, on the coast of the southern Konkun, on the 15th of June 1829, there is a record of 8·133 inches of rain in the 24 hours. In the year 1828, in Bombay, there is an instance of a similar diurnal fall of rain on the 24th of June, viz. 8·67 inches; and in July of the same year, on the 12th and 18th, there fell respectively 7·40 and 7·45 inches of rain.

The mean annual fall of rain for all England, from many years' observations, is 32·2 inches; but the means of different counties vary from 67 in Cumberland to 19 in Essex.

The direction of the wind was carefully recorded three times daily for the years 1826, 1827, 1828, 1829, and 1830. The great features in these observations are the prevalence of winds from the west and westerly quarters, east and easterly points, and the extreme rareness of winds from the north and south, and the points approximating to them, and these features appear to be constant in the several years. In 5229 observations, the wind blew from the west or points adjoining 2409 times; and in this number the south-west (305) and north-west winds (122) amount only to 427, including the record of south-west winds (159) in May, June, and July 1826, which in truth were so westerly, that in the succeeding years in the same months they were classed as westerly winds, their inclination in general being more to the west than to the south of west-south-west, thus leaving 2141 observations of the wind almost exclusively from the west. The records of the easterly winds, including south-

east (103) and north-east (143), in five years, amount to 949: of this number 246 are from the points north-east and south-east, leaving 703 from the east. There is a remarkable paucity of northerly and southerly winds, there being records of the wind blowing from the north only 115 times, and from the south but 36 times. Another remarkable feature is the frequent absence of wind, particularly at sunrise; and more so in the months of January, February, March, October, and November, than in other months of the year. The cessation of wind from the month of May to September inclusive, is comparatively rare; and generally throughout the year the absence of wind at 4 P.M. may be looked upon as unusual. In five years there are 1720 observations of "No wind," and 847 of these belong to sunrise, 452 to 9—10 A.M., and 304 only to 4 P.M., and 117 to 10—11 P.M. in 1830. An inspection of the Tables will show that there is very considerable uniformity in the direction of the wind in the same months in consecutive years. The westerly winds begin to *prevail* in March, alternating with the easterly winds, which blow during the latter part of the night, and up to 7 or 8 A.M. At first they are to the northward of west, but they gradually come round to the west, and for the few last days in May and first week in June they are from the south-west; but when the rains fairly set in, they are limited to west and west-south-west until the beginning of October. In this month they are variable, and the records of "No wind" increase suddenly and rapidly. A few easterly winds, however, indicate the change which is about to take place; they gradually increase, and with those from the north-east and south-east, almost entirely supersede the winds from the westerly points. In March, from the sun's approach, the interior land during the day gets heated; an influx of air from the sea-coast commences after 10 A.M.; but as the earth at this period cools more rapidly than the sea at night, the interior is cooler than the coasts, and there is a reflux of air towards the ocean; the easterly and westerly winds thus alternate day and night. This alternation, however, diminishes in the ratio of the sun's increasing power; and when the earth gets so thoroughly heated that it cannot reduce its temperature by radiation below that of the sea, the consequence is the prevalence of winds from the westerly points to the almost entire exclusion of those from easterly points. In June the west-south-west wind sets in as previously stated.

The winds are rarely remarkable for blowing with very great violence, unless in the terrific but short thunder storms preceding the monsoon. At these periods trees are blown down, thatched houses unroofed, great damage is done by lightning, and the rain falls in a deluge. At Dholpoor in Hindoostan, in May 1805, I saw Lord Lake's camp levelled (except where partially sheltered) in one of these squalls as if by the wand of a magician; and trees which had stood two hundred years were torn up by the roots. Dense clouds of dust always precede the rain and darken the air; and it is amidst this imposing gloom that the lightnings flash with fatal effect.

The principal period of the year in which the wind is marked by its force, is in the latter end of March, all April, and part of May. During these months it is mostly a fresh west, sometimes strong; and I find by a reference to my registers that there

are many instances of its being violent. At these times it is exceedingly exhausting to the frame; and few old Indians are robust enough to bear to sit exposed to its direct action for any continuance. The easterly winds are characterized by their extreme dryness; the lips chap, the exposed parts of the skin are cut, and become harsh and scaly; windows, doors, and joiner's work shrink, and present numerous interstices; and to sleep exposed to the night easterly wind is to risk the loss of a limb or a whole side. With these exceptions the winds are usually agreeable to the feelings and of moderate force.

The hot winds (that is to say, a wind blowing over a heated extensive surface), so well known and complained of in the interior of the Indian Peninsula and in Hindoostan, are of limited duration within my range of observation. They are from the north-north-west to west, and occur in March and April. It is to be observed, that the same westerly wind which on the Ghàts may be passably cool and agreeable, will at Ahmednuggur, and at places more to the eastward, become a hot wind. The inhabitants of Poona and its neighbourhood are little incommoded by hot winds; and in my registers the records of their occurrence, even on my eastern boundary, are too limited to constitute a marked feature.

I must not omit to notice, that in these very months of the hot winds for five years a most unaccountable wind blew for a day or two from the north-north-west to the west-north-west so severely cold as to be injurious to vegetation, and intense enough to benumb the hands and feet. At Yagrah, near the source of the Mota river, on the 11th of March 1825, at sunrise, the young shoots of plants were nipped as if by a frost, although the thermometer was down only to 42°·10 FAHR. On the 9th of March 1826, the thermometer was at 58° at sunrise; the cold intense, no wind, but a westerly wind at 4 P.M. On the 13th of March 1827, at Tacklee near Ahmednuggur, a fresh west-north-west wind was so cold at sunrise, that I could not extend the fingers of my bridle hand, and my people had not been able to sleep during the night from the want of warm covering. In 1828 intense cold occurred on the 2nd of February at Barlonee, on the Seena river, but without wind. On the 29th of February, whilst driving from Poona to Karleh before daylight, my limbs were positively stiffened by a cold north-west wind. In 1829, at Hurreechundurghur, on the 4th of April, in the midst of the hot season, the cold was so great, with a west-north-west wind blowing, that a sheet, blanket, and counterpane were insufficient protection, and I was necessitated to rise in the night and put on a flannel dressing-gown to ensure comfortable feelings. In 1830 this wind occurred on the 2nd of March at Poona, at 11 P.M., and continued all night. It is difficult to assign a cause for these transitory cold winds at the commencement or in the midst of the hot season.

Those curious whirlwinds, noticed by all travellers in Africa, and which in the deserts are not only inconvenient but dangerous, are of common occurrence in Dukhun in the hot months. A score or more columns of dust, in the form of a speaking trumpet or water spout, may be seen at one time chasing over the treeless plains,

marking that vortex of heated air, which in its whirl carries up dust, sand, straw, baskets, clothes, and other light matters, to a height of one or two hundred yards or more. They are not dangerous, but particularly troublesome in a camp, striking the tents, and scattering about all light loose matters on the surface; and the rushing noise with which they come terrifies horses, and induces them to break from their pickets. They are sufficiently powerful also to lift off the grass roof of a hut; and I have known instances of officers' houses having shared the same fate. They appear and disappear with great suddenness; and I have been frequently startled by hearing a loud sound of air rushing from all parts to a central axis, round which it furiously whirls, and on the instant finding myself enveloped in one of these "devils," as they are called by Europeans in India.

During the dry months of December, January, February, and even during March and part of April, electricity is occasionally so prevalent in the air, that removing flannels with quickness from the body in the dark is accompanied with flashes of light; the hair crackles under the comb and emits sparks; suddenly shaking lino-musquito bed-curtains has been known to produce a flash; and stripping down bed-clothes has done the same. From the 8th of March until the 23rd of April 1829, while in tents in the hill fort of Hurreechundurghur, at 3943 feet above the sea, in stripping down the bed-clothes to get into bed I have frequently found my hand in contact with the clothes enveloped in a flame of blue light. On the last date mentioned, at 11 o'clock at night, the flash was so broad, vivid, and repeated at every movement of the bed-clothes, as to excite more than ordinary attention and surprise. I had not the means to determine the hygrometric state of the air at the time; the thermometer at 4 P.M. had stood at 90°·80; no change had taken place in the usual movements of the barometer; the wind up to $9^h\ 30^m$ A.M. had been east-north-east, and from that hour until past midnight had continued at west-north west in gusts: the night had not felt particularly dry; indeed the night of the 21st of April had been so moist as to wet the tents. Electric shocks in filling JONES's barometer in different parts of the country, and the terrific lightning of the storms in May, have been already noticed.

Hail sometimes falls in the *hot* months of March, April, and May, in those thunder storms to which I have alluded. The hail, which in many instances is found to consist of masses of transparent ice, is of considerable magnitude. In the storms of the 21st and 22nd of April 1830 at Poona, the hail-stones were larger than marbles; and they were of a similar size in a hail-storm in the fort of Hurreechundurghur, at 3943 feet above the sea, in the preceding April. I have known a mass of clear ice fall exceeding an inch in diameter, and I have been assured that much larger pieces have been picked up. On one occasion at Poona the hail-stones consisted of globular masses of clear ice, in which was imbedded a star of many points, of *diaphanous* ice like ground glass; and I deemed the fact sufficiently curious to induce me to make drawings of some of the stones.

Dews first appear towards the close of the monsoon, on the last mornings of September after cloudless nights. A precipitation of moisture takes place on similar nights in October and November. In December dews usually become somewhat constant and copious; and they are seen in January and February; but they occur under very anomalous circumstances, the causes of which I cannot explain. In consecutive nights of similar temperature, and similarly cloudless, dew will be found to have been deposited one night and not the following. In September 1827, the journal records "Heavy dew" on the nights of the 23rd, 24th, 25th, 26th, 27th, 28th, 29th, and 30th; they then cease until the 5th of October, on the morning of which there was a little dew; on the 6th there was not any, and on the 7th there was a little. They do not occur again until the 26th; hence to the 1st of November "Dew:" subsequently none until the 1st of December; hence no dew until the 6th of January 1828, when dew was met with *on garden land*, but not on *field land*; such continued to be the case during the whole of January. At Marbeh, Pergunnah Mohol, garden produce was covered with a copious dew every morning; the lands *bordering* the gardens for forty or fifty yards around were slightly sprinkled with it; *but there was not a vestige of it* on the fields constituting the rising ground north and south of the tract of garden land. I had daily experience of these facts from my habits of quail shooting. In the young wheats I observed that the quantity of dew on the plants was in ratio to the proximity of the time at which they had been irrigated. Plants on land, irrigated the day previously, wetted my shoes and cloth pantaloons thoroughly in a few minutes. Plants on land watered two days previously were plentifully covered with dew, but I could walk through two or three fields ere my clothes were fully saturated. Wheat irrigated three or four days previously, and bordering the fields above noticed, had dew on it, but not sufficient to wet me through. Such relative states of moisture in adjoining fields seem to establish the fact of the local character of dews. Aqueous vapour would appear to have been taken up by the action of the sun during the day, suspended over the spot, and deposited at night as dew on the land in proportion to the supply yielded by day, or the different lands radiated their heat in a different manner. My tents were within 200 yards of the fields where I observed these phenomena; but from the 11th to the 30th of January there was not any deposition of dew about them, excepting on the 13th of January only, and the dewing-point was but once within $4^{\circ}\cdot5$ of the point of saturation. In consequence of these observations I was induced to remark particularly the localities of dew at Poona and in its neighbourhood. In September and October I found that when there was not a trace of dew in the cantonment, there would be a deposition on the fields of standing grain half a mile distant; and when there was not any dew either in the cantonment or *in the fields*, it would yet be found on the banks of running rivulets, and on the banks of the Mota Mola river: but with respect to the rivulets, fifteen or twenty feet from the water were the limits of the deposition.

The local character of dew is further attested by the following facts. On the night

of the 28th of February 1828, there was not any deposition of dew at Poona or in its neighbourhood. Before daylight I rode thirty-four miles west-north-west to Karleh, in the hilly tracts, and to my surprise found my baggage, which had been left exposed during the night, dripping wet with a copious deposition. On the 1st of March I reached Bombay at sunrise, and observed all the tents pitched on the esplanade saturated with dew; and they were nightly in this state during the period of my stay in Bombay up to the 10th of March. On the 11th, at sunrise, on my return to Poona, I was at Kundallah, at the top of the Bore Ghàt, thirty-one miles inland from the margin of Bombay Harbour, and at 1700 to 1800 feet above the sea. Dew had not been deposited during the night of the 11th. On the 12th there was not any on the summit of the hill fort of Loghur, near Karleh; none at Poona on the 13th of March; nor have I a record of dew again on the plains of Dukhun, unless near to irrigated lands, until September, although in marching north in April and May, upon the meridian of Poona, there is occasional mention of a moist soft air at sunrise; and when encamped in May on the Ghàts, at Beema Shunkur, 3090 feet above the sea, I was sometimes enveloped in mists rising during the night from the low land of the Konkun, at the level of the sea, passing rapidly to the eastward, but entirely disappearing by 8 o'clock A.M. The first mention of dew on the register after the monsoon of 1828 is on the 23rd of September, and it was very heavy. There was not any on the 24th, 25th, and 26th. On the 27th it fell again copiously, and continued to do so until the 6th of October. It then ceased until the 21st, reappeared, and was deposited with occasional interruptions as in the preceding year. On the 14th of February 1829 there was a remarkable fall of dew at Pait, on the meridian of Poona, and thirty-two miles north of the city: with this exception there is scarcely a record of dew in the whole of that month. From the 10th of December 1828 until the 5th of January 1829, I was in Bombay on the esplanade: there was a nightly deposition of dew, not so copious as I had found it in April and May, but sufficiently abundant on several occasions to drip from the tents in the morning. In 1829 and 1830 the first dew appeared on the 6th of September in both years, and at intervals afterwards as in the preceding years. These notices are sufficient to show the want of uniformity in the appearance of dew. Its occurrence with an absolutely overcast sky is rare; but such was the case on the 23rd of September 1828. There are many instances of its being met with under a misty sky, also under a sky chequered with masses of clouds. For the most part it has been found to form most copiously in clear nights; but an inspection of my registers will show that in two consecutive nights equally clear, and with trifling difference in the thermometer, one night will be characterized by a fall of dew, the other not.

I have thought these details necessary, as a knowledge of the local deposition of dew, and its anomalous occurrence, is of some importance in applying the correction to the specific gravity of air in determining heights barometrically; for in the square of a mile the dewing-points at Marheh on the same morning at sunrise ranged from

30° to 65°!! There are some circumstances in the appearance of dew in Dukhun militating against Dr. Wells's theory of its formation; but more extended and careful observations may possibly show that they resulted from peculiar combinations not affecting his broad principles; and some of the anomalies may be traced to the different power of radiation of heat in different soils.

Fogs are certainly of rare occurrence in the Desh or open country within the limits of my researches, although along the Ghàts they prevail for six months in the year. In the Desh they are only seen in the months of October, November, December, January and February, and then only for a few mornings. By 9½ A.M. they are uniformly dissipated. In 1826 the first record of a fog was on the 8th of October, which was confined to the banks of the river at Poona. The same occurred on the 15th, 21st, and 31st. On the 18th of November of the same year there was a thick fog at Behloondeh, on the meridian of Ahmednuggur. On the 17th of January 1827 a thick fog occurred at Poona, which continued until 9½ A.M. On the 31st of the same month, and on the 1st of February, there was a partial fog until 9½ A.M. At Pairgaon, on the Beema river, on the 29th of November 1827, there was a partial fog until 9½ A.M. On the 31st of December, at the junction of the Beema and Seena rivers, and extending to Wangee, ten miles up the Seena, there was a remarkable fog in a stratum a few feet thick, lying close to the ground, its upper surface being quite flat, and not corresponding to the inequalities of the country. In consequence it frequently occurred, that in passing over slight rises on horseback, I had my head above the fog, while my body was enveloped in it. My view ranged over a sea of mist, and trees and houses appeared to spring from a sheet of water, the surface of which reflected prismatic colours. On the 3rd of October 1828, at Poona, a slight fog occurred; a heavy fog on the 6th, and the same on the 21st. On the 23rd and 24th of November also, at Poona, there was a thick fog. It was during one of these fogs at Poona that I witnessed a *white rainbow*. I had mounted my horse shortly after daybreak in prosecution of my accustomed ride, and galloped a few miles towards the east. Suddenly I found myself emerge from the fog, which terminated abruptly in a wall some hundred feet high. Shortly after sunrise I turned my horse's head homewards, and was surprised to discover, in the mural termination of the fog-bank, a perfect rainbow, defined in its outline, but destitute of prismatic colours. As the sun rose, the bow and fog-bank disappeared. Niebuhr, in his Voyage to Africa, describes a white rainbow; and Mr. St. John, in his Lives of Celebrated Travellers, mentions having seen one, on the 21st of May 1830, in Normandy, on "the morning mist*."

At Poona, on the 12th and 22nd of October 1829, fog until 7 A.M. and 8 A.M.; 23rd of October, partial fog. In 1830 there is not any notice of fog.—Such are my records of fogs in five years in the Desh, amounting only to nineteen times occurrence.

In the hilly tracts, and along the line of the Ghàts, they have been much more

* Vol. iii. p. 121.

frequent. In March, April, and May, for several years, I was encamped for a week or more on the crest of the Ghàts. About the middle of March fogs commence to rise, at uncertain intervals, from the Konkun. As the heat increases, the intervals become shorter; and from the first ten days in May I usually found myself enveloped in a thick fog, three or four times a-week, from dark until 9 or 10 o'clock the next day, by which time the heat of the sun had always redissolved the partially condensed moisture, and cleared the air. These fogs, when they were accompanied by westerly winds, rose rapidly from the Konkun, and flew with great swiftness eastward. At sunset there would not be a speck upon the sky; and within two hours, by a fall in the temperature of the air, the aqueous vapour from the sea, suspended over the Konkun, would be condensed, become visible, and shut out objects from view at a few yards' distance. When there was a want of wind from the west, or light easterly winds prevailed, the condensed vapour did not rise from the Konkun to the Ghàts, but appeared at daybreak lying upon the former, 1000 or 2000 feet below the level of the crest of the latter, like a sea of milk in repose, on which the prismatic colours of the rainbow were occasionally visible after the sun rose. All above would be perfectly bright and clear, and the sky a fine blue. The tops of mountains rose from this singular sea like islands, and the stupendous barriers of the Ghàts looked like a magnificent rocky shore. As the sun got high, the fog would be seen to creep up the chasms of the Ghàts and midway along the slopes of the ranges bounding the valleys, at the top of the Ghàts, and the Konkun would gradually reappear.

It was during such periods that I had several opportunities of witnessing that singular phenomenon the circular rainbow, which from its rareness is spoken of as of possible occurrence only. The stratum of fog from the Konkun on some occasions rose somewhat above the level of the top of a precipice forming the north-west scarp of the hill fort of Hurreechundurghur, from 2000 to 3000 feet perpendicular, without coming over upon the table land: I was placed at the edge of the precipice just without the limits of the fog, and with a cloudless sun at my back at a very low elevation.

Under such a combination of favourable circumstances, the circular rainbow appeared quite perfect, of the most vivid colours, one half above the level on which I stood, the other half below it. Shadows in distinct outline of myself, my horse, and people appeared in the centre of the circle as in a picture, to which the bow formed a resplendent frame. My attendants were incredulous that the figures they saw under such extraordinary circumstances could be their own shadows, and they tossed their arms and legs about, and put their bodies into various postures, to be assured of the fact by the corresponding movements of the objects within the circle; and it was some little time ere the superstitious feeling with which the spectacle was viewed wore off. From our proximity to the fog, I believe the diameter of the circle at no time exceeded fifty or sixty feet. The brilliant circle was accompanied with the usual outer bow in fainter colours. I witnessed these phenomena on the

29th of April, the 9th, 11th, and 12th of May 1829, on the hill fort of Hurreechundurghur.

I made some observations on solar and terrestrial radiation in 1828 and 1829, and had purposed extending them through several months; but unfortunately the severe labour of my statistical duties in those years did not admit of my devoting the necessary time to the interesting inquiry. In 1830, however, I persisted in investigating the subject day and night during the whole year, but as this paper is already too voluminous, I must reserve the details for a future communication. I will simply remark, that a thermometer on the grass covered with black wool at 2 P.M. on the 25th of November 1828, at Poona, rose to 164° FAHR., whilst a thermometer in my library stood at 76°·6; the force of the solar power, therefore, was 87°·4, far exceeding the maximum of any observations that have come under my notice: and I find that grass was frequently exposed to a range of more than 111° FAHR. between sunrise and 2^h 30^m P.M.

The opacity of the atmosphere in the hot months is very remarkable. In looking from the crest of the Ghàts over the Konkun at sunrise, the sky would be free from a cloud, and every object in the Konkun 3000 or 4000 feet below the spectator distinctly visible in the intervals of the fogs previously noticed: as the day advanced and the heat increased, the air would get misty, but without a cloud in the sky, and by 1 or 2 o'clock objects of great magnitude only would be visible in the Konkun, seen as through a diaphanous medium. The upper surface of this stratum of hot air was horizontal and quite defined. I found it very rarely reach to the height of 4000 feet, and I could invariably foretell the temperature of the coming afternoon above the Ghàts, by observing at 9 or 10 A.M. the height of the upper line of the heated atmosphere of the Konkun. If very high at those hours, compared with the preceding day, the temperature would be high; and vice versâ. In the Desh or open country above the Ghàts, the heated air rises for a few feet from the ground in wavy lines; and objects seen through the atmosphere in this state have an undulatory flickering motion.

HUMBOLDT most truly says, that in judging of temperature, nothing is more deceitful than the testimony of the senses: we can judge of the difference of climates only by numerical calculations. Having felt the full force of this dictum, I have thought it necessary to expatiate fully on the meteorology of Dukhun; and it now only remains for me to show how far the preceding numerical indications are coincident with salubrity of climate. This point I shall illustrate by a few facts equally brief and satisfactory. I was six years and one month in Dukhun employed in my statistical labours: my followers in the field, with their families, always exceeded one hundred persons, and in monsoon quarters the number was rarely below forty. During the whole period, and amongst such a number of persons, there was not a single casualty of an adult, and only one of an infant shortly after its birth; and but one case of disease that I could not cure myself without professional aid,—a degree of healthiness which probably few other countries can equal. Dr. WALKER, long civil-

surgeon in the city of Ahmednuggur, (exclusive of losses from spasmodic cholera,) found the casualties in that city to be only 1·82 per cent., or 1 in 55·1 persons; and including cholera, 2·48 per cent., or 1 in 40·2 persons. Dr. LAWRENCE, in charge of a regiment of natives 1000 strong, lost only 0·85 parts of an integer per cent., or about 5 men in every 600 per annum during the years the regiment was in Dukhun!

In conclusion, it may be desirable to give an abstract of the facts established, and the principal matters noticed in the preceding paper, viz. the entire removal of HUMBOLDT's doubts, founded on the authority of HORSBURGH, of the suspension of the atmospheric tides during the monsoon in Western India: the existence of four atmospheric tides in the twenty-four hours, two diurnal and two nocturnal, each consisting of a maximum and a minimum tide: the occurrence of these tides within the *same* limit hours as in America and Europe: the greatest *mean* diurnal oscillations taking place in the coldest months, and the smallest tides in the damp months, of the monsoon in Dukhun; whilst at Madras, the smallest oscillations are in the *hottest* months, and in Europe it is supposed the *smallest* oscillations are in the *coldest* months: the regular diurnal and nocturnal occurrence of the tides without a single case of intervention, whatever the thermometric or hygrometric indications might be, or whatever the state of the weather, storms and hurricanes even only modifying and not interrupting them: the anomalous fact of the mean diurnal oscillations being greater at Poona at 1823 feet, than at the level of the sea in a lower latitude at Madras: the fact of the diurnal tides at a higher elevation than Poona being *less*, whilst the nocturnal tides were *greater* than at Poona: the seasons apparently not affecting the limit hours of the tides: the maximum mean pressure of the atmosphere being greatest in December or January, then gradually diminishing until July or August, and subsequently increasing to the coldest months: the very trifling diurnal and annual oscillations compared with those of extra-tropical climates: the *annual* range of the thermometer less in Dukhun than in Europe, but the *diurnal* range much greater: the maximum mean temperature in April or May, gradually declining until December or January: the *observed* mean temperature of places on the continent of India much higher than the *calculated* mean temperature agreeably to MEYER's formula: annual mean dewing-point higher at $9^h\ 30^m$ than at sunrise or 4 P.M.: highest dewing-points in the monsoon, and lowest in the cold months: considerable difference in the dewing-points within very short distances: remarkable contrast between the dewing-points in Bombay and Dukhun: dew frequently local and occurring under anomalous circumstances: rain in Dukhun only 28 per cent. of the fall in Bombay, ninety or a hundred miles to the westward: winds principally from the westerly and easterly points, rarely from the northerly or southerly points, and the absence of wind frequent: electricity very abundant under certain circumstances: fogs rare, and always dissipated by 9—10 A.M.: very remarkable *circular* and also white rainbows: solar radiation very great: and finally, I must not omit to notice the singular opacity of the atmosphere in the hot weather, and the occurrence of the mirage.

Table I.

Oscillations of the Barometer in Dukhun, East Indies, between the parallels of latitude 17° 25′ and 19° 27′ N., and longitude 73° 30′ and 75° 53′ E., at a mean elevation of 1800 feet above the sea; the whole reduced to 32° Fahr., with correction for the brass scale.

	1827.							
	Rise of Barometer from sunrise to 9—10 A.M.		Fall of Barometer from 9—10 A.M. to 4—5 P.M.					
	Monthly Mean.	Therm. attached.	Max.	Therm. attached.	Min.	Therm. attached.	Monthly Mean.	Therm. attached.
	in	°	in.	°	in.	°	in.	°
Jan.	+·0483	+ 8·5	−·1442	+ 7·5	−·0664	+ 5·0	−·1134	+ 5·9
Feb.	+·0483	+14·4	−·1709	+ 5·5	−·0891	+ 7·	−·1257	+ 8·7
March.	+·0562	+12·9	−·1892	+10·0	−·0722	+10·0	−·1248	+ 8·7
April.	*+·0645	+10·05	−·1282	+10·0	−·0218	+ 1·5	−·0836	− 0·05
May.	+·0408	+ 5·7	−·1180	+ 7·4	−·0153	+ 5·2	−·0624	+ 7·4
June.	+·0334	+ 2·94	−·1591	+11·3	−·0316	− 1·5	−·0902	+ 3·41
July.	+·0331	+ 2·72	−·0930	+ 5·4	−·0195	+ 3·5	−·0489	+ 1·64
Aug.	+·0323	+ 3·02	−·0827	+ 2·5	−·0150	− 0·8	−·0600	+ 1·31
Sept.	+·0327	+ 3·65	−·1259	− 1·4	−·0368	+ 2·8	−·0813	+ 2·82
Oct.	+·0412	+ 4·63	−·1472	+ 1·2	−·0567	− 3·6	−·1147	+ 2·52
Nov.	+·0590	+11·0	−·1695	+ 4·5	−·0956	+10·3	−·1444	+10·4
Dec.	+·0775	+13·1	−·1835	+11·1	−·1161	+14·0	−·1616	+13·6
Year.	+·0473	+ 7·27	−·1892	+10·0	−·0150	− 0·8	−·1025	+ 5·99

	1828.							
	Rise of Barometer from sunrise to 9—10 A.M.		Fall of Barometer from 9—10 A.M. to 4—5 P.M.					
	Monthly Mean.	Therm. attached.	Max.	Therm. attached.	Min.	Therm. attached.	Monthly Mean.	Therm. attached.
	in.	°	in.	°	in.	°	in.	°
Jan.	+·0713	+12·1	−·1856	+10·8	−·0753	+ 8·5	−·1483	+13·6
Feb.	+·0658	+13·07	−·1791	+17·9	−·1048	+10·8	−·1505	+14·04
March.	+·0439	+ 8·13	−·1575	+ 8·5	−·1038	+ 8·5	−·1386	+ 9·55
	†+·0360	+ 8·92	−·1302	+ 5·6	−·0938	+ 6·1	−·1123	+ 7·6
April.	+·0585	+ 7·92	−·1642	+ 7·2	−·1093	+ 8·5	−·1334	+ 9·97
May.	+·0500	+ 9·27	−·1270	+13·8	−·0441	+ 5·5	−·0836	+10·55
June.	+·0658	+ 4·84	−·1229	+ 6·2	−·0252	+ 3·8	−·1007	+ 2·5
July.	+·0133	+ 3·04	−·0862	+ 0·2	−·0220	− 1·1	−·0471	+ 0·56
Aug.	+·0162	+ 2·67	−·1249	+ 4·2	−·0381	+ 0·5	−·0706	+ 1·21
Sept.	+·0419	+ 3·15	−·1178	+ 3·0	−·0672	+ 0·7	−·0910	+ 2·23
Oct.	+·0530	+ 3·12	−·1366	+ 2·5	−·0155	+ 2·3	−·1106	+ 2·46
Nov.	+·0448	+ 5·19	−·1627	+ 5·0	−·0562	+ 1·0	−·1277	+ 3·31
Dec.	‡+·0530	+ 7·8	−·1533	+ 8·9	−·0945	+ 8·6	−·1141	+ 8·48
Year.	+·0481	+ 6·71	−·1856	+10·8	−·0155	+ 2·3	−·1093	+ 6·36

	1829.							
	Rise of Barometer from sunrise to 9—10 A.M.		Fall of Barometer from 9—10 A.M. to 4—5 P.M.					
	Monthly Mean.	Therm. attached.	Max.	Therm. attached.	Min.	Therm. attached.	Monthly Mean.	Therm. attached.
	in.	°	in.	°	in.	°	in.	°
Jan.	+·0401	+11·34	−·1606	+14·5	−·0934	+ 2·7	−·1358	+ 9·47
Feb.	+·0422	+13·56	−·1648	+11·5	−·0765	+ 7·6	−·1083	+ 8·32
March.	+·0437	+ 9·57	−·1641	+10·7	−·0343	+ 1·1	−·1024	+ 4·25
April.	+·0514	+10·91	−·1371	+ 6·0	−·0607	+ 9·5	−·0981	+ 3·6
May.	+·0514	+10·54	−·1192	− 4·8	−·0523	+ 3·7	−·0903	+ 2·42
June.	+·0234	+ 2·74	−·1366	+ 3·7	−·0351	+ 0·7	−·0734	+ 1·55
July.	+·0363	+ 2·51	−·1091	− 1·0	−·0281	+ 0·9	−·0654	+ 0·75
Aug.	+·0251	+ 3·07	−·1045	+ 2·8	−·0521	− 1·0	−·0866	+ 0·8
Sept.	+·0330	+ 4·75	−·1073	+ 3·5	−·0460	+ 0·0	−·0772	+ 1·43
Oct.	+·0350	+ 5·56	−·1446	+ 5·0	−·0594	+ 3·5	−·1116	+ 4·01
Nov.	+·0364	+ 6·53	−·1403	+ 4·0	−·0680	+ 4·5	−·1067	+ 4·7
Dec.	+·0399	+ 8·72	−·1435	+ 6·0	−·0659	+ 5·0	−·1338	+ 5·74
Year.	+·0382	+ 7·48	−·1648	+11·5	−·0281	+ 0·9	−·0991	+ 3·92

	1830.							
	Fall of Barometer from 9—10 A.M. to 4—5 P.M.						Rise of Barometer from 4—5 P.M. to 10—11 P.M.	
	Max.	Therm. attached.	Min.	Therm. attached.	Monthly Mean.	Therm. attached.	Monthly Mean.	Therm. attached.
	in.	°	in.	°	in.	°	in.	°
Jan.	−·1643	+ 5·5	−·1211	+ 8·2	− ·136	+ 5·8	....	......
Feb.	−·1781	+ 7·2	−·0692	+ 3·7	− ·140	+ 6·5	+ ·088	− 8·1
March.	−·1663	+10·0	−·0493	+ 9·9	− ·133	+ 9·8	+ ·097	−12·7
April.	−·1950	+ 7·6	−·0887	+ 8·7	·143	+ 7·7	+ ·108	−11·1
May.	−·1799	+ 6·0	−·0622	− 0·2	− ·132	+ 5·8	+ ·114	− 9·0
June.	−·1583	+ 5·3	−·0544	+ 1·5	− ·106	+ 3·7	+ ·105	− 7·4
July.	−·1117	+ 3·3	−·0327	− 0·5	− ·075	+ 1·1	+ ·094	− 3·0
Aug.	−·1224	+ 2·1	−·0463	+ 1·0	− ·085	+ 2·3	+ ·082	− 4·5
Sept.	−·1480	+ 5·5	−·0519	+ 2·2	− ·090	+ 2·1	+ ·074	− 4·7
Oct.	−·1478	+ 3·0	−·0966	+ 4·0	− ·125	+ 2·9	+ ·084	− 4·1
Nov.	−·1561	+ 6·0	−·0624	+ 6·7	− ·125	+ 6·5	+ ·082	− 8·2
Dec.	−·1372	+ 5·5	−·0740	+ 4·5	− ·110	+ 4·9	+ ·045	− 6·3
Year.	−·1950	+ 7·6	−·0327	− 0·5	−·1166	+ 4·9	+·0884	− 7·2

The mean rise of the barometer from sunrise to 9—10 A.M. for 3 years is ·0445, thermometer +7°·15.

The mean fall of the barometer from 9—10 A.M. to 4—5 P.M. for 4 years is ·1066, thermometer +5°·21.

The mean rise of the barometer from 4—5 P.M. to 10—11 P.M. for 1 year is ·0884, thermometer −7°·2.

* 1827, April, in Bombay, not included in the means.
† 1828, March, ten days, in Bombay, not included in the means.
‡ 1828, December, in Bombay, not included in the means. 1829, February, sixteen days, at Pait, at 2531 feet above the sea. 1829, March, April, and May, at 3943 feet above the sea. 1829, December, nineteen days, at Chamblee, at 2416 feet above the sea.

TABLE II.—Mean diurnal and nocturnal oscillations of the barometer, and difference tioned places within the Northern Tropic on the Continent of India; reduced

	Calcutta, 1829, 1830, 1831.		Madras, maximum and minimum every tenth day, 1823.				Bombay, 1829.		Poona, 1830. 1823 feet above the level of the sea.						Hill Fort of Hurreechundurghur, 1829. 3900 feet above the level of the sea.			
	Fall from 9h. 40m. to 4 P.M.		Fall from 9—10 A.M. to 4—5 P.M.		Fall from 10 P.M. to 5 A.M.		Fall from 9 A.M. to 3 P.M.		Fall from 9—10 A.M. to 4—5 P.M.		Rise from 4—5 P.M. to 10—11 P.M.		Rise from sunrise to 9—10 A.M.		Rise from sunrise to 9—10 A.M.		Fall from 9—10 A.M. to 4—5 P.M.	
	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.
	in.	°	in.	°	in.		in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
Jan.	−·123	+20·7	−·072	+11·0	−·004	....	−·099	+5·4	−·136	+ 5·8	....	....	....	....	....	....	....	....
Feb.	−·117	+18·5	−·070	+10·0	−·029	....	−·091	+3·7	−·140	+ 6·5	+·088	− 8·1	....	....	....	....	....	....
Mar.	−·125	+14·0	−·076	+ 7·0	−·026	......	* −·1123 −·102	+7·6 +4	−·133	+ 9·8	+·097	−12·7	....	....	·0437	9·57	·1024	4·25
April.	−·124	+14·6	−·081	+ 9·0	−·027	......	† −·0836 −·089	+0·5 +3·1	−·143	+ 7·7	+·108	−11·1	....	....	·0514	10·91	·0981	3·68
May.	−·115	+13·7	−·081	+ 9·0	−·014	....	−·071	+2·3	−·132	+ 5·8	+·114	− 9·0	....	....	·0514	10·54	·0903	2·42
June.	−·095	+ 7·6	−·092	+ 9·0	−·026	....	−·054	+2·2	−·106	+ 3·7	+·105	− 7·4	·0234	2·74	....	....	....	....
July.	−·090	+ 6·1	−·097	+ 7·0	−·009	....	−·046	+1·2	−·075	+ 1·1	+·094	− 3·0	·0363	2·51	....	....	....	....
Aug.	−·099	+ 5·9	−·105	+ 7·0	−·028	....	−·063	+1·4	−·085	+ 2·3	+·082	4·5	·0251	3·07	....	....	....	....
Sept.	−·101	+ 6·2	−·094	+ 6·0	−·024	....	−·074	+2·1	−·090	+ 2·1	+·074	− 4·7	·0330	4·75	....	....	....	....
Oct.	−·110	+ 8·4	−·068	+ 8·0	−·033	....	....	....	−·125	+ 2·9	+·084	− 4·1	·0350	5·56	....	....	....	....
Nov.	−·107	+13·4	−·071	+ 8·0	−·010	....	....	....	−·125	+ 6·5	+·082	− 8·2	·0364	6·53	....	....	....	....
Dec.	−·114	+17·1	−·071	+ 9·0	−·019	....	‡ −·1141	+8·48	−·110	+ 4·9	+·045	− 6·3	·0399	8·72	....	....	....	....
Mean Tide	−·110	+12·2	−·079	+ 8·5	−·021	....	−·075	+2·82	−·1166	+ 4·9	+·0884	− 7·2	....	....	·0488	10·34	·0969	3·45

The Calcutta observations were made in the Surveyor-General's office; those at Madras, by Mr. GOLDINGHAM, at the Observatory; in Bombay, by Captain GEORGE JERVIS, at the Engineer Institution; at Poona and

* Ten days' observations in Bombay, in 1828, made by Colonel SYKES from 9—10 A.M. to 4—5 P.M.: Rise from sunrise to 9—10 A.M. ·0360; Therm. +8°·92.

† April 1827, in Bombay.—Observations made by Colonel SYKES, in tents, from 9—10 A.M. to 4—5 P.M.: Rise from sunrise to 9—10 A.M. ·0645; Therm. +10°·05.

‡ Observations made in Bombay, 1828, by Colonel SYKES, from 9—10 A.M. to 4—5 P.M.: Rise from sunrise to 9—10 A.M. ·0530; Therm. +7°·8, in tents.

of thermometer attached, at different levels above the sea, at the undermen- to 32° Fahr.

Mahabuleshwur, the source of the Kristna River, 1828, 1829, at 4500 feet above the level of the sea.								Kotagherry on the Nielgherry Mountains, 1836, at 6407 feet above the level of the sea.								
Rise from sunrise to 9—10 A.M.		Fall from 9—10 A.M. to 4 P.M.		Rise from 4 P.M. to 10 P.M.		Fall from 10 P.M. to sunrise.		Rise from sunrise to noon.		Fall from noon to sunset.		Rise from sunset to 9—12 P.M.		Fall from 9—12 P.M. to sunrise.		
Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	
in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.		in.		
+·0498	+3·99	−·0735	+4·76	+·0291	− 6·74	−·0054	−2·01	....	....	....	....	....	....	....	....	Jan.
+·0478	+8·90	−·0666	+6·5	+·0362	−12·92	−·0174	−2·48	+·033	+11·8	−·037	−3·2	....	....	....	....	Feb.
+·0456	+6·86	−·0827	+2·23	+·0534	− 6·79	−·0163	−2·3	+·073	+14·5	−·044	−5·5	·045	....	·074	....	Mar.
+·0627	+5·26	−·0835	+2·58	+·0443	− 5·96	−·0235	−1·84	+·031	+10·5	−·042	−4·6	·056	....	·045	....	April.
+·0536	+3·56	−·0757	+1·33	+·0445	− 4·02	−·0224	− ·87	+·033	+10·9	−·046	−4·8	·043	....	·030	....	May.
+·0392	+ ·31	−·0528	+ ·49	+·0365	− ·33	−·0229	− ·47	+·075	+ 4·3	−·080	−2·5	·028	....	·023	....	June.
....	....	−·0556	+ ·85	....	....	....	....	....	....	....	....	....	....	....	....	July.
....	....	−·0503	+ ·64	....	....	....	....	....	....	....	....	....	....	....	....	Aug.
....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	Sept.
....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	Oct.
+·0357	+1·46	−·0801	+3·22	+·0632	− 3·21	−·0188	−1·39	....	....	....	....	....	....	....	....	Nov.
+·0468	+3·15	−·0738	+3·55	+·0443	− 4·64	−·0173	−2·06	....	....	....	....	....	....	....	....	Dec.
+·0476	+4·18	−·0694	+2·61	+·0439	− 5·58	−·0180	−1·68	+·0490	+10·4	−·0498	−4·0	·0430	....	·0433	....	Mean Tide.

Hurreechundurghur, by Colonel Sykes; at Mahabuleshwur, at the convalescent station, by Dr. Walker; and at Kotagherry by Mr. Dalmahoy. The whole are unpublished, with the exception of those taken at Madras and Calcutta. From the hours at which Captain Jervis observed, and the small oscillation of the thermometer, I have not thought it worth while to reduce his observations to 32° Fahr. My own observations in Bombay are reduced.

Table III.

Table of some of the Anomalies in the period of the ebb and flow of the atmospheric tides in Dukhun, together with their Stationary Periods during 1830 at Poona.

Date. 1830.	Maximum diurnal tide 9—10 A.M.		Minimum diurnal tide 4—5 P.M.		Maximum nocturnal tide 10—11 P.M.	
	Exact hour at which the tide turned, together with the stationary period.	Differ. of attached Therm.	Exact hour at which the tide turned, together with the stationary period.	Differ. of attached Therm.	Exact hour at which the tide turned, together with the stationary period.	Differ. of attached Therm.
Feb. 5.	Turned before 10^h A.M.		Turned at 4^h P.M.; rise of ·002 only in 90^m.	+0·5	10^h to 10^h 30^m; quite stationary; fall of ·002 in 15^m.	−1·0
6.	Turned at 10^h 15^m; stationary 35^m.	+0·5	4^h P.M.; rise of ·008 in 75^m.	+0·2	Turned at 10^h 30^m; fall to 10^h 45^m = ·008.	0·0
8.	9^h 45^m to 10^h 15^m; quite stationary	+0·5	Turned at 4^h 30^m; rise of ·005 in 15^m.	0·0	Turned at 10^h 45^m; fall to 11^h P.M. = ·001.	0·0
9.	9^h 30^m to 10^h A.M.; quite stationary	+0·4	Turn at 4^h 15^m; rise to 4·30 = ·004; then quite stationary till 5^h P.M.	−0·5	Turn at 10^h 15^m; fall to 10^h 45^m = ·004.	0·0
14.	9^h 45^m to 10^h 20^m; quite stationary	+1·0	Turn at 4^h 45^m; rise to 5·45 = ·005.	−0·3	Turn at 11^h P.M.	0·0
20.	9^h 30^m to 10^h A.M; quite stationary; fall to 11^h A.M., = ·008.	+1·5	Turn at 4^h; rise to 5·30 = ·008.	−0·5	10^h 30^m; fall to 10^h 45^m = ·006.	0·0
March 11.	9^h 15^m; fall to 9^h 45^m = ·010.	+2·0	4^h to 4^h 30^m; quite stationary.	0·0	10^h 30^m; fall to 10^h 45^m = ·003.	0·0
19.	9^h 30^m to 10^h; quite stationary	+0·5	4^h; rise to 4·40 = ·024.	−2·0	10^h; fall to 10^h 30^m = ·001.	0·0
April 11.	9^h 30^m; fall to 10^h = ·001.	+2·5	4^h; rise to 4^h 45^m = ·001.	−1·2	10^h to 10^h 45^m; stationary; fall to 11^h = ·006.	−0·3
17.	9^h 45^m; fall to 10^h = ·002.	+0·5	4^h 30^m; rise to 5^h P.M. = ·002.	−0·2	10^h 30^m to 11^h P.M.; stationary.	0·0
May 10.	9^h 30^m to 10^h 15^m; stationary.	+1·0	4^h 30^m; rise to 4^h 45^m = ·005.	−0·5	10^h 15^m to 10^h 45^m; stationary; fall to 11^h 15^m = ·005.	0·0
June 9.	10^h A.M.	+2·0	4^h P.M.; rise to 4^h 15^m = ·004.	0·0	12^h P.M., after heavy storm from N E. at 7^h 0^m and 8^h 30^m; tide not suspended during storm.	+0·3
10.	9^h 30^m to 10^h; stationary.	+1·0	4^h 15^m; rise to 4^h 30^m * = ·008.	0·0	10^h 45^m; fall to 11^h = ·010.	+0·5
14.	9^h; fall to 10^h = ·017.	+1·5	4^h 30^m; rise to 4^h 45^m = ·004.	0·0	10^h 30^m to 11^h; stationary.	0·0
July 30.	9^h 30^m to 10^h 15^m; stationary.	+1·0	4^h 15^m to 4^h 40^m; stationary; rise to 5^h = ·002.	0·0	10^h; fall to 10^h 30^m = ·001.	0·0
August 5.	10^h 15^m A.M.	0·0	4^h 15^m; rise to 5^h = ·003.	0·0	10^h 15^m; fall to 11^h = ·003.	+0·2
20.	9^h 30^m to 10^h; stationary; fall to 10^h 30^m = ·003.	+0·5	4^h 30^m to 5^h; stationary.	−0·5	11^h P.M.; fall to 11^h 15^m = ·001.	0·0
30.	9^h 45^m; fall to 10^h 15^m = ·004.	+1·0	4^h 45^m; rise to 5^h 30^m = ·013.	−0·5	10^h 45^m.	0·0
Sept. 5.	9^h 30^m to 10^h; stationary.	+0·5	4^h 30^m to 4^h 45^m; rise of ·001 only.	0·0	11^h 20^m; fall to 11^h 30^m = ·001.	0·0
11.	9^h 30^m to 10^h 15^m; stationary.	+0·5	4^h 15^m; rise to 4^h 30^m = ·003.	−0·3	10^h 45^m; fall to 11^h 15^m = ·003.	−0·1
October 4.	9^h 30^m; fall to 10^h = ·002.	+1·0	4^h; rise to 4^h 30^m = ·001.	−0·5	11^h to 11^h 30^m; stationary.	−0·5
12.	9^h 30^m; fall to 10^h 10^m = ·002.	+1·0	4^h 15^m; rise to 4^h 30^m = ·004.	−0·5	†9^h P.M.; fall to 11^h P.M. = ·012.	−0·5
13.	9^h 30^m; fall to 10^h = ·003.	+0·2	4^h 30^m P.M.	0·0	9^h 30^m to 10^h; stationary; fall to 10^h 30^m = ·002.	0·0
14.	9^h 30^m; fall to 10^h — ·001.	+1·0	4^h 30^m P.M.; rise to 5^h = ·003.	0·0	10^h to 11^h; stationary.	−0·8
Nov. 3.	10^h; fall to 10^h 15^m = ·003.	+0·5	4^h to 4^h 15^m; stationary.	0·0	11^h P.M.; rise to 11^h 30^m = ·001.	0·0
Dec. 6.	9^h 45^m to 10^h 10^m; stationary.	0·0	4^h P.M.	0·0	11^h P.M.; fall to 11^h 45^m = ·003.	0·0
15.	9^h 45^m; fall to 10^h 15^m = ·004.	+1·1	4^h 15^m to 5^h P.M.; stationary.	0·0	11^h 30^m; fall to 11^h 45^m = ·001.	0·0

I have no instance of a stationary period of 5^h 30^m, nor of two hours even, as observed by Dr. Balfour in Calcutta in 1795; and I strongly suspect that these lengthened periods would not have been on record had Dr. Balfour's barometer *read off* to *thousandths* of an inch instead of *hundredths*.

* Storm at 4 P.M.

† Storms round the horizon at 4^h 30^m.

TABLE IV.

Barometrical Observations made at Hay Cottage, Poona, with CARY's Barometer No. 2, during the Monsoon of 1827, reduced to 32° FAHR., with correction for the expansion of the brass scale. Height above the sea 1823 feet. Latitude 18° 30′ 40″ N. Longitude 74° 05′ 53″ E.

1827.	Maximum.		Mean.						Minimum.		Monthly mean height.		Monthly range.		Mean diurnal oscillation.			
	9–10 A.M.		Sunrise.		9–10 A.M.		4–5 P.M.		4–5 P.M.						Rise from sunrise to 9–10 A.M.		Fall from 9–10 A.M. to 4–5 P.M.	
	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
June.	27·9155	82·70	27·7390	76·11	27·7724	79·05	27·6822	82·46	27·6240	76·3	27·7272	80·76	·291	− 6·4	·0334	+ 2·94	·0902	+ 3·41
July.	27·9092	78	27·7650	73·94	27·7981	76·66	27·7492	78·30	27·6267	78·30	27·7731	77·48	·2825	+ ·3	·0331	+ 2·72	·0489	+ 1·64
August.	27·9827	76·50	27·8274	72·88	27·8597	75·90	27·7997	77·21	27·7180	75·20	27·8292	76·55	·2647	− 1·3	·0323	+ 3·02	·0600	+ 1·31
Sept.	27·9918	78	27·8826	73·49	27·9153	76·72	27·8340	79·04	27·6877	75	27·8746	77·88	·304	− 3	·0327	+ 3·65	·0813	+ 2·82
October.	28·1343	76·40	28·0041	73·08	28·0453	77·71	27·9306	80·23	27·8724	84	27·9874	78·97	·262	− 7·60	·0412	+ 4·63	·1147	+ 2·52
For the Monsoon.	28·1343	76·40	27·8436	73·90	27·8781	77·21	27·7991	79·45	27·6240	76·3	27·8383	78·33	·5103	− ·1	·0345	+ 3·31	·0790	+ 2·24

Under the head "Monthly range" and "Therm. attached," in the bottom line "For the Monsoon," is seen ·5103 for the Barom. and ·1 for the Therm. It means that between the two periods, when the Barom. ranged ·5103, the Therm. at the two periods differed only ·1, one tenth of a degree: and this observation is applicable to all the barometrical tables.

TABLE V.

Barometrical Observations made at Hay Cottage, Poona, with CARY's Barometer No. 2, during the Monsoon, and for the month of November, 1828, reduced to 32° FAHR., with correction for the expansion of the brass scale. Height above the sea 1823 feet. Latitude 18° 30′ 40″ N. Longitude 74° 05′ 53″ E.

1828.	Maximum.		Mean.						Minimum.		Monthly mean height.		Monthly range.		Mean diurnal oscillation.			
	9–10 A.M.		Sunrise.		9–10 A.M.		4–5 P.M.		4–5 P.M.						Rise from sunrise to 9–10 A.M.		Fall from 9–10 A.M. to 4–5 P.M.	
	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
June.	27·8328	79·90	27·7253	77·88	27·7911	82·72	27·6904	85·20	27·5999	79·50	27·7407	83·96	·2329	·40	+·0658	+4·84	−·1007	+2·5
July.	27·7781	81·50	27·6891	75·35	27·7024	78·69	27·6553	79·25	27·5630	77·50	27·6788	78·97	·2151	4	+·0133	+3·34	−·0471	+ ·56
August.	27·8475	78·80	27·7482	75·49	27·7644	78·16	27·6938	79·37	27·5881	77	27·7291	78·76	·2594	·10	+·0162	+2·67	−·0706	+1·21
Sept.	27·8942	76	27·7533	73·80	27·7952	76·95	27·7043	79·18	27·5711	80·70	27·7497	78·06	·3231	4·70	+·0419	+3·15	−·0910	+2·23
October.	28·0700	75	27·8420	73·79	27·8950	76·91	27·7844	79·37	27·5044	80·70	27·8397	78·14	·5656	5·70	+·0530	+3·12	−·1106	+2·46
Nov.	28·0823	76	27·9502	71·15	27·9950	76·34	27·8733	79·65	27·7587	81·30	27·9341	77·99	·3236	5·30	+·0448	+5·19	−·1277	+3·31
For the Monsoon	28·0700	75	27·7516	75·26	27·7896	78·68	27·7056	80·47	27·5044	80·70	27·7476	79·58	·5656	5·70	+·0380	+3·42	−·0840	+1·70

The great monthly range in October is to be attributed to the unusual number and the late period of the occurrence of the thunder-storms at the breaking up of the monsoon.

TABLE VI.

Barometrical Observations made at Hay Cottage, Poona, with CARY'S Barometer No. 2, during the Monsoon of 1829, and during November and December, reduced to 32° FAHR., with correction for the expansion of the brass scale.

1829.	Maximum. 9—10 A.M. Barom.	Maximum. 9—10 A.M. Therm. attached.	Mean. Sunrise. Barom.	Mean. Sunrise. Therm. attached.	Mean. 9—10 A.M. Barom.	Mean. 9—10 A.M. Therm. attached.	Mean. 4—5 P.M. Barom.	Mean. 4—5 P.M. Therm. attached.	Minimum. 4—5 P.M. Barom.	Minimum. 4—5 P.M. Therm. attached.	Monthly mean height. Barom.	Monthly mean height. Therm. attached.	Monthly range. Barom.	Monthly range. Therm. attached.	Mean diurnal oscillation. Rise from sunrise to 9—10 A.M. Barom.	Mean diurnal oscillation. Rise from sunrise to 9—10 A.M. Therm. attached.	Mean diurnal oscillation. Fall from 9—10 A.M. to 4—5 P.M. Barom.	Mean diurnal oscillation. Fall from 9—10 A.M. to 4—5 P.M. Therm. attached.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
June.	28·0026	81·30	27·7729	75·42	27·7963	78·16	27·7229	79·71	27·6219	77·80	27·7596	78·93	·3806	·60	+·0234	+2·74	—·0734	+1·55
July.	27·9754	76·60	27·7768	73·62	27·8131	76·13	27·7477	76·88	27·6000	82·30	27·7804	76·50	·3754	5·70	+·0363	+2·51	—·0654	+ ·75
August.	27·9777	75·30	27·8426	72·40	27·8677	75·47	27·7811	76·27	27·6692	76	27·8244	75·87	·3085	·70	+·0251	+3·07	—·0866	+ ·80
September.	28·0146	76·50	27·9100	72·47	27·9430	77·22	27·8658	78·65	27·7883	79	27·9044	77·93	·2263	2·50	+·0330	+4·75	—·0772	+1·43
October.	28·0867	75·50	27·9441	72·79	27·9791	78·35	27·8675	82·36	27·7380	84	27·9233	80·35	·3487	2	+·0350	+5·56	—·1116	+4·01
Nov. 6 days.	28·0904	76·40	28·0166	69·30	28·0530	75·83	27·9463	80·70	27·9155	79·50	27·9896	78·28	·1749	3·10	+·0364	+6·53	—·1067	+4·70
Dec. 12 days.	28·1940	70·50	28·0956	63·19	28·1355	71·92	28·0017	77·66	27·9732	75·50	28·0686	74·99	·2208	5	+·0399	+8·72	—·1338	+5·74
For the Monsoon.	28·0867	75·50	27·8492	73·34	27·8798	77·06	27·7970	78·77	27·6000	82·30	27·8384	77·91	·4867	—6·8	+·0306	+3·73	—·0828	+1·71

TABLE VII.

Barometrical Observations made at Hay Cottage, Poona, with CARY'S Barometer No. 2, during the year 1830, reduced to 32° FAHR., with correction for the expansion of the brass scale.

1830.	Maximum. 9—10 A.M. Barom.	Maximum. 9—10 A.M. Therm. att.	Mean. 9—10 A.M. Barom.	Mean. 9—10 A.M. Therm. att.	Mean. 4—5 P.M. Barom.	Mean. 4—5 P.M. Therm. att.	Mean. 10—11 P.M. Barom.	Mean. 10—11 P.M. Therm. att.	Minimum. 4—5 P.M. Barom.	Minimum. 4—5 P.M. Therm. att.	Monthly mean height. Barom.	Monthly mean height. Therm. att.	Monthly range. Barom.	Monthly range. Therm. att.	Mean diurnal and nocturnal oscillation. Fall from 9—10 A.M. to 4—5 P.M. Barom.	Mean diurnal and nocturnal oscillation. Fall from 9—10 A.M. to 4—5 P.M. Therm. att.	Mean diurnal and nocturnal oscillation. Rise from 4—5 P.M. to 10—11 P.M. Barom.	Mean diurnal and nocturnal oscillation. Rise from 4—5 P.M. to 10—11 P.M. Therm. att.	Maximum diurnal oscillation. Barom.	Maximum diurnal oscillation. Therm. att.	Minimum diurnal oscillation. Barom.	Minimum diurnal oscillation. Therm. att.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
January.	28·242	73·6	28·155	72·5	28·019	78·3	......	......	27·945	77·8	28·0870	75·4	·297	4·2	—·136	+5·8	......	......	·1643	+ 5·5	·1211	+8·2
February.	28·126	74	28·072	75·6	27·932	82·1	28·020	74	27·860	83·7	28·0020	78·85	·266	9·7	—·140	+6·5	+·088	— 8·1	·1781	+ 7·2	·0692	+3·7
March.	28·105	75·3	28·019	81	27·886	90·8	27·983	78·1	27·824	95·3	27·9525	85·9	·281	20	—·133	+9·8	+·097	—12·7	·1663	+10	·0493	+9·9
April.	28·028	82·5	27·979	84·1	27·836	91·8	27·939	80·7	27·764	86	27·9075	87·95	·264	3·5	—·143	+7·7	+·108	—11·1	·1950	+ 7·6	·0887	+8·7
May.	28·044	83·5	27·912	83·8	27·780	89·6	27·894	80·6	27·682	92·2	27·8460	86·7	·362	8·7	—·132	+5·8	+·114	— 9	·1799	+ 6	·0622	— ·2
June.	27·936	85·2	27·821	81·8	27·715	85·5	27·820	78·1	27·602	89	27·7680	83·65	·334	3·9	—·106	+3·7	+·105	— 7·4	·1583	+ 5·3	·0544	—1·5
July.	27·896	74	27·804	76·4	27·729	77·5	27·823	74·5	27·570	75	27·7666	76·95	·326	1	—·075	+1·1	+·094	— 3	·1117	+ 3·3	·0327	— ·5
August.	27·945	77	27·883	75·7	27·798	78	27·870	73·5	27·728	77·5	27·8405	76·85	·217	·5	—·085	+2·3	+·082	— 4·5	·1224	+ 2·1	·0463	—1
September.	28·080	76·3	27·970	77·3	27·880	79·4	27·954	74·7	27·756	81·5	27·9250	78·35	·324	5·2	—·090	+2·1	+·074	— 4·7	·1480	+ 5·5	·0519	—2·2
October.	28·028	82	27·986	79·2	27·861	82·1	27·945	78	27·786	83·4	27·9235	80·65	·242	1·4	—·125	+2·9	+·084	— 4·1	·1478	+ 3	·0968	—4
November.	28·182	75·3	28·081	73·9	27·956	80·4	28·038	72·2	27·811	85·5	28·0185	77·15	·371	10·2	—·125	+6·5	+·082	— 8·2	·1561	+ 6	·0624	—6·7
December.	28·184	71·3	28·123	72·6	28·013	77·5	28·058	71·2	27·957	78·6	28·0680	75·05	·227	7·6	—·110	+4·9	+·045	— 6·3	·1372	+ 5·5	·0740	—4·5
For the Year.	28·242	73·6	27·9837	77·82	27·8676	82·75	27·9404	75·96	27·570	75	27·9254	80·28	·672	1·4	—·1166	+4·9	+·0884	— 7·2	·1950	+ 7·6	·0327	— ·5
For the Monsoon.	28·080	76·3	27·8928	78·08	27·7966	80·5	27·8824	75·76	27·570	75	27·8447	79·29	·510	1·3	—·0962	+2·42	+·0878	— 4·74	·1583	+ 5·3	·0327	— ·5

On the 9th of June, after a heavy thunder-storm from the N.E., at 8½ P.M., the night-tide did not turn until 12 o'clock. On the 12th of October, at sunset, there were several storms round the horizon, and the night-tide turned at 9 P.M. instead of continuing to rise as usual until 10 or 11 P.M.

TABLE VIII.

Barometrical Observations made in Bombay, at the level of the sea, in certain months during the years 1827, 1828, 1829. Latitude 18° 58′ N. Longitude 72° 38′ E.

	Maximum.		Mean.						Minimum.		Monthly mean height.		Mean diurnal oscillation.				Monthly range.	
	9–10 A.M.		Sunrise.		9–10 A.M.		4–5 P.M.		4–5 P.M.				Rise from sunrise to 9–10 A.M.		Fall from 9–10 A.M. to 4–5 P.M.			
	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.	Barom.	Therm. attached.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
April 1827, 9 days.	29·9183	90	29·7979	78·65	29·8625	88·75	29·7789	88·72	29·7617	87·10	29·8207	88·73	+·0646	+10·10	−·0836	−·03	·1563	12·90
Dec. 1828, 19 days.	29·9487	76·30	29·7800	67·47	29·8330	75·25	29·7189	83·75	29·6676	79·40	29·7759	79·51	+·0530	+ 7·80	−·1141	+3·48	·2811	3·10
Jan. 1829, 5 days.	29·8558	7·2	29·7885	64·4	29·8249	73·4	29·7151	76·8	29·6835	76·8	29·7700	75·1	+·0364	+ 9	−·1098	+3·4	·1723	5·6

TABLE IX.

Barometrical Observations made at Mahabuleshwur, the source of the Kristna river; the whole reduced to 32° FAHR., with correction for the brass scale; from the Registers kept by J. O. WALKER, Esq., Surgeon in charge of the Convalescent Hospital. Latitude 17° 58′ 33″ N. Longitude 73° 29′ 53″ E. Elevation above the sea 4500 feet.

1828. 1829.	Maximum.		Mean.								Minimum.		Monthly mean height.		Monthly range.		Mean diurnal and nocturnal oscillation.							
	9¼ A.M.		Sunrise.		9¼ A.M.		4 P.M.		10 P.M.		4 P.M.						Rise from sunrise to 9¼ A.M.		Fall from 9¼ A.M. to 4 P.M.		Rise from 4 P.M. to 10 P.M.		Fall from 10 P.M. to sunrise.	
	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.	Barom.	Therm. att.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
Nov. 1828.	25·6170	64·7	25·5051	61·69	25·5408	63·15	25·4607	66·37	25·5239	63·08	25·3347	63	25·5007	64·76	·2823	−1·7	+·0357	+1·46	−·0801	+3·22	+·0632	− 3·21	−·0188	−1·39
Dec. 1828.	25·6156	62·2	25·4881	58·64	25·5349	61·79	25·4611	65·34	25·5054	60·70	25·3770	65	25·4980	63·56	·2386	+2·8	+·0468	+3·15	−·0738	+3·55	+·0443	− 4·64	−·0173	−2·06
Jan. 1829.	25·5889	64·8	25·4524	59·01	25·5022	63	25·4287	67·76	25·4578	61·02	25·3667	65·2	25·4654	65·38	·2222	+0·4	+·0498	+3·99	−·0735	+4·76	+·0291	− 6·74	−·0054	−2·01
February.	25·5564	69	25·4378	56·15	25·4856	65·05	25·4190	71·55	25·4552	58·63	25·3471	71·2	25·4523	68·30	·2093	+2·2	+·0478	+8·90	−·0666	+6·5	+·0362	−12·92	−·0174	−2·48
March.	25·6707	75·2	25·5720	66·71	25·6176	73·57	25·5349	75·80	25·5883	69·01	25·4647	76	25·5762	74·68	·2060	+0·8	+·0456	+6·86	−·0827	+2·23	+·0534	− 6·79	−·0163	−2·30
April.	25·6236	77·5	25·5172	69·16	25·5799	74·42	25·4964	77	25·5407	71·04	25·4349	75·5	25·5381	75·71	·1887	−2·0	+·0627	+5·26	−·0835	+2·58	+·0443	− 5·96	−·0235	−1·84
May.	25·6076	77·8	25·4512	70·57	25·5048	74·13	25·4291	75·46	25·4730	71·44	25·3387	75·8	25·4669	74·79	·2689	−2·0	+·0536	+3·56	−·0757	+1·33	+·0445	− 4·02	−·0224	− ·87
June.	25·5087	68·5	25·3644	67·53	25·4036	67·84	25·3508	68·33	25·3873	68	25·2515	66·8	25·3772	68·08	·2572	−1·7	+·0392	+ ·31	−·0528	+ ·49	+·0365	− ·33	−·0229	− ·47
July.	25·4136	64·7	....	......	25·3471	65·01	25·2915	65·86	....	......	25·2277	68·1	25·3193	65·43	·1859	+3·4	....	......	−·0556	+ ·85	....	....	....	......
August.	25·4303	64·8	....	......	25·4126	64·28	25·3623	64·92	....	......	25·3470	65·1	25·3874	64·60	·0223	+0·3	....	......	−·0503	+ ·64	....	....	....	......
																	+0·476	+4·18	−·0694	+2·61	+·0439	− 5·58	−·0180	−1·68

Table X.

Barometrical Observations made at the Hill Fort of Hurreechunderghur in the months of March, April, and May 1829, reduced to 32° Fahr., with correction for the expansion of the brass scale. Height above the sea 3943 feet. Latitude 19° 23′ 23″ N. Longitude 73° 40′ 19″ E.

1829.	Maximum.		Mean.						Minimum.		Monthly mean height.		Monthly range.		Mean diurnal oscillation.				Maximum diurnal oscillation.		Minimum diurnal oscillation.	
	9¾ A.M.		Sunrise.		9¾ A.M.		4 P.M.		4 P.M.						Rise from sunrise to 9¾ A.M.		Fall from 9¾ to 4 P.M.					
	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°
March, 22 days.	26·029	81·30	25·9256	70·43	25·9693	80	25·8669	84·25	25·7781	85·70	25·9181	82·12	·2509	+4·40	+·0437	+ 9·57	−·1024	+4·25	·1641	+10·70	·0343	+1·1
April, 29 days.	25·9855	86·80	25·8807	69·69	25·9321	80·6	25·8340	84·28	25·7766	82	25·8831	82·17	·2089	−4·80	+·0514	+10·91	−·0981	+3·60	·1371	+ 6·0	·0607	+9·5
May, 19 days.	25·9590	85·40	25·8311	68·70	25·8825	79·24	25·7922	81·66	25·7361	78·8	25·8373	80·45	·2229	−6·6	+·0514	+10·54	−·0903	+2·42	·1192	− 4·8	·0523	+3·7

Table XI.

Barometrical Means for the Monsoon months during the years 1827, 1828, 1829, and 1830, at Hay Cottage, Poona; the whole reduced to 32° Fahr., with correction for the brass scale.

	Maximum.		Mean.								Minimum.		Monthly mean height.		Monthly range.		Mean diurnal and nocturnal oscillation.						Maximum diurnal oscillation.		Minimum diurnal oscillation.	
	9–10 A.M.		Sunrise.		9–10 A.M.		4–5 P.M.		10–11 P.M.		4–5 P.M.						Rise from sunrise to 9–10 A.M.		Fall from 9–10 A.M. to 4–5 P.M.		Rise from 4–5 P.M. to 10–11 P.M.					
	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.	Barom.	Ther. att.
	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in.	°	in	°	in.	°	in.	°	in.	°	in.	°	in.	°
1827.	28·1343	76·4	27·8436	73·90	27·8781	77·21	27·7991	79·45	....	......	27·6240	76·3	27·8383	78·33	·5103	−·1	·0345	3·31	·0790	2·24	....	......	......	......	......	......
1828.	28·0700	75	27·7516	75·26	27·7896	78·68	27·7056	80·47	....	......	27·5044	80·7	27·7476	79·58	·5656	+5·7	·0380	3·42	·0840	1·70	....	......	......	......	......	......
1829.	28·0867	75·5	27·8492	73·34	27·8798	77·06	27·7970	78·77	....	......	27·6000	82·3	27·8384	77·91	·4867	−6·8	·0306	3·73	·0828	1·71	....	......	......	......	......	......
1830.	28·0080	76·3	....	.....	27·8928	78·08	27·7966	80·5	27·8824	75·76	27·5700	75	27·8447	79·29	·5100	+1·3	....	......	·0962	2·42	·0878	4·74	·1583	+5·3	·0327	−·5
Means.	28·0972	75·8	27·8184	74·17	27·8601	77·76	27·7746	79·80	27·8824	75·76	27·5771	78·57	27·8172	78·78	·5181	+ ·1	·0343	3·48	·0855	2·02	·0878	4·74	·1583	+5·3	·0327	−·5

TABLE XII.

Mean temperature by two of DOLLOND's Thermometers for the year 1825 in Dukhun, between the parallels of latitude 18° 2 and 19° 10′ 31″ North, and longitude 73° 35′ and 74° 49′ East, at a mean elevation above the sea of 1700 feet.

1825.	Sunrise.				9¼ A M.				4 P M.				Place of observation.
	Number of observations.	Therm. No. 1.	Number of observations	Therm. No 2	Number of observations.	Therm. No. 1.	Number of observations.	Therm. No. 2	Number of observations.	Therm No 1	Number of observations.	Therm. No. 2.	
January.	12	63·32	......	....	29	70·01	29	70·75	31	80·61	31	79·24	Poona.
February.	24	65·13	......	....	27	73·83	26	74·78	28	86·60	27	82·20	Poona.
March.	25	65·77	8	68·88	29	79·03	28	79·02	29	88·88	25	87·13	En route; Poona.
April.	29	72·64	29	72·60	29	84·38	29	84·02	30	89·68	30	89·17	En route.
May.	30	76·25	30	76·73	30	85·79	30	86·30	31	91·45	31	91·41	En route.
June.	30	79·24	30	81·17	30	85·32	30	86·65	30	90·73	30	91·49	Serroor.
July.	31	77·04	31	78·67	31	81·08	31	81·89	31	83·70	31	84·00	Serroor.
August.	30	76·98	30	78·43	30	80·55	30	81·63	30	84·46	30	84·50	Serroor.
September.	30	76·13	30	77·26	29	79·43	29	80·44	29	82·35	30	82·69	Serroor.
October.	27	77·22	27	77·40	31	80·37	31	81·37	31	85·07	31	85·77	Serroor.
November.	30	74·72	30	74·69	30	76·81	30	77·55	30	81·61	30	81·30	Serroor.
December.	31	53·81	31	53·47	31	67·11	31	67·19	31	79·98	31	80·04	En route.
Mean for the year.		71·52				78·64				85·43			

In February, to ascertain the effect of difference of position, Thermometer No. 1. was placed in a room having a western aspect, Thermometer No. 2. being in a room facing the north: the difference of mean temperature for the month was 4° 40′ at 4 P.M.

TABLE XIII.

Indications of two DOLLOND's Thermometers in Dukhun for the year 1826, between the parallels of latitude 18° and 19° 08′ North, and longitude 73° 25′ and 74° 50′ East, at a mean elevation of 1800 feet above the sea.

1826.	Maximum.						Mean.						Minimum.						Number of observations with each Thermometer			Mean for the Month.		Monthly range.		Maximum diurnal range.	Minimum diurnal range.	Place of observation.
	Sunrise.		9¼ A.M.		4 P.M.		Sunrise		9¼ A M.		4 P.M.		Sunrise.		9¼ A M.		4 P M											
	Ther No. 1.	Ther No. 2.	Ther No. 1.	Ther No 2.	Ther No 1.	Ther No. 2.	Ther. No. 1.	Ther No 2	Ther. No 1	Ther. No. 2.	Ther No 1	Ther. No. 2.	Ther No 1	Ther No. 2.	Ther. No 1	Ther No. 2.	Ther No. 1.	Ther. No. 2.	Sunrise.	9¼ A.M.	4 P.M.	Ther. No. 1.	Ther. No. 2.	Ther No. 1.	Ther. No. 2.			
Jan.	69·50	69	77	78·50	82·60	83·10	53·45	53·12	68·36	68·32	78·36	78·18	40·50	41·20	57·60	56	71·80	71·60	30	29	28	65·90	65·65	42·10	40·90	35·70	10·70	En route.
Feb.	65	64·50	79·10	78	92·20	92	57·81	57.69	69·48	69·64	85·64	85·70	50·60	50·50	60·30	60	74·20	75	24	28	27	71·77	71·69	41·60	41·50	37·10	17·20	En route.
Mar.	78	77·80	85·80	87	93·30	94.40	68·82	68·76	79·46	79·61	86·41	86·34	50·50	50·50	64·50	63·30	81	81	30	28	29	77·61	77·55	42·80	43·90	37·30	6·00	En route.
April.	80·80	80·50	89·90	90·80	93·90	94·20	77·65	77·45	84·35	84·75	85·39	85·23	72·20	72	80·60	81	82	82	30	29	30	81·52	81·34	21·70	22·20	16·10	2·90	Bombay.
May.	82·10	82·30	88	88	91·80	92	80·23	80·19	84·94	85·30	86·33	86·38	77·30	77·30	79·50	79	81	81	23	24	28	83·28	83·28	14·50	14·70	10·10	1·30	Bombay. En route.
June.	80·50	81·20	82	82·10	86·40	86·40	76·54	76·77	78·70	78·88	80·87	80·93	73·40	73·20	75·50	75·90	73·70	74·10	30	29	30	78·70	78·85	13·0	13·20	7·20	1	Poona.
July.	79·00	79·40	80·50	80·50	84·50	84·30	75·09	75·25	76·71	76·83	77·68	77·85	73	72·90	74·30	73·40	74·40	74·40	31	29	31	76·38	76·55	11·50	11·40	6·80	·80	Poona.
Aug.	77·40	77·50	78·30	78·80	83·90	83	75·07	74·93	76·83	77·06	78·93	78·99	73·20	73	74·90	75·00	75·60	75·80	31	31	31	77	76·96	9·70	10	7·30	·60	Poona.
Sept.	76·10	76	77·50	77·90	83	83·50	74·45	74·38	76·29	76·48	78·68	78·74	72	72	73·50	73·60	74·80	75	30	30	30	76·56	76·56	11	11·50	6·90	1	Poona.
Oct.	82·60	80·80	84·30	84·10	88·30	88	77·18	75·18	79·27	79·60	83·51	83·64	71·30	71	75·10	76·10	79·30	79·30	31	31	31	80·34	78·91	17	17	11·90	2·10	Poona.
Nov.	75·00	74	81	81	85·10	85·30	70·80	70·45	75·63	75·82	80·86	80·97	61·50	62·20	71·20	71·40	74	73·90	25	30	29	75·83	75·71	23·60	23·10	21·80	3·40	En route.
Dec.	74·30	74·20	78·50	78·60	84	84	67·92	67·83	73·80	73·80	79·33	79·37	56·50	56·50	68·10	68·70	76	76	25	29	31	73·62	73·60	27·50	27·50	23·60	4·40	En route. Poona.
Year.	82·60	82·30	89·90	90·80	93·90	94·40	71·25	71	76·81	77	81·83	81·77	40·50	41·20	57·60	56	71·80	71·60	340	347	355	76·54	76·38	53·40	53·20	37·30	·60	

Table XIV.

Indications of two Dollond's Thermometers in Dukhun for the year 1827, between the parallels of latitude 17° 25′ and 19° 27′ North, and longitude 73° 25′ and 75° 53′ East, at a mean elevation of 1700 feet above the sea.

1827.	Maximum.						Mean.						Minimum.						Number of observations of each Thermometer.			Mean for the Month.		Monthly range.		Maximum diurnal range.	Minimum diurnal range.	Place of observation.
	Sunrise.		9¾ A.M.		4 P.M.		Sunrise.		9¾ A.M.		4 P.M.		Sunrise.		9¾ A.M.		4 P.M.											
	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Sunrise.	9¾ A.M.	4 P.M.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.			
Jan.	73·10	73	75	75	80·50	80·80	67·58	66·96	70·10	70·09	75·93	76·32	63	62·50	65·20	64·50	68·60	68·30	31	31	31	71·75	71·64	17·50	18·30	12·50	3·50	Poona.
Feb.	70·80	69	81	79·50	89	88·80	64·11	63·44	74·99	74·65	83·62	83·90	55·20	55·80	70·50	70·50	79·50	80	21	28	28	73·86	73·67	33·80	33	28·30	11·60	En route.
Mar.	78	77·80	85·50	85·70	96·60	96·80	66·80	66·77	79·28	79·11	88·07	87·98	57	57·50	73	73	81·20	81·50	18	31	31	77·43	77·37	39·60	39·30	29·40	9·20	En route.
April.	87	85	86·80	86·50	95	89·30	80·32	78·59	83	82·37	91·30	85·80	73·70	71·60	79	77	84·50	84	13	14	14	85·81	82·19	21·30	17·50	12·80	5·10	Poona. Bombay.
May.	84	81	87	87	93·30	93	79·23	78·01	83·22	81·95	89·54	86·35	74	74	81	80	83	81	26	27	27	84·38	82·18	19·30	19	17·70	6·30	Poona. En route.
June.	81·90	81	85	82·80	94	90·10	77·69	77·32	79·60	79·20	82·61	81·17	72·80	73	75	75	74·20	73·80	30	30	30	80·15	79·24	21·20	17·10	12	1·20	Poona.
July.	76·50	76·80	78·40	79	81·50	81·50	75·35	75·51	77·10	77·50	78·65	78·71	73·80	73·50	75	75·50	75·40	75·50	31	31	31	77	77·11	7·70	8	6	·80	Poona.
Aug.	77·20	76·90	79	79	82	82	74·55	74·32	76·40	76·69	77·74	77·61	73·30	73	74·70	74	74·80	74·50	31	31	31	76·14	75·96	8·70	9	8·20	·40	Poona.
Sept.	77·50	77	79·20	79	84	85·50	75·44	74·85	77·21	77·29	79·35	79·23	74	73	75	74	74·70	74	30	30	30	77·39	77·04	10	12·50	8·40	·60	Poona.
Oct.	79	78·80	82	83	84	85	75·62	74·74	78·03	78·57	80·63	81·63	71	69	74·70	75	77·60	77	30	31	31	78·12	78·18	13	16	9	2	Poona.
Nov.	75	75	87·20	87·20	89·70	90	66·05	65·89	76·65	76·82	85·92	86·06	56·30	56	64·50	65	75·50	75	23	30	28	75·98	75·97	33·40	34	28 30	3·29	En route.
Dec.	71·70	71·50	82	81·80	91·80	92	59·50	58·92	73·10	73·36	86·21	86·28	49·50	48	65	66	82	82	21	30	30	72·85	72·60	42·30	44	39·50	10·30	En route.
Year.	87	85	87·20	87·20	96·60	96·80	71·85	71·27	77·39	77·30	83·30	82·58	49·50	48	64·50	64·50	68·60	68·30	305	344	342	77·57	76·93	47·10	48·80	39·50	·40	

During part of April, May, and June, Thermometer No. 2. was placed in the heat of the day in a large room, having the windows and doors closed. A diminution of the mean temperature at 4 P.M. of 5° 50′ in April, and 3° 19′ in May was the consequence.

TABLE XV.

Indications of two DOLLOND's Thermometers for the year 1828, in Dukhun, between the parallels of Latitude 17° 40′ and 19° 11′ N., and Longitude 73° 25′ and 75° 53′ E., at a mean elevation of 1800 feet above the sea.

1828.	Maximum.						Mean.						Minimum.						Number of observations with each Thermometer.			Monthly mean.		Monthly range.		Maximum diurnal range.	Minimum diurnal range.	Place of observation.
	Sunrise.		9¾ A.M.		4 P.M.		Sunrise.		9¾ A.M		4 P.M.		Sunrise.		9¾ A.M.		4 P.M.											
	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.	Sunrise.	9¾ A.M.	4 P.M.	Ther. No. 1.	Ther. No. 2.	Ther. No. 1.	Ther. No. 2.			
Jan.	71°	70°	80°	80°	92·30°	92·50°	64·10°	63·64°	74·82°	75°	87·85°	87·96°	58°	57·80°	69°	69°	83·30°	83°	25	31	31	75·97°	75·80°	34·30°	34·70°	29·80°	14°	En route.
Feb.	75·80	76	82	82	94·10	94	63·93	63·90	75·38	75·60	88·83	88·90	56	56·50	69·20	68·80	83·50	83·50	17	28	27	76·38	76·40	38·10	37·50	34·8	8·30	En route.
Mar. {	76	76	83·10	83	88	88·50	73·19	72·69	81·39	81·33	86·65	86·67	72·50	71·50	77	77	85	85	19	21	19	79·92	79·68	15·50	17	16·70	12·30	Bombay.
	79·10	80	86	86	97	97·50	76·55	76·76	83·06	83·13	91·95	91·72	72·50	72·50	77·30	76·50	87	87	9	9	7	84·25	84·24	24·50	25	19·50	12·70	En route.
April.	80·40	80	91	91	99·30	99	74·07	74·06	82·68	82·69	92·54	92·51	70·80	71	77·50	77	87	87	14	27	27	83·30	83·28	28·50	28	24·50	9·60	En route.
May.	84·80	83	92	92·50	101	100·80	74·40	73·97	82·33	82·46	93·04	93·08	62·90	62·90	75	75	83·50	83	26	31	30	83·72	83·52	38·10	37·90	28·90	6·20	En route.
June.	86·50	84·80	89·10	91	97·60	99	79·84	79·43	83·41	83·16	86·63	86·58	76·80	76·80	78·30	78	79·60	79·50	29	30	29	83·23	83·03	20·80	22·20	13·60	2·30	Poona.
July.	78·10	78·10	82·70	82	85·70	85	76·11	76·12	78·16	78·02	79·49	79·48	73·30	73	74·60	74·80	75·20	75·20	29	28	31	77·80	77·80	12·40	12	8·40	·70	Poona.
Aug.	79	77·80	80·30	79·50	83·10	82	76·49	75·84	78·46	77·97	79·53	79·10	74·10	74·20	76	75·50	77·20	77	17	30	30	78·01	77·47	9	7·80	5·40	1·40	Poona.
Sept.	77·40	77·70	79·30	78·80	83·40	83	75·07	74·96	77·05	77·02	78·80	78·62	72·70	72	75·50	75·50	75·20	74·60	29	30	30	76·93	76·79	10·70	11	9·80	·50	Poona.
Oct.	79·30	78·80	81	81	84	84	75·40	75·20	77·43	77·24	80·03	80·22	68·60	69	73·20	73	75	75	31	30	31	77·71	77·70	15·40	15	10·70	·40	Poona.
Nov.	77	75·50	80	83	83	86·50	73·35	65·93	76·26	78·02	79·54	82·25	66·20	56	72·20	71·50	75·40	74·80	29	30	30	76·44	74·09	16·80	30·50	10·10	1·60	Poona.
Dec. {	67	61·60	72·40	72·70	77·30	75·60	60·57	59·20	68·61	68·12	76·05	75·05	56	57	67·30	67·30	75	75·10	4	6	6	68·31	67·12	21·30	17·40	19	9·70	En route.
	71	71·50	77	77	86·30	86·50	69	69·14	74·89	75·25	81·40	81·65	63·50	63·50	71·80	72·30	76·70	77	19	21	21	75·20	75·39	22·80	23	15·60	8·70	Bombay.
Year.	86·50	84·80	92	92·50	101	100·80	72.21	71·25	77·99	78·05	84·08	84·20	56	56	67·30	67·30	75·20	74·60	297	352	349	78·14	77·72	45	44·80	34·8	·40	

* On the 7th of May, at 3½ P.M., the thermometer rose to 105° at an elevation of 3123 feet above the level of the sea.

The discrepancies between Thermometer No. 1. and No. 2. originate in Thermometer No. 2. having been placed outside the house, under a grass roof, at times.

The observations in Bombay have not been included in the means.

TABLE XVI.

Showing the maximum, minimum, and mean temperature, the maximum, minimum, and mean diurnal range, and the extreme monthly and annual range of the thermometer for the year 1829 in Dukhun, between the parallels of latitude 18° 10′ and 19° 23′ North, and longitude 73° 20′ and 74° 30′ East.

1829. Months.	Place of observation.	Height in feet above the sea.	Extreme temperatures. Maximum.	Extreme temperatures. Minimum.	Monthly mean temperature.	Maximum diurnal range.	Minimum diurnal range.	Mean diurnal range.	Extreme monthly range.
		feet.	°	°	°	°	°	°	°
January.	En route	....	82	45·10	67·70	30·90	9	20·80	36·90
February.	En route	....	88·90	47·30	70·60	32·40	10·40	21·40	41·60
March.	Tents in Hill Fort of Hurreechunderghur	3943	88·80	68·80	78·31	17·90	6	14·01	20
April.	Tents in Hill Fort of Hurreechunderghur	....	94·20	60	78·07	20·80	8·70	14·72	34·20
May.	Tents in Hill Fort of Hurreechunderghur	....	87·30	64·80	76·11	19·50	8	13·35	22·50
June.	Hay Cottage, Poona	1823	86	71·50	76·80	8·50	4·50	6·33	14·50
July.	Hay Cottage, Poona	....	81·50	70·50	75·50	7	4	5·62	11
August.	Hay Cottage, Poona	....	79	70·50	73·80	8	·60	4·31	8·50
September.	Hay Cottage, Poona	....	85	69	75·43	12·50	2·30	6·48	16
October.	Hay Cottage, Poona	....	89·50	66·50	78·40	17	6·50	12·74	23
November.	Hay Cottage, Poona	....	83·50	68	75·67	15	10	12·50	15·50
	Sasswur Government House	2416	80	49·50	64·65	26·50	10	21·62	30·50
December.	Chamblee, in tents	2416	84	43	66·71	37·50	8	24·92	41
	Hay Cottage, Poona	1823	85	55	71·20	30	16	22·26	30
Year.			94·20	43	74·80	37·50	·60	12·90	51·20

The mean temperature of the year was reduced several degrees from former years, in consequence of the whole of the observations for the hot months having been made in the Hill Fort of Hurreechunderghur in the Western Ghâts, at an elevation of 3943 feet above the level of the sea. The weather also during the Monsoon was cooler than usual.

TABLE XVII.

Hygrometric Observations made with DANIELL's Hygrometer in Dukhun during the year 1826.

1826.	Maximum.									Mean.									Minimum.									Monthly mean.			Monthly range of hygrometer.
	Sunrise.			9—10 A.M.			4—5 P.M.			Sunrise.			9—10 A.M.			4—5 P.M.			Sunrise.			9—10 A.M.			4—5 P.M.						
	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	
	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°
April.	75	81		79	87		78	85·5		72·31	79·34		73·77	85·42		72·46	85·70		68	79		67	83		65	90		72·84	83·48	8·988	14
May.	78	82		79	87		79	87		74·84	80·73		75·83	86·22		76·10	86·63		70	79		73	87		73	84		75·59	84·52	9·748	9
June.	73	77		74	77		74	78		69·84	75·76		70·15	78·38		69·53	80·26		66	76		65	81		65	81		69·84	78·13	8·218	9
July.	73	74	*9·181	75	77		74	78		70·71	74·96		72·26	77		72·29	77·85		68	75		70	77		69	79		71·75	76·6	8·775	7
Aug.	72	74·5		75	76·5	‡9·752	74	77		70·42	74·71		71·90	77·46		71·16	79·08		68	74·5		69	78·5		69	83		71·16	77·08	8·602	7
Sept.	72	75		74	78·5		75	78·5		69·86	74·10		70·56	77		71·33	78·90		67	72·5		67	77		65	80·5		70·58	76·66	8·447	10
Oct.	73	77·5		73	77·5		76	84·5	‖ 9·945	67·19	76·22		64·48	81·27		59·19	85·12		56	73		53	84·5		46	86		63·62	80·87	6·703	30
Nov.	69	71·5		74	78·5		73	78·5		62·04	72·69		65·88	76·63		63·74	81·85		47	73		55	80		45	87		63·88	77·06	6·810	29
Dec.	66	68·5		68	72		67	76		56·04	67·21		57·70	75		55·50	80·57		44	56		47	73·5		46	83		56·41	74·26	5·388	24
Year.	78	82	†10·617	79	87	§10·941	79	87	¶10·941	66·58	73·66	7·473	67·56	77·53	7·634	66·10	80·52	7.258	44	56	3·673	47	73·5	3·987	45	87	3·587	66·75	77·23	7·455	32

The observations of April and May were taken in Bombay, the remaining in Dukhun, (from June to October, inclusive, at Poona,) at a mean elevation of 1800 feet above the sea.
The weight of moisture is in a cubic foot of air, in grains troy and decimals.

* At Poona. † Bombay. ‡ At Poona. § Bombay. ‖ At Poona. ¶ Bombay.

TABLE XVIII.

Hygrometric Observations made with DANIELL's Hygrometer in Dukhun during the year 1827.

1827.	Maximum.									Mean.									Minimum.									Monthly mean.			Monthly range of hygrometer.	
	Sunrise.			9—10 A.M.			4—5 P.M.			Sunrise			9—10 A M			4—5 P M.			Sunrise.			9—10 A.M.			4—5 P.M.							
	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther att.	Weight of moist.	Hyg.	Ther. att	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.		
	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.		
Jan.	64	67·5		64	68		67	75		50·64	66·64		50·48	72·12		49·83	78·85		29	62		33	70		37	80		50·31	72·53	4·385	38	
Feb.	51	70		55	80		44	84		43·09	64·19		46·10	77·39		44	84		37	66·5		40	79·5		44	84		44·39	75·19	3·588	18	
Mar.	58	65		63	77·5		62	81		47·61	67·80		50·06	81·66		……	……		32	59		41	81		62	81		48·83	74·73	4·167	31	
Apr. *	65	84·5		61	86·5		57	96·5		56·07	79·92		54·38	86·11		55·57	95·64		43	76		43	85		54	93·5		55·34	87·22	5·077	22	Poona.
	79	83		80	89		80	91		75·66	80·05		77·50	89·87		78·22	89·27		73	77		75	91·5		75	90		77·12	86·39	10·243	7	Bombay.
May.	69	78		73	83		70	82		66·75	79·22		64·66	84		62·23	91·03		61	79		51	83		52	96		64·54	84·75	6·858	22	
June.	75	80		75	84		76	79		72·20	77·26		72·83	79·88		72·43	82·83		69	72·5		65	85		64	93		72·48	79·99	8·931	12	
July.	73	75·5		74	78		73	79		71·35	74·95		72·41	77·46		71·32	78·69		70	73·5		70	76		69	78		71·69	77·03	8·751	5	
Aug.	72	75		73	78		73	80		70·45	74·08		71·19	76·74		71·45	77·83	8·692	68	73·5		67	77		68	79·5		71·03	76·21	8·574	6	
Sept.	72	76·5		74	79		75	79		70·53	74·71		71·30	77·51		71·50	79·50		68	73		68	77·5		66	83·5		71·11	77·24	8·583	9	
Oct.	62	73		64	75		62	82		61·66	71·75		60	78·57		54·38	82·83		61	70·5		52	81		45	80		58·68	77·71	5·773	19	
Nov.	66	69·5		71	73·5		72	74		52·04	64·50		58·83	76·97		55·75	84·93		42	59		47	75		45	88		55·54	75·46	5·226	30	
Dec.	62	89		69	79·5		63	92		48·80	59·46		56·63	75·63		48·50	84·68		40	53		46	70·5		37	83		51·31	72·59	4·535	32	
Year.	75	80	9·686	75	84	9·614	76	79	10·049	59·26	71·20	5·940	60·74	78·50	6·140	59·72	83·71	5·893	29	62	2·146	33	70	2·442	37	83	2·721	59·60	77·55	5·940	47	

N.B. The weight of moisture is in a cubic foot of air, in grains troy and decimals.

* Ten days' observations were made in Bombay, and put in juxta-position with the remaining observations of the month made at Poona, to show the remarkable contrast between the dewing-points at the two places.

Table XIX.

Hygrometric Observations made with Daniell's Hygrometer in Dukhun during the year 1828.

1828.	Maximum.									Mean.									Minimum.									Monthly mean.			Monthly range of the hygrometer.
	Sunrise.			9–10 A.M.			4–5 P.M.			Sunrise.			9–10 A.M.			4–5 P.M.			Sunrise.			9–10 A.M.			4–5 P.M.						
	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	
	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°
Jan.	62	66·5	6·517	69	78·5	7·988	63	88·5	6·471	48	61·77	4·163	60·1	75·65	6·054	50	89·05	4·205	32	61	2·394	49	70	4·228	34	91	2·429	52·7	75·49	4·736	37
Feb.	52	68·5	4·677	62	79	6·362	53	92	4·641	39	60·92	3·017	46·96	75·82	3·916	43·4	81·02	3·423	27	57·5	2·032	34	78	2·489	29	90	2·034	43·12	72·58	3·441	35
March. Poona.	66	73·5	7·340	63	84	6·524	64	94·5	6·618	47·85	73·92	4·082	42·93	83·43	3·346	42·12	90·48	3·200	35	73	2·600	34	84	2·461	32	88	2·273	44·3	82·59	3·527	34
March. In Bombay.	72	75	8·873	72	83	8·739	81	87	11·534	66·66	72·16	7·507	68·55	81·20	7·927	77·71	83·21	10·497	60	72·5	6·072	62	82·8	6·318	74	85·5	9·280	70·97	78·85	8·521	21
Mean of 3 months.	66	73·5	7·340	69	78·5	7·988	64	94·5	6·618	44·95	65·53	3·730	50	78·5	4·291	45·17	86·83	3·609	27	57·5	2·032	34	84	2·461	29	90	2·034	46·7	76·89	3·875	35·33

Nine days' observations were made in Bombay in March, the remaining days of the month at Poona.

The weight of moisture in a cubic foot of air is in grains troy and decimals.

The observations in Bombay are not included in the means.

Table XX.

Hygrometric Observations made with Daniell's Hygrometer in Dukhun during the year 1829.

1829.	Maximum.									Mean.									Minimum.									Monthly mean.			Monthly range.	Maximum diurnal range.	Minimum diurnal range.
	Sunrise.			9–10 A.M.			4–5 P.M.			Sunrise.			9–10 A.M.			4–5 P.M.			Sunrise.			9–10 A.M.			4–5 P.M.								
	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.			
	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	°
June.	75	78·50		75	81		77	80		70·88	74·55		72·16	77·4		72·63	78·42		69	72·5		71	77		69	76·5		71·89	76·79	8·814	8	3	1
July.	72	72		74	76·5		76	80·5		69·88	72·98		71·03	75·29		71·61	75·86		68	72		68	79		69	78·5		70·84	74·71	8·550	7	5	0
Aug.	73	73		73	73		73	73		69·1	71·88		70·48	74·63		70·86	75·38		67	71		68	76·5		69	78·5		70·14	73·96	8·398	6	4	0
Sept.	70	73		71	73·5		73	77·5		68·03	72·25		69·11	76·75		71·06	77·66		65	70·5		67	81		68	77·5		69·4	75·55	8·142	8	6	1
Oct.	73	78·5		74	81		77	85·5		67·25	72·42		66·1	79·22		64·15	82·76		58	68		50	78·5		44	84		65·83	78·13	7·077	33	26	2
Nov. *	62	69·5		54	79·5		...	...		59·25	69·58		51·3	79·1		......	......		56	68·5		46	79		...	......		......	......		...	......	...
Nov. †	51	60		60	73·5		54	74		41·47	55·07		47·21	70·03		43·5	75·43		35	52·5		37	74		38	75		44·06	66·84	3·603	25	10·5	5
Dec. ‡	65	65		69	75		...	...		49·5	55·93		55·84	72·31		......	......		34	47		40	76		...	......		......	......		...	......	...
Dec. §	53	65		54·5	72		...	...		45·25	63·56		46·56	76·65		......	......		41	62		40	77		...	......		......	......		...	......	...
Monsoon.	75	78·5	9·715	75	81	9·669	77	80	10·327	69·03	72·82	8·075	69·77	76·66	8·230	70·06	78·02	8·264	58	68	5·754	50	78·5	4·290	44	84	3·480	69·62	75·83	8·191	33	26	

* At Hay Cottage. Poona. † Government House at Sasswur, at 2416 feet above the sea. ‡ At Chamblee near Sasswur, at 2416 feet above the sea. § At Hay Cottage. Poona.

Table XXI.

Hygrometric Observations made with Daniell's Hygrometer in Dukhun during the year 1830 at Poona.

1830.	Maximum.									Mean.									Minimum.									Monthly range of hygrometer.	Maximum diurnal range.	Minimum diurnal range.
	Sunrise.			9—10 A.M.			4—5 P.M.			Sunrise.			9—10 A.M.			4—5 P.M.			Sunrise.			9—10 A.M.			4—5 P.M.					
	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.	Hyg.	Ther. att.	Weight of moist.			
	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°	°	gr.	°		
Jan.	51	65		65	72					45·7	65		47·8	73·4					38	61		38	72					27		
Feb.	......	......		66	72		......	......		......	......		48·7	77		......	......		......	......		40	80		......	......		26		
Mar.	45	71		62	78		......	......		......	......		49·9	80·8		......	......		......	......		40	84		......	......		22		
April.	69	75		73	81		75	81		65	74·3		62·7	82·3		66·5	84·7		61	63		44	85		63	84		31		
May.	......	......		74	83		76	88		......	......		66·6	85·2		64·9	89·5		......	......		60	92		54	93		22		
June.	......	......		76	76		75	76		......	......		72	82·3		71	84		......	......		64	86		61	91		15		
July.	......	......		76	76		75	75		......	......		73·2	76·4		72·8	78·4		......	......		71	80		69	77		7		
Aug.	......	......		74	74		77	77		......	......		70·7	75·4		71·6	77·1		......	......		68	78		67	79		10		
Sept.	......	......		76	77		75	75		......	......		68·5	76·4		68·5	79·5		......	......		65	74		58	80		19		
Oct.	......	......		76	77		78	81		......	......		68·4	79		66·6	80·9		......	......		53	79		54	84		24		
Nov.	40	48		62	76		......	......		......	......		52·5	74·5		......	......		......	......		45	74		......	......		22		
Dec.	55	55		......	......		......	......		43·7	52·3		......	......		......	......		31	50		......	......		......	......		24		
Year.	69	75	8·041	76	76	10·107	78	81	10·638	......	......		61·9	78·4	6·351	68·8	82	7·885	31	50	2·358	38	72	2·865	54	93	4·888	47		

The weight of moisture in a cubic foot of air is expressed in grains troy and decimals.

The extreme difficulty of obtaining the dewing-point at 4 P.M. in the dry months, frequently interrupted the observations at that hour. Observations were not regularly taken at sunrise.

Table XXII.

Weight in grains troy of the quantity of aqueous vapour contained in a cubic foot of air at the undermentioned places, 1828.

Dates.	Names of Places.	Sunrise.			9—10 A.M.			4—5 P.M.		
		Daniell's Hygrom.	Therm. attached.	Weight of vapours.	Daniell's Hygrom.	Therm. attached.	Weight of vapours.	Daniell's Hygrom.	Therm. attached.	Weight of vapours.
March 10.	Bombay, level of sea	72°	75°	8·873 gr.	71°	81·5°	8·487 gr.	80°	86°	11·205 gr.
11.	Kundallah, top of Bhore Ghat, 1744 feet	36	72	2·691	40	78	3·004	....	....	....
11.	Karleh, 2015 feet	....	....	....	....	....	....	40	87	2·954
12.	Hill fort of Loghur, 3381 feet	27	67	1·995	....	....	....	....	....	....
12.	Karleh, 2015 feet	....	....	....	34	84	2·461	36	88	2·611
13.	Poona, Hay Cottage, 1823 feet	....	....	....	44	84	3·480	35	89	2·518
14.	Poona, Hay Cottage, 1823 feet	35	73	2·601	37	84	2·715	32	88	2·273

Table XXIII.

Register of the Ombrometer from December 1825 to December 1826.

Dates. 1826.	Dec.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.
1.	....	....	....	....	....	....	....	....	·10	....	·34	....
2.	....	....	....	....	....	....	....	....	·10	·12	·02	....
3.	....	....	....	....	....	....	....	....	·03	·02	·42	....
4.	....	....	....	....	....	....	....	....	·11	·04	·12	....
5.	....	....	....	....	....	....	....	....	·06	....	·01	....
6.	....	....	....	....	....	....	·01	2·58	·07	....	....	....
7.	....	....	....	....	....	....	·14	·10	·08	·03	....	....
8.	....	....	....	....	....	....	·09	·18	·05	·14	....	....
9.	....	....	....	....	....	....	·21	·06	·03	....	....	....
10.	....	....	....	....	....	....	·01	....	....	·01	....	·20
11.	....	....	....	....	....	....	....	·22	....	·49	....	·01
12.	....	....	....	....	....	....	·02	·24	....	·03	....	....
13.	....	....	....	....	....	....	·53	·11	....	....	....	....
14.	....	....	....	....	....	....	....	·06	....	·02	....	·04
15.	....	....	....	....	....	....	·01	·07	·02	·07	....	....
16.	....	....	....	....	....	....	....	·02	....	·02	....	·10
17.	....	....	....	....	....	....	·06	·04	....	....	....	·70
18.	....	....	....	....	....	....	....	·16	....	....	....	....
19.	....	....	....	....	....	....	....	·52	....	·03	....	....
20.	....	....	....	....	....	....	....	·23	....	·03	....	·76
21.	....	....	....	....	....	....	·03	·48	....	·12	....	·33
22.	....	....	....	....	....	....	1·05	·74	·15	·11	·96	·16
23.	....	....	....	....	....	....	....	·87	·09	·14	·02	·03
24.	....	....	....	....	....	·45	·03	·07	·06	·08	....	....
25.	....	....	....	....	....	....	1·01	1·32	·08	....	....	....
26.	....	....	....	....	....	·27	·10	·28	....	....	·01	....
27.	....	....	....	....	....	·02	....	....	....	....	....	....
28.	....	....	....	....	....	·37	....	....	....	....	....	....
29	....	....	....	....	....	·17	....	·05	....	·04	....	....
30.	....	....	....	....	....	1·43	....	·03	....	....	....	....
31.	....	....	....	....	....	·70	....	....	....	....	....	....
Total	....	....	....	....	....	3·41	3·30	8·43	1·03	1·54	1·90	2·33

Total fall of rain 21·94 inches.

The Register, during the Monsoon, was kept at Poona every year.

Table XXIV.

Register of the Ombrometer from December 1826 to December 1827, in Dukhun.

Dates. 1827.	Dec.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.
1.	....	....	....	....	....	....	....	·22	·01	·06	....	....
2.	....	....	....	·03	....	....	....	·01	·04	....	·05	....
3.	....	....	....	....	....	·04	·17	·01	·23	....	·11	....
4.	....	....	....	....	....	....	·39	·02	·03	·08	....	....
5.	....	....	....	....	....	....	·60	·01	·22	·06	....	....
6.	....	....	....	....	....	....	....	·06	·02	....	....	....
7.	·39	....	....	....	....	....	·17	·03	·07	·03	·03	....
8.	....	....	....	....	....	....	·15	·01	·24	....	·48	....
9.	....	....	....	....	....	....	....	·05	·40	....	·31	....
10.	.01	·01	....	....	....	....	·74	·02	·11	·27	·09	....
11.	....	....	....	....	....	....	1·32	....	·03	·10	·07	....
12.	....	....	....	....	....	....	1·51	·03	·09	·60	·38	....
13.	....	....	....	....	....	....	1·68	....	....	·22	·37	....
14.	....	·09	....	....	....	....	·10	·03	·04	....	·28	....
15.	....	2·17	....	....	....	....	....	·13	....	·04	·02	....
16.	....	·02	....	....	....	....	·18	·16	·22	·10	·33	....
17.	....	....	....	....	....	....	·02	·07	·07	·13	....	....
18.	...	....	....	....	....	....	·15	·02	·03	....	1·81	....
19.	....	....	....	....	....	....	·31	·01	....	·17	....	....
20.	....	....	....	....	....	....	·04	·07	....	·04	....	....
21.	....	....	....	·01	....	....	1·23	·08	·01	....	....	....
22.	....	....	....	....	....	....	·15	·01	·01	....	....	....
23.	....	....	....	....	....	....	....	·34	·01	....	....	....
24.	....	....	....	....	....	....	·07	·20	....	....	....	....
25.	....	....	....	....	....	....	·28	·02	....	....	....	....
26.	....	....	....	....	....	....	·04	·11	....	2·54	....	·09
27.	....	....	....	....	....	....	·51	·05	·13	·07	....	·06
28.	....	....	....	....	....	....	·34	·01	....	....	....	....
29.	....	....	....	....	....	....	2·57	·01	....	....	....	....
30.	....	....	....	....	....	....	·75	....	....	....	....	....
31.	....	....	....	....	....	....	....	....	....	....	....	....
Total	·40	2·29	....	·04	....	·04	13·47	1·79	2·01	4·51	4·33	·15

Total fall of rain 29·03 inches.

Table XXV.

Register of the Ombrometer from December 1827 to December 1828, in Dukhun.

Dates. 1828.	Dec.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.
1.	....	....	....	....	....	....	·15	....	....	·29	·11	....
2.	....	....	....	....	....	....	....	·03	·02	1·17	....	....
3.	....	....	....	....	....	....	·17	·13	....	1·53	....	....
4.	....	....	....	....	....	....	·57	·03	....	·15	....	....
5.	....	....	....	....	....	....	....	·12	....	·32	·71	....
6.	....	....	....	....	....	....	....	....	·02	·02	....	....
7.	....	....	....	....	....	....	....	·06	·12	....	....	....
8.	....	....	....	....	....	....	....	....	·04	....	....	....
9.	....	....	....	....	....	....	....	....	·02	·06	·21	....
10.	....	....	....	....	....	....	....	....	·28	·03	·68	....
11.	....	....	....	....	....	....	....	·08	....	·20	....	....
12.	....	....	....	....	....	....	....	·03	....	·01	....	....
13.	....	....	....	....	....	....	....	....	....	....	·15	....
14.	....	....	....	....	....	....	....	....	....	·01	1·47	....
15.	....	....	....	....	....	....	·04	·12	·07	....	·08	....
16.	....	....	....	....	....	....	....	·07	....	·01	·03	....
17.	....	....	....	....	....	....	·17	·43	....	·03	1·96	....
18.	....	....	....	....	....	....	....	·06	....	·31	·03	....
19.	....	....	....	....	....	....	·01	1·16	·05	1·48	·69	·23
20.	....	....	....	....	....	1·50	....	·93	·01	1·15	·22	1·81
21.	....	....	....	....	....	....	....	·30	....	·06	....	....
22.	....	....	....	....	....	....	....	·31	....	....	....	....
23.	....	....	....	....	....	....	·52	·70	....	·04	....	....
24.	....	....	....	....	....	....	....	·04	....	·04	....	....
25.	....	....	....	....	....	·45	....	·08	....	·01	....	....
26.	....	....	....	....	....	....	....	....	....	....	....	....
27.	....	....	....	....	....	....	....	·19	....	....	....	....
28.	....	....	....	....	....	....	....	....	·02	....	....	....
29.	....	....	....	....	....	....	....	·06	·28	....	....	....
30.	....	....	....	....	....	....	....	1·17	2·24	....	....	....
31.	....	....	....	....	....	....	....	1·48	·18	....	....	....
Total	....	....	....	....	....	1·95	1·63	7·58	3·35	6·92	6·34	2·04

Total fall of rain 29·81 inches.

Table XXVI.

Register of the Ombrometer in Dukhun during the year 1829.

Dates. 1829.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
1.	....	....	....	....	....	....	·10	....	....	....	....	....
2.	....	....	....	....	....	....	·03	....	....	....	....	....
3.	....	....	....	....	....	....	·18	....	....	....	....	....
4.	....	....	....	....	....	....	·31	·05	....	....	....	....
5.	....	....	....	....	....	....	....	....	·02	....	....	....
6.	....	....	....	....	....	....	....	·03	....	·49	....	....
7.	....	....	....	....	....	....	....	·64	·07	....	....	....
8.	....	....	....	....	....	....	·09	·12	·04	....	....	....
9.	....	....	....	....	·05	....	·09	....	·07	·37	....	....
10.	....	....	....	....	....	·53	·08	·10	·11	·07	....	·50
11.	....	....	....	....	....	....	....	1·09	....	·09	....	·70
12.	....	....	....	....	....	·12	....	·07	....	....	....	....
13.	....	....	....	....	·04	1·30	·10	·11	....	....	....	....
14.	....	....	....	....	....	....	·03	·13	·02	....	....	....
15.	....	....	....	....	....	....	....	·07	....	·03	....	....
16.	....	....	....	....	....	·05	·04	....	....	·08	....	....
17.	....	....	....	....	....	·12	·16	....	....	....	....	....
18.	....	....	....	....	....	....	....	....	....	....	....	....
19.	....	....	....	....	....	....	....	....	....	....	....	....
20.	....	....	....	....	....	·19	....	·36	....	·64	....	....
21.	....	....	....	....	·05	1·19	·70	....	....	·04	....	....
22.	....	....	....	....	·35	·15	·05	....	....	....	....	....
23.	....	....	....	....	1·90	·11	....	·08	....	....	....	....
24.	....	....	....	....	·10	....	·14	·12	....	....	....	....
25.	....	....	....	....	...	·50	·24	....	....	....	....	....
26.	....	....	...	....	....	·40	·65	....	....	....	....	....
27.	....	....	....	....	·25	....	1·16	....	....	....	....	....
28.	....	....	....	....	....	·02	·17	·04	....	....	....	....
29.	....	....	....	....	....	·17	·02	·18	....	....	....	....
30.	....	....	....	....	....	·01	....	·02	....	....	....	....
31.	....	....	....	....	....	....	·04	....	....	....	....	....
Total	....	....	....	....	2·74	4·86	4·38	3·21	·33	1·81	....	1·20

Total 18·53 inches.

Table XXVII.

Register of the Ombrometer at Poona during the year 1830.

Dates. 1830.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
1.	....	....	....	....	....	....	....	·39	·03	·40	....	....
2.	....	....	....	....	....	....	·02	....	....	·04	....	....
3.	....	....	....	....	....	....	....	....	....	....	....	....
4.	....	....	....	....	.02	....	....	·40	·02	....	....	....
5.	....	....	....	....	....	....	....	·10	....	....	....	....
6.	....	....	....	....	....	·02	....	·10	....	....	....	....
7.	....	....	....	....	·04	·02	....	....	....	·06	....	....
8.	....	....	....	....	....	·04	....	·12	·08	·17	....	....
9.	....	....	....	....	·03	·50	....	....	....	·11	....	....
10.	....	....	....	....	....	·20	....	....	....	·24	....	....
11.	....	....	....	....	....	·10	....	....	·05	·23	....	....
12.	....	....	....	....	....	1·19	....	....	·01	1·81	....	....
13.	....	....	....	....	·06	·04	....	·07	....	·01	....	....
14.	....	....	....	....	....	....	....	....	....	....	....	....
15.	....	....	....	....	....	·01	1·81	....	....	....	....	....
16.	....	....	....	....	....	....	....	·07	....	....	....	....
17.	....	....	....	....	....	....	....	·08	....	....	....	....
18.	....	....	....	....	....	....	....	·02	....	....	....	....
19.	....	....	....	....	....	....	....	....	....	....	....	....
20.	....	....	....	....	....	....	....	....	....	....	....	....
21.	....	....	....	·43	·28	·32	....	....	....	....	....	....
22.	....	....	....	....	....	·47	....	·04	....	....	....	....
23.	....	....	....	·30	·33	·33	....	....	....	....	....	....
24.	....	....	....	·19	....	2·31	....	·17	....	....	....	....
25.	....	....	....	·09	....	....	2·41	·04	....	....	....	....
26.	....	....	....	....	....	....	·16	....	....	....	....	....
27.	....	....	....	·03	....	....	·52	....	....	....	....	....
28.	....	....	....	....	·03	·02	·02	....	....	....	....	....
29.	....	....	....	....	....	....	....	·01	·10	....	....	....
30.	....	....	....	....	....	....	....	....	....	....	....	....
31.	....	....	....	....	....	....	·41	....	....	....	....	....
Total	....	....	....	1·04	·79	5·57	5·35	1·61	·29	3·07	....	....

Table XXVIII.

Tabular view of the fall of rain in Dukhun from 1826 to 1830, both inclusive.

Months.	1826.	1827.	1828.	1829.	1830.	Total in five years.
	inches.	inches.	inches.	inches.	inches.	inches.
January.	......	2·29	......	......	......	2·29
February.	......	......	......	......	......	......
March.	......	·04	......	......	......	·04
April.	......	......	......	......	1·04	1·04
May.	3·41	·04	1·95	2·74	·79	8·93
June.	3·30	13·47	1·63	4·86	5·57	28·83
July.	8·43	1·79	7·58	4·38	5·35	27·53
August.	1·03	2·01	3·35	3·21	1·72	11·32
September.	1·54	4·51	6·92	·33	·29	13·59
October.	1·90	4·33	6·34	1·81	3·07	17·45
November.	2·33	·15	2·04	......	......	4·52
December.	·40	......	......	1·20	......	1·60
Total	22·34	28·63	29·81	18·53	17·83	117·14

23·43 inches mean annual fall.

Table XXIX.

Tabular view of the fall of rain in Bombay from 1817 to 1829, inclusive, in the monsoon.

Months.	1817.	1818.	1819.	1820.	1821.	1822.	1823.	1824.	1825.	1826.	1827.	1828.	1829.	Total.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.
Jan.	....	....	....	....	....	....	....	....	....	....	....	....	....	
Feb.	....	....	....	....	....	....	....	....	....	....	....	....	....	
Mar.	....	....	....	....	....	....	....	....	....	....	....	....	....	
April.	....	....	....	....	....	....	....	....	....	....	....	....	....	
May.	....	....	....	....	....	....	....	....	....	....	....	....	....	
June.	45·72	22·54	15·95	18·82	15·18	29·21	21·76	3·89	24·25	17·75	49·15	23·53	27·86	
July.	23·67	17·69	30·66	28·37	20·60	26·39	15·96	8·07	25·17	26·97	10·29	52·75	19·78	
Aug.	9·34	28·45	20·24	19·49	28·52	33·83	19·70	17·86	12·94	8·40	10·51	17·22	12·40	
Sept.	24·87	10·39	10·11	10·66	18·29	22·16	4·28	1·78	9·68	23·50	10·16	22·08	4·95	
Oct.	·19	2·07	·14	....	·40	·82	....	2·37	....	1·23	·92	6·40	....	
Nov.	....	....	....	....	....	....	....	....	....	....	....	....	....	
Dec.	....	....	....	....	....	....	....	....	....	....	....	....	....	
Total	103·79	81·14	77·10	77·34	82·99	112·61	61·70	34·33	72·24	77·85	81·03	121·98	64·99	

80·69 inches mean annual fall in the monsoon.

Table XXX.

Prevailing Winds in Dukhun in the year 1826.

1826.	North.				North-east.				East.				South-east.				South.				South-west.				West.				North-west.				No wind.				No. of obs.
	Sun-rise.	9–10 A.M.	4 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	
Jan.			2	2					3	7	8	13	3	3		6	2			2					4	3	8	15	2			2	18	15	15	48	88
Feb.					1	1		2	5	9	1	15		1		1									2		18	20			1	1	20	17	8	45	84
Mar.									1	3		4													3	9	22	34		1	3	4	26	16	4	46	88
April.	4	1		5	3			3									3			3		1	1	2		14	28	42			1	1	20	14		34	90
May.					3			3	1		1	2	1			1	4	1			5	6	6	17		7	17	24	4	1	1	6	9	9		18	75
June.																		2			16	23	21	60	2	6	9	17					10			10	90
July.																					22	29	28	79		2	3	5					9			9	93
Aug.																						1		1	13	30	31	74					18			18	93
Sept.																									20	28	30	78					10	2		12	90
Oct.	1	7	5	13		1	5	6		4		4														2	7	9					30	17	11	58	90
Nov.	2	3	2	7	4	4	4	12	5	12	11	28	1	4	3	8			2	2													17	7	7	31	88
Dec.		3	3	6	2	1	3	6	5	9	7	21	3	12	12	27														1	1	2	21	4	5	30	92
Total.	7	14	12	33	13	7	12	32	20	44	23	87	8	20	15	43	9	3	2	14	43	60	56	159	44	101	173	318	6	3	7	16	208	101	50	359	1061

Table XXXI.

Prevailing Winds in Dukhun in the year 1827.

1827.	North.				North-east.				East.				South-east.				South.				South-west.				West.				North-west.				No wind.				No. of obs.
	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	
Jan.						3	5	8	2	8	8	18		3	1	4									1	4	7	12		1	1	2	28	12	9	49	93
Feb.	1	1	1	3	2	1	1	4	2	7	10	19			1	1									3	1	2	6	1			1	19	18	13	50	84
Mar.	2	2	1	5	1	2		3	3	10	1	14		1		1		1	1	2			1	1	8	6	17	31					17	9	9	35	92
April.	1			1			1	1	1	3		4										1	1	2	4	14	23	41			1	1	19	7		26	76
May.																									19	27	27	73	1	1	1	3	9	2		11	87
June.			1	1																	1		1	2	17	26	27	70					12	4	1	17	90
July.																									24	31	31	86					7			7	93
Aug.																									20	29	31	80					11	2		13	93
Sept.																			1	1					10	26	27	63					20	4	2	26	90
Oct.	1			1		1		1		2	4	6		2	3	5									3	12	10	25		1	1	2	26	13	12	51	91
Nov.			1	1					9	22	25	56	1			1									1			1	1			1	15	8	3	26	86
Dec.	1	1		2		2		2	10	11	9	30	7	5	5	17	2	3	1	6			1	1	1			1	2	2		4	8	7	15	30	93
Total.	6	4	4	14	3	9	7	19	27	63	57	147	8	11	10	29	2	4	3	9	1	1	4	6	111	176	202	489	5	5	4	14	191	86	64	341	1068

Table XXXII.

Prevailing Winds in Dukhun in the year 1828.

1828.	North.				North-east.				East.				South-east.				South.				South-west.				West.				North-west.				No wind.				No. of obs.
	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	
Jan.	5	3	4	12					1	14	5	20	1		1	2	3	6	2	11			2	2					1			1	21	8	15	44	92
Feb.	1		1	2	2	2	1	5		5	2	7	1			1									5	3	11	19	1	2		3	14	16	13	43	80
Mar.	1			1					5	15		20											1	1	6	6	11	23			5	5	19	10	11	40	90
April.							1	1	4	6	4	14													16	12	21	49		4	3	7	6	7		13	84
May.	3	1		4	1	1		2	1			1	1		1	2					1		2	3	9	13	26	48	9	7	1	17	6	9		15	92
June.															1	1					1	1	2	4	12	23	24	69					17	6	1	24	88
July.																									18	29	31	78					12			12	90
Aug.																									11	25	28	64					10	5	2	17	81
Sept.																									8	25	28	61					22	5	2	29	90
Oct.			1	1		1		1	3	11	11	25			2	2			1	1	1	1	1	3		5	5	10		1	1	2	27	13	5	45	90
Nov.		1		1		2	4	6	7	22	19	48														1	1	2		2	1	3	20	4	6	30	90
Dec.									27	26	6	59													1		5	6			15	15	1	3	4	8	88
Total.	10	5	6	21	3	6	6	15	48	99	47	194	3		5	8	3	6	3	12	3	2	8	13	86	142	191	419	11	16	26	53	175	86	59	320	1055

Table XXXIII.

Prevailing Winds in Dukhun in the year 1829.

1829.	North.				North-east.				East.				South-east.				South.				South-west.				West.				North-west.				No wind.				No. of obs.
	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	Sun-rise.	9–10 A.M.	4–5 P.M.	To-tal.	
Jan.	2	2	2	6	1	2	1	4	9	15	8	32		1		1									1	6	10	17		1	2	3	17	3	7	27	90
Feb.	3	7	5	15		1		1	3	5	4	12		3	5	8		1		1					3	5	8	16	1	2	1	4	18	4	4	26	88
Mar.		2	1	3					12	22	4	38										1		1	3	2	19	24	1		1	2	15	4	4	23	91
April.			1	1	2	3		5	4	7		11													15	13	28	56		1	1	2		8	6	14	89
May.					1	1		2	2	4		6									4	10	11	25	15	21	25	61			3	3	12	3		15	87
June.																						1	1	2	15	14	15	44					11	6	2	*19	88
July.																					1	5	4	10	19	27	30	76					10	3		†13	91
Aug.																									12	21	24	57					16	3	1	20	87
Sept.																					1			1	11	20	27	58					16	4		20	78
Oct.	1	1	4	6		4	4	8		6	6	12			1	1									6	4	8	18	2	1	1	4	21	13	6	40	90
Nov.		1		1					2	26	9	37										1		1			3	3					28	2	17	47	88
Dec.						1	1	2	2	20	15	37		2		2											2	2					23	5	13	41	85
Total.	6	13	13	32	4	12	6	22	34	105	46	185		6	6	12		1		1	6	18	16	40	100	133	199	432	4	5	9	18	187	58	60	305	1047

* Four indications of wind, violent and variable. † One indication of wind, variable.

TABLE XXXIV.

Prevailing Winds in Dukhun in the year 1830.

1830.	North.					North-east.					East.					South-east.					South.					South-west.					West.					North-west.					No wind.					No. of obs.
	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—4 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	
January.	...	...	...	...	...	...	6	8	...	14	...	12	10	...	22	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	...	2	...	...	...	...	...	28	14	9	*	51	89
February.	...	...	...	...	...	...	2	3	...	5	1	7	1	1	10	...	...	1	...	1	...	...	...	...	...	...	3	...	...	3	3	3	3	3	12	...	...	2	3	5	24	10	14	9	57	93
March.	...	...	...	...	...	...	2	...	...	2	...	2	1	...	3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	4	9	24	7	44	1	1	...	1	3	5	17	3	9	34	86
April.	...	...	0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	1	...	2	7	6	27	12	52	...	1	...	...	1	8	22	...	12	42	97
May.	...	..	1	...	1	...	1	3	1	5	...	3	...	...	3	3	2	...	...	5	...	...	...	...	...	...	1	6	...	7	*	15	20	11	36	...	1	...	5	6	*	8	1	9	18	77
June.	...	...	...	...	...	...	1	...	...	1	...	...	1	...	1	...	...	4	...	4	...	...	...	...	...	...	5	12	2	19	*	14	13	14	41	...	...	...	1	1	*	4	...	7	11	78
July.	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	5	6	...	12	*	8	7	19	34	...	...	...	...	...	1	...	...	8	9	55
August.	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	*	7	9	2	18	*	19	16	4	39	...	...	...	...	...	*	2	2	7	11	58
September.	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	10	11	3	25	1	16	15	7	39	...	...	...	...	...	7	3	3	14	27	91
October.	...	...	2	...	2	...	3	6	...	9	...	10	3	3	16	...	1	...	...	1	...	...	...	...	...	...	...	1	...	1	...	1	4	2	7	...	...	1	...	1	10	17	15	23	65	102
November.	...	3	4	...	7	...	4	6	...	10	...	11	3	4	18	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	3	1	4	...	1	1	1	3	3	10	12	12	37	79
December.	...	1	4	...	5	...	4	5	...	9	...	12	5	...	17	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	..	3	...	4	1	...	1	...	2	*	14	12	7	33	70
Total year.	...	4	11	...	15	...	23	31	1	55	1	57	24	8	90	3	3	5	...	11	...	...	...	...	...	2	32	46	7	87	16	91	137	80	314	2	4	5	11	22	88	121	71	117	395	974

* No observation.

TABLE XXXV.

Abstract of the prevailing Winds in Dukhun during the years 1826, 1827, 1828, 1829, and 1830.

Years.	North.					North-east.					East.					South-east.					South.					South-west.					West.					North-west.					No wind.					No. of obs.
	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	Sunrise.	9—10 A.M.	4—5 P.M.	10—11 P.M.	Total.	
1826.	7	14	12	...	33	13	7	12	...	32	20	44	23	...	87	8	20	15	...	43	9	3	2	...	14	43	60	56	...	159	44	101	173	...	318	6	3	7	...	16	208	101	50	...	359	1061
1827.	6	4	4	...	14	3	9	7	...	19	27	63	57	...	147	8	11	10	...	29	2	4	3	...	9	1	1	4	...	6	111	176	202	...	489	5	5	4	...	14	191	86	64	...	341	1068
1828.	10	5	6	...	21	3	6	6	...	15	48	99	47	...	194	3	...	5	...	8	3	6	3	...	12	3	2	8	...	13	86	142	191	...	419	11	16	26	...	53	175	86	59	...	320	1055
1829.	6	13	13	...	32	4	12	6	...	22	34	105	46	...	185	...	6	6	...	12	...	1	...	...	1	6	18	16	...	40	100	133	199	...	432	4	5	9	...	18	187	58	60	...	305	1047
1830.	...	4	11	...	15	...	23	31	1	55	1	57	24	8	90	3	3	5	...	11	...	...	...	...	...	2	32	46	7	87	16	91	137	80	324	1	4	5	11	21	86	121	71	117	395	998
Total.	29	40	46	...	115	23	57	62	1	143	130	368	197	8	703	22	40	41	...	103	14	14	8	...	36	55	113	130	7	305	357	643	902	80	1982	27	33	51	11	122	847	452	304	117	1720	5229

In 1830 the observations at sunrise were for the most part omitted, and observations at 10—11 P.M. substituted.

XI. *Geometrical Investigations concerning the Phenomena of Terrestrial Magnetism.*

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Received December 18th, 1834,—Read February 5th and 12th, 1835.

THOUGH the experiments of Michel and Coulomb have satisfactorily determined the law according to which magnetic forces vary as the distance of the needle acted on is made to vary, yet, so far as I know, no one has attempted to deduce from that law any method of accounting for the phenomena of terrestrial magnetism. Till that is done, however, we cannot assure ourselves whether the poles (I use the term to designate the centres from which the forces emanate,) be two or more; nor even whether it be necessary to consider them infinite in number and distributed over the whole surface or through the whole mass of the magnet. The agreement of the results as to quantity with the actual phenomena would be decisive in favour of any hypothesis. The *necessary consequences* have not, however, yet been deduced from any one hypothesis whatever: and even had it been otherwise, there is so much uncertainty attached to magnetical observations, and so many anomalous and unaccountable discrepancies and disturbances continually mingling in the registered results, that it is not possible, in the present state of the numerical data, to bring any hypothesis fairly to the trial, however complete the mathematical development of its consequences may be.

Notwithstanding the great difficulty of conducting a series of observations in a perfectly unexceptionable manner, and the utter impossibility, with our present knowledge, of properly determining the correction to be made at any given place and period with any given instrument, there are yet several features in the phenomena which are of too decided a character to be overlooked in comparing the results of any theory with observation. We may not, indeed, be able to avoid considerable discrepancies in our comparison, but still there should at least be a general tendency towards agreement, and in no case a direct reversal of the phenomena presenting itself as the result of any hypothesis which prefers its claims to our adoption. In respect to terrestrial magnetism, no direct attempt has, however, been made to embody the results of any hypothesis in a series of appropriate formulæ; and hence the conjectures which have been made respecting such agreements have been made from extremely vague and inconclusive considerations.

The duality of the terrestrial magnetic poles is the oldest hypothesis, and perhaps that whose consequences will be found most easy to examine. The hasty comparisons

that have been made between its *supposed* results and the observations made on the needle at certain places, and especially respecting the Halleyan lines, and HANSTEEN'S poles of greatest intensity, have caused the hypothesis to be rejected by many persons, who, if they had looked more closely into the question, could not have failed to discover that their conclusions were altogether premature, and probably erroneous. I speak now of the broad features of the phenomena compared with a popular rather than a calculated series of deductions from the hypothesis. Whether, however, when the results come to be more closely tested by an appeal to the numerical values of the quantities in question, the same accordance would be found, is a question altogether different: and it is one which we are not at present in a condition, for want of numerical data, upon which to offer a distinct opinion, much less are we entitled to express a positive decision concerning it.

The present series of papers is chiefly intended to deduce the mathematical consequences of the theory of two poles situated arbitrarily within the earth, and especially to investigate the singular points and lines which result from the intersection of the earth's surface with other surfaces related to the magnetic poles. Amongst these are the magnetic equator, the points at which the needle is vertical, the lines of equal dip, the Halleyan lines, the isodynamic lines, and the Hansteen poles. If it shall appear in the course of these developments that the general features of all these are roughly represented by the hypothesis of two poles, then it will be a strong argument in favour of that simple theory; but should a result, in any one of these cases, follow from that hypothesis which is very decidedly opposed to the corresponding observed phenomena, we shall be compelled, if our observations are authentic and to be depended on as unaffected by an extreme degree of foreign influence, to abandon it altogether.

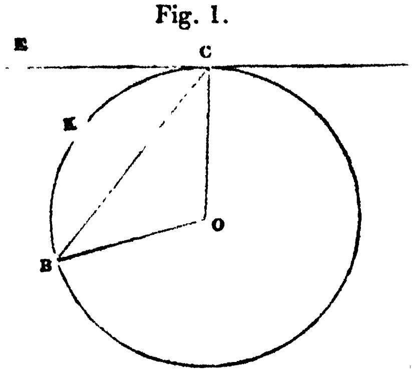
Our hope of being able to separate the disturbing from the primary forces must depend altogether upon their relative quantities. The success of astronomical research has hinged wholly on the relative smallness of the disturbances in comparison with the primary forces that govern the motions of a planet: and if the same order of magnitude should exist in the magnetic forces in question, the same success will, there is every reason to hope, follow in due time. If not, the research should be placed at once amongst the *desperata*. However, till some method is discovered of ascertaining whether such is the case or not, we should leave no effort untried either to accomplish the proposed object, or to render manifest its impracticability. With that view the present investigations, which are conducted in a manner altogether untried before by any one, are offered to the attention of geometers, as being calculated, besides exhibiting the general consequences of the dual hypothesis, in some degree to point out where we should look for the influence of foreign forces, and especially showing that in reference to one great circle, the want of symmetry in the results at positions taken symmetrically with respect to it, gives us great cause to suspect the action of such foreign forces.

The nature of this paper does not allow of much numerical experiment upon the observation-data; but still, in illustration of the method of determining the position of the magnetic axis, I have entered into a little. The results are not very favourable to the hypothesis; but when it is considered that the observations were selected almost at hazard, all made with different instruments, by various persons, and in geological regions extremely dissimilar, we could hardly, in the confessedly imperfect state of the art of observation, expect to obtain satisfactory results. Taking all things into account, the results are, unfavourable though they be, as satisfactory as we could expect. However, whatever conclusion may be drawn from them, they furnish at least a pattern for calculating any better and more consistent observations we may at any future time be able to procure; and if any that are beyond question can be obtained, it will enable us to bring the *linearity* of the magnetic axis to an indisputable test. The *duality*, should the linearity be established, can be at once put to the test by means of the process in Section III.

Now that this method of investigation is proposed, it will doubtless occur to some of my readers that a more *direct* course would have been to assume the undetermined coordinates $a_{,}\, b_{,}\, c_{,}$, $a_{,,}\, b_{,,}\, c_{,,}$, of the two poles, and express the equation of the sphere in reference to the same axes, and hence the directive effect of the two poles upon a needle placed at a point $x\,y\,z$ on the spherical surface. Such a method, they will believe, must also have occurred to me as the most natural; but if they will take the trouble to form the equations of condition that this method will require, they will see the utter impracticability of effecting the reductions under the mere motive of making an experiment upon the results of an hypothesis when no confidence was felt in the numerical data which entered into the formula. The reason for adopting the less direct, but incomparably more simple, preliminary test illustrated by Sections XI. and XII. will then be sufficiently obvious.

I.—Given the dip and variation of the magnetic needle and the geographical coordinates of the place of observation, to find the geographical coordinates of the place where the needle, sufficiently prolonged, will intersect the surface of the earth again.

Fig. 1.

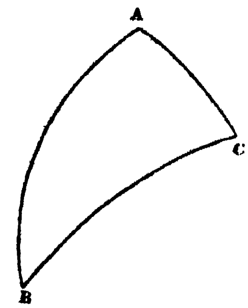


We assume, for reasons too well known to need specification here, that the orthogonal projection of the dipping-needle upon the horizontal plane gives the position of the horizontal needle; or, which comes to the same, that the dipping-needle and the horizontal needle are in the same vertical plane. This plane cuts the sphere in a great circle, which we shall for the present suppose to coincide with the plane of the paper, and to be represented in the annexed figure by C K B. Let E C be the

intersection of the plane B K C with the horizontal plane, and let C B be the line along which the dipping-needle disposes itself.

Join C O, O B, (O being the centre of the circle and, of consequence, of the sphere): then the arc C K B measures the angle C O B, which is twice the angle E C B, or twice the dip of the free needle. This arc, then, is known from observations at several particular places on the earth's surface.

Fig. 2.



Next, let the spherical triangle A B C denote that whose vertex A is the geographical pole of the earth; C the place of observation; and the angle B that determined as above from observations made at C: and let the angle A C B denote the observed variation of the horizontal needle at C. Then we have the sides A C, C B, and included angle A C B, from which to determine the colatitude A B and polar angle B A C.

We have therefore the polar spherical coordinates of the point B, the polar distance A B at once, and the polar angle by adding B A C to the longitude of C with its proper sign.

I shall designate the coordinates of C and B by $\alpha_{,} \beta_{,}$ and $\alpha_{,,} \beta_{,,}$ respectively, as is done in my paper on Spherical Geometry in the twelfth volume of the Edinburgh Transactions; α denoting the polar distance and β the longitude of the point.

II.—Given the dip, variation, and geographical coordinates of the place of observation, to express the equations of the line in which the dipping-needle disposes itself.

Let $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$ denote the coordinates of two points in space: then the equations of the straight line through them are

$$x = \frac{a_{,,} - a_{,}}{c_{,,} - c_{,}} z - \frac{a_{,,} c_{,} - a_{,} c_{,,}}{c_{,,} - c_{,}}$$

$$y = \frac{b_{,,} - b_{,}}{c_{,,} - c_{,}} z - \frac{b_{,,} c_{,} - b_{,} c_{,,}}{c_{,,} - c_{,}}.$$

But in the present case the points $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$ are on the surface of the sphere; and if we consider the axis of the sphere to be the axis of z, the intersection of the meridian with the equator to be the axis of y, and that of the meridian at right angles to it with the equator to be the axis of x, then $\alpha_{,} \beta_{,}$ and $\alpha_{,,} \beta_{,,}$ being, as before stated, the coordinates of the extremities of the chord in which the dipping-needle disposes itself, we shall have, for determining the equations of the needle, the following values of the constants:

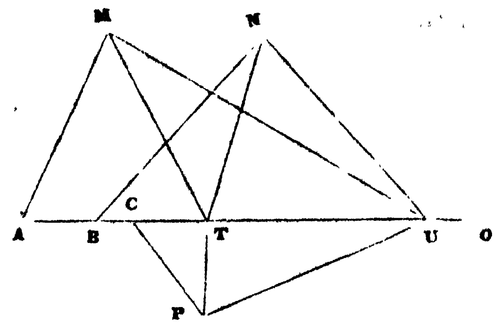
$c_{,} = r \cos \alpha_{,}$	$c_{,,} = r \cos \alpha_{,,}$
$b_{,} = r \sin \alpha_{,} \cos \beta_{,}$	$b_{,,} = r \sin \alpha_{,,} \cos \beta_{,,}$
$a_{,} = r \sin \alpha_{,} \sin \beta_{,}$	$a_{,,} = r \sin \alpha_{,,} \sin \beta_{,,}$.

Hence the equations of the needle take the following forms:

$$x = \frac{\sin \alpha_{\prime\prime} \sin \beta_{\prime\prime} - \sin \alpha_{\prime} \sin \beta_{\prime}}{\cos \alpha_{\prime\prime} - \cos \alpha_{\prime}} z - \frac{\sin \alpha_{\prime\prime} \sin \beta_{\prime\prime} \cos \alpha_{\prime} - \sin \alpha_{\prime} \sin \beta_{\prime} \cos \alpha_{\prime\prime}}{\cos \alpha_{\prime\prime} - \cos \alpha_{\prime}} r \quad . \quad . \quad (1.)$$

$$y = \frac{\sin \alpha_{\prime\prime} \cos \beta_{\prime\prime} - \sin \alpha_{\prime} \cos \beta_{\prime}}{\cos \alpha_{\prime\prime} - \cos \alpha_{\prime}} z - \frac{\sin \alpha_{\prime\prime} \cos \beta_{\prime\prime} \cos \alpha_{\prime} - \sin \alpha_{\prime} \cos \beta_{\prime} \cos \alpha_{\prime\prime}}{\cos \alpha_{\prime\prime} - \cos \alpha_{\prime}} r. \quad . \quad (2.)$$

III.—Let M, N, P be the centres of three dipping-needles at known positions on the surface of the earth, and denote the poles by T and U. Then the needles will arrange themselves so as that each shall be in a plane passing through T U; and hence each needle prolonged will cut the magnetic axis T U in some point, as A, B, C, respectively.



Take any point, O, in T U, and refer all the points to this origin; denote the several distances O U, O T, O C, O B, O A, by u, t, c, b, a respectively; the angles M A O, N B O, and P C O, by A, B, C; and the distances M A, N B, P C, by f, g, h.

Then we have

$$\left.\begin{aligned} MT^2 &= (a-t)^2 - 2f(a-t)\cos A + f^2 \\ MU^2 &= (a-u)^2 - 2f(a-u)\cos A + f^2 \\ NT^2 &= (b-t)^2 - 2g(b-t)\cos B + g^2 \\ NU^2 &= (b-u)^2 - 2g(b-u)\cos B + g^2 \\ PT^2 &= (c-t)^2 - 2h(c-t)\cos C + h^2 \\ PU^2 &= (c-u)^2 - 2h(c-u)\cos C + h^2. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

Again, by the properties of a needle subjected to the action of the magnetic poles T and U, we have (those needles being small in comparison with its distance from those poles,) the following proportions:

$$\left.\begin{aligned} TA : AU &:: TM^3 : MU^3 \\ TB : BU &:: TN^3 : NU^3 \\ TC : CU &:: TP^3 : PU^3. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

Inserting in (4.) the values of the lines T A, &c. given in (3.), we get the three equations

$$\frac{a-t}{a-u} = \left\{\frac{(a-t)^2 - 2f(a-t)\cos A + f^2}{(a-u)^2 - 2f(a-u)\cos A + f^2}\right\}^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

$$\frac{b-t}{b-u} = \left\{\frac{(b-t)^2 - 2g(b-t)\cos B + g^2}{(b-u)^2 - 2g(b-u)\cos B + g^2}\right\}^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

$$\frac{c-t}{c-u} = \left\{\frac{(c-t)^2 - 2h(c-t)\cos C + h^2}{(c-u)^2 - 2h(c-u)\cos C + h^2}\right\}^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

IV.—Given the equations of the dipping-needle at three given places on the surface of the earth to find the magnetic poles.

Let M, N, P be the places on the surface of the earth, and denote the needles as follows, viz.

$$\text{M A by } x = a' z + \alpha' \text{ and } y = b' z + \beta' \quad \ldots \quad (8.)$$

$$\text{N B by } x = a'' z + \alpha'' \text{ and } y = b'' z + \beta'' \quad \ldots \quad (9.)$$

$$\text{P C by } x = a''' z + \alpha''' \text{ and } y = b''' z + \beta''', \quad \ldots \quad (10.)$$

and denote the magnetic axis itself (T U) by

$$x = \overline{a}\, z + \overline{\alpha} \text{ and } y = \overline{b}\, z + \overline{\beta}. \quad \ldots \quad (11.)$$

Then, since the line (11.) intersects the lines (8.), (9.), (10.), we have the three equations of condition

$$(\alpha' - \overline{\alpha})\,(b' - \overline{b}) = (\beta' - \overline{\beta})\,(a' - \overline{a}) \quad \ldots \quad (12.)$$

$$(\alpha'' - \overline{\alpha})\,(b'' - \overline{b}) = (\beta'' - \overline{\beta})\,(a'' - \overline{a}) \quad \ldots \quad (13.)$$

$$(\alpha''' - \overline{\alpha})\,(b''' - \overline{b}) = (\beta''' - \overline{\beta})\,(a''' - \overline{a}). \quad \ldots \quad (14.)$$

Taking now as the unknown coordinates of the magnetic poles, T and U, the symbols $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$, we have $\overline{a}\, \overline{b}$, $\overline{\alpha}\, \overline{\beta}$, given functions of $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$; and hence we have in the equations just given three equations for the discovery of these six quantities which determine the poles. The three other requisite equations are thus derived:

By means of (11.) combined separately and successively with (8.), (9.), (10.), we can find the coordinates of the points A, B, C in terms of $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$ and given quantities $a'\, \alpha'\, b'\, \beta'$, &c.; and the coordinates of T and U are themselves $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$. We hence have the several quantities $a - t$, $b - t$, $c - t$, $a - u$, $b - u$, and $c - u$ in terms of $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$. Also the distances M A, N B, P C, that is, f, g, h, are also given in terms of the coordinates of M and A, N and B, P and C respectively, and hence in terms of $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$ and given quantities. And lastly, the equations of the lines T U and M A, N B, P C being given in terms of $a_{,} b_{,} c_{,}$ and $a_{,,} b_{,,} c_{,,}$ and known quantities, the cosine of the inclinations of T U to each of them, that is, of the angles A, B, C, are given so as to involve no quantities but known ones and the coordinates of the magnetic poles. It hence follows, that all the terms which enter into the composition of the equations (5.), (6.), (7.), are functions of the coordinates of the poles and of given quantities. The three remaining requisite equations for the actual determination of the magnetic poles are furnished, then, by those equations marked (5.), (6.), and (7.); the whole six equations which we have laid down being each, obviously, independent of the others under every combination.

V.—The preceding processes show that the determination of the magnetic poles, their duality being admitted, as well as the equality of their intensity, can be effected from three observations of the magnetic needle as to dip and azimuth; and hence

that the problem is now reduced to a purely arithmetical state, requiring only the application of known processes, and perfectly capable of execution, for the actual assignment of the positions of the poles themselves. A very slight attention to the processes themselves must, however, convince us that the operations will be very laborious; but at the same time, the symmetrical forms in which the two triads of equations are presented, might induce us to hope that a greater degree of simplification would result in the final formulæ than our passage through so many operations could at first sight lead us to expect. Such, indeed, proves to be the case; and the results are not altogether destitute of elegance, as well as simplification. Fortunately, however, there is no necessity to even attempt the solution under the present aspect of the problem; and having learnt from it, in its present state, the number of observations necessary for the determination of the poles, we shall exchange it for another, which is in some degree different as to general plan, and considerably more simple in its requisite calculations.

VI.—A necessary consequence of the hypothesis upon which we are proceeding, is,—*that all the needles must intersect the magnetic axis.* If, then, we assume the coordinates of the two poles $a_{\prime}\, b_{\prime}\, c_{\prime}$ and $a_{\prime\prime}\, b_{\prime\prime}\, c_{\prime\prime}$, we have the equation of the magnetic axis as before,

$$\left.\begin{aligned} x &= \frac{a_{\prime\prime} - a_{\prime}}{c_{\prime\prime} - c_{\prime}}\, z - \frac{a_{\prime\prime} c_{\prime} - a_{\prime} c_{\prime\prime}}{c_{\prime\prime} - c_{\prime}} \\ y &= \frac{b_{\prime\prime} - b_{\prime}}{c_{\prime\prime} - c_{\prime}}\, z - \frac{b_{\prime\prime} c_{\prime} - b_{\prime} c_{\prime\prime}}{c_{\prime\prime} - c_{\prime}}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (15.)$$

And if we take the equations of four magnetic needles, as

$$\left.\begin{aligned} x &= a' z + \alpha' \text{ and } y = b' z + \beta' \\ x &= a'' z + \alpha'' \text{ and } y = b'' z + \beta'' \\ x &= a''' z + \alpha''' \text{ and } y = b''' z + \beta'' \\ x &= a^{\text{IV}} z + \alpha^{\text{IV}} \text{ and } y = b^{\text{IV}} z + \beta^{\text{IV}}, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (16\text{—}19.)$$

the intersections of (15.) with (16—19.) give four equations, of condition similar to those of (12.), (13.), (14.), from which to determine $a_{\prime}\, b_{\prime}\, c_{\prime}$ and $a_{\prime\prime}\, b_{\prime\prime}\, c_{\prime\prime}$, viz.

$$\left(\alpha' - \frac{a_{\prime\prime} c_{\prime} - a_{\prime} c_{\prime\prime}}{c_{\prime\prime} - c_{\prime}}\right)\left(b' - \frac{b_{\prime\prime} - b_{\prime}}{c_{\prime\prime} - c_{\prime}}\right) = \left(\beta' - \frac{b_{\prime\prime} c_{\prime} - b_{\prime} c_{\prime\prime}}{c_{\prime\prime} - c_{\prime}}\right)\left(a' - \frac{a_{\prime\prime} - a_{\prime}}{c_{\prime\prime} - c_{\prime}}\right). \quad . \quad . \quad (20\text{—}24.)$$

These four equations will determine four of the coordinates, as $a_{\prime}\, b_{\prime}$ and $a_{\prime\prime}\, b_{\prime\prime}$, leaving the two others indeterminate. But still the law of force furnishes four other equations from which to determine the two quantities $c_{\prime}$ and $c_{\prime\prime}$; that is, a redundancy of equations, from which redundancy the remaining number may be taken as checks of accurate calculation if the principle be admitted, or as tests of the truth of the principle when we are assured of the accuracy of calculation and of observation.

By this method a greater uniformity of process, and a perfect symmetry in respect to the quantities involved, are obtained; but still the process is very laborious, and it

is probable that the resulting equation will be of a higher degree than really belongs to the problem in its direct form. If so, it will contain foreign factors, which it may be difficult to detect and peculiarize, so as to separate them from the proper solutions of the problem. The method, besides, is not *essentially* different from the last.

Another difficulty also presents itself here; nor is it the only one. The mere intersection of the magnetic axis with the magnetic needle is not a test of the duality in point of number, nor of the equality of intensity in point of force, nor is it confined to any law of variation of force whatever; and hence the mere intersection is not of itself *sufficient* for the determination of the question respecting the duality or the relative intensity of the poles. Still it is one of the *necessary conditions*, though not the *only one*, by which the hypothesis is to be tested; since the poles, being of any number, and of any intensities whatever, *if situated in the same straight line*, will cause the needle to intersect that line, and hence render that phenomenon incapable of determining the number, intensity, or position of the poles; yet wherever this intersection is not fulfilled the duality of the poles cannot be admitted, nor yet the position of the poles, however many they may be, be in one straight line. The determination of $a_{\prime}\, b_{\prime}\, c_{\prime}\, a_{\prime\prime}\, b_{\prime\prime}\, c_{\prime\prime}$ from the equations (20—24.) cannot then be effected completely.

We shall hence proceed in the following manner. A straight line, which constantly touches three given straight lines, but undergoing all the changes compatible with that triple contact, describes the hyperboloid of one sheet. This surface being of the second order, will be cut by a fourth given line in two points; and hence there are two positions which a line resting upon four other lines can take. If, then, we imagine these four lines to be four different positions of the magnetic needle, and the line which rests upon them to be the magnetic axis, we shall perceive at once that in case of any number of poles of any variety of intensity, and acting under any law of variation of force depending upon distance, the magnetic axis can be determined in position from *four* observations of the magnetic needle; and, therefore, of course, in the case which we are examining, where the poles are two, the intensities equal, and the law of force that determined by MICHEL and COULOMB.

Let us take as the equations of the magnetic axis and four of the needles, respectively, the following:

$$\left.\begin{array}{l} x = \bar{a}\ \ z + \bar{\alpha} \text{ and } y = \bar{b}\ \ z + \bar{\beta} \\ x = a'\ z + \alpha' \text{ and } y = b'\ z + \beta' \\ x = a''\ z + \alpha'' \text{ and } y = b''\ z + \beta'' \\ x = a'''\ z + \alpha''' \text{ and } y = b'''\ z + \beta''' \\ x = a^{\text{iv}}\ z + \alpha^{\text{iv}} \text{ and } y = b^{\text{iv}}\ z + \beta^{\text{iv}}. \end{array}\right\} \quad \ldots\ldots\ldots \quad (25\text{—}29.)$$

Then the condition, that the first of these intersects each of the others simultaneously, gives the four equations,

$$(\alpha' - \bar{\alpha})\,(b' - \bar{b}) = (\beta' - \bar{\beta})\,(a' - \bar{a}) \quad \ldots\ldots\ldots \quad (30.)$$

$$(\alpha'' - \overline{\alpha})(b'' - \overline{b}) = (\beta'' - \overline{\beta})(a'' - \overline{a}) \quad . \quad . \quad . \quad . \quad . \quad (31.)$$

$$(\alpha''' - \overline{\alpha})(b''' - \overline{b}) = (\beta''' - \overline{\beta})(a''' - \overline{a}) \quad . \quad . \quad . \quad . \quad . \quad (32.)$$

$$(\alpha^{\text{iv}} - \overline{\alpha})(b^{\text{iv}} - \overline{b}) = (\beta^{\text{iv}} - \overline{\beta})(a^{\text{iv}} - \overline{a}). \quad . \quad . \quad . \quad . \quad . \quad (33.)$$

This reduction is easily effected by subtracting the first from each of the others, in which case we obtain equations of the first degree, giving each of the other three the quantities $\overline{a}\ \overline{b}\ \overline{\alpha}\ \overline{\beta}$ in terms of the fourth, as of $\overline{\alpha}$. These substituted in any one of the four equations give a quadratic equation involving $\overline{\alpha}$; and hence we obtain two values of $\overline{\alpha}$, and hence again of $\overline{b}$, of $\overline{\alpha}$, and of $\overline{\beta}$. We should then obtain, by a simple and direct process, the *equations of the magnetic axis.*

The next inquiry is into the signification of this double result. Are there two magnetic axes which fulfill the condition? If so, are they both occupied by magnets? Or if not, why is one to be selected in preference to the other? Can they both belong to *every* quadruple combination of the magnetic needle?

The last question may be answered at once. If they both belonged to all the combinations of the needles, then they must form two of the directrices of a rule surface, to which the needles themselves were always tangents. The third directrix not being yet fixed, there is no inconsistency in the conclusion thus derived; for the needles are at liberty to rest upon any magnetic surface, whatever be the number or intensity of the poles, or whatever be the parameter which determines the particular stratum of surface which corresponds to the place of observation on the sphere. There is hence nothing to prevent their belonging to every position of the magnetic needle, so far as we at present can discover from the conditions in their arbitrary form. How far this is consistent with the particular data is another question, and will be presently discussed.

There is no *necessity* that they should be both occupied by magnets; and it is at once giving up the duality of the poles, and even their being situated in a right line, to make such an hypothesis. They are both, it is true, solutions of the algebraical problem which we have proposed; but as the algebraical rarely includes *all* the conditions of the physical problem, it is easy to suppose that one of these solutions may be foreign to the inquiry, without violating our knowledge of the nature of the relations subsisting between the algebraical and physical problem. To prove that it actually *is* a foreign result must be subsequent to the determination of the particular values of the coefficients of the quantities involved in the inquiry. All we can say at present is, that there are two axes which fulfill the algebraical condition; but as there is only one which enters into the physical hypotheses, one of these two algebraical axes must be rejected. We cannot, however, ascertain which, except by other conditions than have yet been taken into the formula. For the present, then, we can only compute them both, and take that which best answers to those other physical conditions of which the algebraical problem has taken no account.

VII.—Having obtained the equations of the magnetic axis by the determination of $\overline{a}\ \overline{b}\ \overline{\alpha}\ \overline{\beta}$, we may put the rectilinearity of the positions of the poles to the test at once. For if we take a fifth needle

$$x = a^{v} z + \alpha^{v} \text{ and } y = b^{v} z + \beta^{v} \quad \ldots \ldots \ldots \quad (34.)$$

and combine it with (29.), viz.

$$x = \overline{a}\, z + \overline{\alpha} \text{ and } y = \overline{b}\, z + \overline{\beta},$$

so as to ascertain whether they intersect or not, by means of the equation of condition

$$(\alpha^{v} - \overline{\alpha})\,(b^{v} - \overline{b}) = (\beta^{v} - \overline{\beta})\,(a^{v} - a) \quad \ldots \ldots \quad (35.)$$

The approximation to a fulfilment of this condition for $\alpha^{v}\ a^{v}$, $b^{v}\ \beta^{v}$, derived from all the other observations upon which reliance can be placed, and to the extent of the probable accuracy of those observations, will establish this hypothesis to the same extent.

In order to estimate the real amount of the error in the application of this equation of condition, we must recollect that the formula itself is derived from that which gives the shortest distance between two given lines, viz.

$$\text{D (dist.)} = \pm \frac{(\alpha^{v} - \overline{\alpha})\,(b^{v} - \overline{b}) - (\beta^{v} - \overline{\beta})\,(a^{v} - \overline{a})}{\sqrt{(a^{v} - a)^{2} + (b^{v} - b)^{2} + (a\, b^{v} - a^{v}\, b)^{2}}}. \quad \ldots \ldots \quad (36.)$$

Hence, in order to estimate the number of miles which the needle would be from fulfilling the condition, we must calculate the denominator of the fraction (36.), and divide the result of (35.) by it. By then calculating the angle which D would subtend at the place of observation, we shall be in some degree prepared to judge whether such an error might possibly have arisen from unskilful observation, the imperfect structure of the instrument, from any probable geological or meteorological causes, or from any temporary local disturbance. Or, conversely, were this line satisfactorily determined by a great number of tests, and the instrument and observer well prepared for the task, then we should be able in some degree to estimate the amount of the disturbing forces that climate, geological structure, and local attraction do actually exert at that place, and perhaps in some cases also to form a probable conjecture respecting the separate contribution of each of these causes to the total amount of the disturbance.

VIII.—Let us now suppose the magnetic axis satisfactorily determined and tested; and proceed to inquire whether the poles be two or more, and whether equal or unequal in the intensity of their action: and in the first place we shall suppose the intensities equal.

Recurring to the figure in (III.) and the conditions that are tabulated, as (5.), (6.), (7.), we have the quantities designated as a, b, c, A, B, C, f, g, h, and the corresponding ones for any number of points $M_{\prime}$ $N_{\prime}$ $P_{\prime}$ $Q_{\prime\prime}$, &c. from actual observation and the calculated equation of the magnetic axis. From any two of these, as (5.), (6.), we then shall be able to compute t and u. The remaining equations, whatever be their

number, will serve as tests of the truth of the dual hypothesis with the poles of equal intensity.

It will be the most convenient method of taking the point O, to assume for it the intersection of the magnetic axis by the perpendicular from the centre of the sphere. Having, then, the distances of the poles from this point, and the equations of the line in which they lie, we can easily determine their coordinates, the great object of our inquiry.

It can be no objection to this process that it necessarily requires the solution of equations of a high order, since it is only the solution of it in the case of given numeral coefficients, and not with literal coefficients. The method of effecting these solutions with rapidity and precision is now well known, and need not here be dwelt upon. We shall have occasion hereafter to employ them in the numerical solution of this special problem*.

IX.—If we suppose the intensities unequal, we can assume their ratios to be that of $F_{,}$ to $F_{,,}$, or R. The relation upon which (5.), (6.), (7.),... are founded no longer holds good in this case. Nevertheless, by reference to (XIV.) we see that the difference is only in the numeral coefficients of the equations, and not in the form or the number of terms, or in any circumstance that alters in the slightest degree the labour or the difficulty of the actual solution. We have however, in consequence of the new quantity R which is thus introduced, to employ one equation more derived from observation†, and one only. Hence still in this case, too, four observations are sufficient, not only for the determination of the actual position of the poles, but also to furnish a test of the accuracy of the hypothesis. As a method, then, this also is complete, and the problem is fully brought within the reach of known and familiar operations.

X.—As a specimen of the method of computing the equations of the magnetic needle, I have given calculations for Chamisso Island, Valparaiso, Paramatta, Port Bowen, Paris, and Boat Island; and that the whole process may be distinctly seen, I have also given the equations of the magnetic axis itself as deduced from the equations of the first four needles, and a comparison of the result with the Paris needle. That result is not very favourable to the theory, provided the observations themselves are considered trustworthy. But since those philosophers who have had most experience in the use of magnetical instruments, and especially of the dipping-needle, are most strongly convinced that there are errors attached to all our present *instruments* and *modes of observation* whose amount vitiates any result obtained by them,

* I refer, of course, to Mr. HORNER's method, published in the Philosophical Transactions for 1819, and in the fifth volume of Professor LEYBOURN's Mathematical Repository. It is unnecessary to add, that all the *effective* methods of solution of algebraical equations that have since appeared have been but imitations of Mr. HORNER's, however much the *notation* and *form* of the reasoning employed in them may differ from his.

† Or if we seek to determine the actual value of $F_{,}$ and $F_{,,}$ we shall want all the four complete observations.

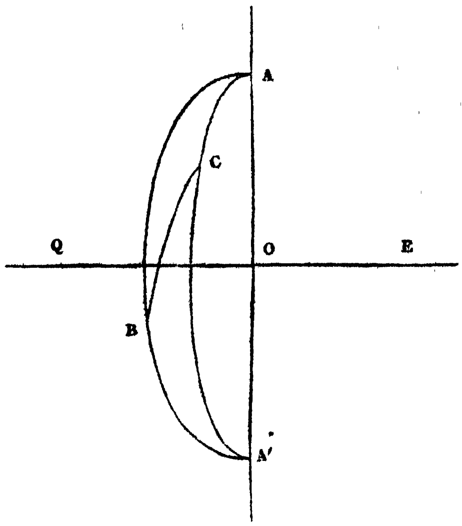
I have not thought it necessary to add any further discussion of the question in the present stage of my investigations, in as much as till the results can be ensured as unaffected by extraneous sources of error, all methods must alike be useless, since they are alike dependent upon data that are at least unsatisfactory if not erroneous. There is some reason, however, to hope, since the attention of the scientific world is now so intensely turned to researches of this nature, that there will at length be discovered some methods of observing which shall be free from this class of errors. However, till this is done, it would be useless to attempt the discussion of the present or any other method of mathematical investigation into its numerical details: and the utmost we can now perform is to lay down *methods of investigation* by which, when satisfactory experimental data are obtained, the question may be brought to a decisive test at once.

I should also state here, that in consequence of the great labour attending the calculations of the axes, I have been led to examine the method of construction by descriptive geometry, (especially on account of the facility and the considerable degree of certainty which may be attached to its solutions,) of the problem of *describing a line which shall rest upon four given lines.* In any case where the data are so uncertain as the present, such a method is sufficiently accurate, since in very few cases will the errors of construction be probably near so great as the errors in the data themselves. The geometrical problem itself is not in practice so simple as could be desired, at least by any method yet made public, but still it offers far greater facilities than the algebraical one. It has, moreover, one important advantage which the algebraical has not, viz. the ready and visual exhibition of those cases which are unfit for this method, or those in which a small error in the data will greatly increase in the result. It should hence be always used before the algebraical. No doubt the algebraical method may be rendered subservient to the same purpose, but then the method is intolerably operose, and hence, practically, almost useless.

By employing such constructions, I find that there is a greater degree of approximation in the few magnetic axes which I have determined by them from existing data than appears compatible with any other theory of the constitution of the terrestrial magnet than that which considers the magnetic force situated in two isolated centres or poles. By this I would not be understood to say that the approximation is close, but simply that, in comparison with all the positions which lines may take, there seems to be *one region* of space, in reference to the coordinate planes or planes of projection, into which they dispose themselves, but dispose themselves very irregularly in it.

As I expect to be favoured, by the kindness of my distinguished colleague Mr. Christie, with the results of the observations of the late lamented Captain Foster, I shall probably resume this branch of the subject at an early period; and hence any further details respecting these constructions which may appear necessary will be with more propriety included in a future than in the present paper.

The equations of the magnetic needle for different places, in reference to rectangular coordinates, when the geographical coordinates, dip, and variation are given.



Let A O A′ be the meridian of Greenwich, E O Q the equator, C the position of a place where a magnetic observation is made, A C B the variation, and B C equal to twice the dip of the needle. Then B is the point on the surface of the earth towards which the dipping-needle is directed, and that in which the straight line which coincides with the needle intersects the earth a second time.

Estimating (as is done in my paper on Spherical Loci before referred to) the positions of places on the surface of the earth by means of the polar angle C A O and radius-vector C A, we have the coordinates of C directly from observation; and by means of the triangle A C B, whose sides B C, C A, and included angle A C B are given, we can compute the coordinates of B. Denote the polar distance and polar angle of C by $\alpha_{\prime}$ $\beta_{\prime}$, and those of B by $\alpha_{\prime\prime}$ $\beta_{\prime\prime}$.

In the next place, by the employment of equations (1.), (2.) of II., we may obtain the equations of the needle, referred to three rectangular axes, the coordinate planes of which are the meridian of Greenwich, the meridian of $\pm 90°$, and the equator. The results for the six different places before mentioned are given in the last column of the following Table. The construction of the Table itself is indicated at the head of each column, in a way that renders further explanation unnecessary.

Place, Date, Observer, and Authority.	Geographical coordinates of the place.	Observed position of the needle.	Spherical coordinates of the place of observation.	Spherical coordinates of the second intersection.	Rectangular equations of the needle.
Port Bowen. 1824. Capt^s^ PARRY & FOSTER. Phil. Trans. 1826.	Lat. 73° 14′ 0″ N. Long. 88 54 0 W.	Dip. 81° 1′ 24″ N. Var. 124 0 0 W.	$\alpha_{\prime} =$ 16° 46′ 0″ $\beta_{\prime} = -$ 88 54 0	$\alpha_{\prime\prime} =$ 165° 25′ 32″ $\beta_{\prime\prime} = -$ 256 3 55	$x = -\ {\cdot}278746\,z - {\cdot}023702\,r$ $y = {\cdot}034611\,z - {\cdot}027756\,r$
Boat Island. 1821. Captain PARRY. Voyage, I.	Lat. 68° 59′ 13″ N. Long. 53 12 56 W.	Dip. 82° 53′ 40″ N. Var. 72 2 0 W.	$\alpha_{\prime} =$ 21° 0′ 47″ $\beta_{\prime} = -$ 53 12 56	$\alpha_{\prime\prime} =$ 151° 5′ 36″ $\beta_{\prime\prime} = =$ 204 45 57	$x = -\ {\cdot}270447\,z - {\cdot}034011\,r$ $y = {\cdot}122598\,z - {\cdot}122598\,r$
Chamisso Island. 1827. Captain BEECHEY. Voyage, App. p. 732.	Lat. 66° 12′ 0″ N. Long. 161 46 0 W.	Dip. 77° 39′ 0″ N. Var. 32 0 30 W.	$\alpha_{\prime} =$ 23° 48′ 0″ $\beta_{\prime} = -$ 161 46 0	$\alpha_{\prime\prime} =$ 133° 34′ 36″ $\beta_{\prime\prime} =$ 0 27 25	$x = -\ {\cdot}081565\,z - {\cdot}051635\,r$ $y = -\ {\cdot}690471\,z + {\cdot}248470\,r$
Paris. 1829. M. ARAGO. Annuaire, 1830.	Lat. 48° 50′ 0″ N. Long. 2 20 0 E.	Dip. 67° 41′ 18″ N. Var. 22 12 5 W.	$\alpha_{\prime} =$ 41° 10′ 0″ $\beta_{\prime} =$ 2 20 0	$\alpha_{\prime\prime} =$ 96° 10′ 34″ $\beta_{\prime\prime} = -$ 162 10 56	$x = {\cdot}384722\,z - {\cdot}262819\,r$ $y = 1{\cdot}864500\,z - {\cdot}745899\,r$
Valparaiso. 1821. Captain BASIL HALL. "Magnetism," Enc.Met.	Lat. 33° 1′ 0″ S. Long. 288 29 0 E.	Dip. 38° 46′ 0″ S. Var. 14 43 0 E.	$\alpha_{\prime} =$ 123° 1′ 0″ $\beta_{\prime} =$ 288 29 0	$\alpha_{\prime\prime} =$ 156° 58′ 20″ $\beta_{\prime\prime} =$ 85 20 23	$x = -\ 3{\cdot}159630\,z - 2{\cdot}518180\,r$ $y -\ {\cdot}621395\,z + {\cdot}206769\,r$
Paramatta. 1823. Sir THOMAS BRISBANE. Phil. Trans. 1829, pt. 3.	Lat. 33° 48′ 50″ S. Long. 151 1 34 E.	Dip. 62° 57′ 0″ S. Var. 8 47 41 E.	$\alpha_{\prime} =$ 123° 48′ 50″ $\beta_{\prime} =$ 151 1 34	$\alpha_{\prime\prime} =$ 109° 46′ 44″ $\beta_{\prime\prime} = -$ 21 23 25	$x = -\ 3{\cdot}418875\,z - 1{\cdot}500120\,r$ $y = 7{\cdot}349865\,z + 3{\cdot}363313\,r$

XII.—We may now proceed to compute the equations of that straight line (or lines, for there are two, but determined by the same series of processes,) which rests upon any four of lines determined as those in the last section were found. Thus we shall take as an instance Chamisso, Valparaiso, Paramatta, and Port Bowen, the equations of which are given in the Table.

Assume as the equations of the magnetic axis the two following:

$$x = a z + \alpha,$$
$$y = b z + \beta,$$

and denote the several equations of the needles by equations of the same form, but with the constants accented, viz. $a' \, b'$, $\alpha' \beta'$: then the conditions of intersection will, in each case, be expressed by the equation

$$(\alpha' - \alpha)(b' - b) - (\beta' - \beta)(a' - a) = 0.$$

The insertions of the actual values in the four cases mentioned above being made, and the vincula thrown open, we obtain

$$\begin{aligned}
&\cdot055920 + \cdot690471\,\alpha - \cdot081565\,\beta + \cdot248470\,a + \cdot051635\,b + \alpha b - a\beta = 0,\\
-&\cdot911471 - \cdot621395\,\alpha - 3\cdot159630\,\beta + \cdot206769\,a + 2\cdot518180\,b + \alpha b - a\beta = 0,\\
&\cdot473095 - 7\cdot349865\,\alpha - 3\cdot418875\,\beta + 3\cdot363313\,a + 1\cdot500120\,b + \alpha b - a\beta = 0,\\
-&\cdot008188 - \cdot034611\,\alpha - \cdot278746\,\beta - \cdot027756\,a + \cdot023702\,b + \alpha b - a\beta = 0.
\end{aligned}$$

Subtracting each of the last three of these from the first, we obtain

$$\begin{aligned}
b &= \cdot391938 + \cdot531872\,\alpha + 1\cdot247954\,\beta + \cdot017024\,a,\\
b &= -\cdot288008 + 3\cdot550860\,\alpha + 2\cdot304000\,\beta - 2\cdot150420\,a,\\
b &= -2\cdot295058 - 25\cdot957900\,\alpha - 7\cdot059053\,\beta - 9\cdot888800\,a.
\end{aligned}$$

Subtract the two latter from the former of these, then there result

$$a = -\cdot271254 - 2\cdot674164\,\alpha - \cdot838603\,\beta, \text{ and}$$
$$a = -\cdot313706 + 1\cdot392881\,\alpha + \cdot487231\,\beta.$$

From equating which values of a we obtain $\beta = \cdot032019 - 3\cdot067660\,\alpha$.

Insert this value of β in that of a, and we find $a = -\cdot298105 - \cdot101755\,\alpha$.

In a similar manner, from the proper substitutions, $b = \cdot426822 - 3\cdot288083\,\alpha$.

And inserting all these values in the first of the four equations, that of the Chamisso needle, we obtain

$$\alpha^2 - \cdot071009\,\alpha = \cdot003821, \text{ and hence } \alpha = \cdot035505 \pm \cdot071274.$$

The two pairs of equations which result from this calculation, then, as those of the magnetic axis, are

$$\left.\begin{aligned} x &= -\cdot308970\,z + \cdot106779\,r,\\ y &= \cdot075732\,z - \cdot295538\,r; \end{aligned}\right| \text{ and } \begin{aligned} x &= -\cdot294465\,z - \cdot035769\,r,\\ y &= \cdot544430\,z + \cdot141744\,r; \end{aligned}$$

according as the + or − sign is taken in the valuation of α.

In the same way we may proceed to find the magnetic axis which would accord with any other four observations, and by a comparison of these ascertain whether the

discrepancies were such as to admit of account from the errors of observation and the imperfection of instruments. However, it is much simpler to ascertain whether the axis thus determined agrees with the observation made at a fifth or a sixth place. Let us take the Parisian needle as an instance.

By recurring to equation (36.), and putting the values of $\overline{a}\,\overline{b}$ and $\overline{\alpha}\,\overline{\beta}$ just determined and those of $a^v\, b^v$ and $\alpha^v\, \beta^v$ found in the table of section XI. for Paris, we have the least distance between the Parisian needle and the magnetic axis either $\cdot173341\, r$ or $\cdot189540\, r$, according as the $+$ or $-$ sign above mentioned is employed.

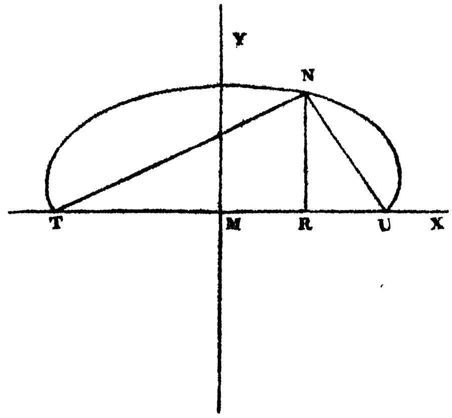
These are between a fifth and a sixth part of the terrestrial radius. We may now, were it necessary, seek the coordinates of the points in which the line of shortest distance intersects the two different magnetic axes and the Paris needle, and thence the equations of the lines drawn from those points to Paris, and thence again the angle formed by the Paris needle, and each of the other lines: that is, we should find the error of observation in the Paris needle if we suppose the magnetic axis correctly determined from the other four needles. But it is unnecessary to go through the computations, as it is easy to see that this angle will not be very different from (but its difference, whatever it be, will be greater than it,) $\tan^{-1} \cdot173341$, and $\tan^{-1} \cdot189540$, that is, about 10° or 11°. The discrepancies in every other case that I have tried are as great as, and in most of them still greater than, in that just examined. The further prosecution of this branch of the inquiry, with our present data, must therefore be abandoned.

XIII.—So far as *method* is concerned, the previous processes are perfectly adapted to decide the question of the duality and the equality of intensity of the magnetic poles. In the absence, however, of data upon which full reliance can be placed, we are not at present able to apply that method to the actual circumstances of the earth. It hence becomes desirable to examine certain other phenomena, to ascertain whether, in their general bearings and character, they also are compatible with the hypothesis of two such centres of force. These are:—the points at which the needle becomes horizontal, constituting the magnetic equator; the points at which the needle becomes vertical; the curves of equal dip; the Halleyan lines, or curves of equal variation; the lines of equal magnetic intensity, or isodynamic lines of Hansteen; and those particular cases where the isodynamic line is reduced to a point, which constitutes the "pole" in the language of Hansteen and a considerable number of modern writers. The first two of these I shall examine in the present, and the remaining ones in a subsequent paper. I then propose to enter into a minute numerical discussion of such observations as I may be able to obtain in the interim; and of course attempt to ascertain how far any one may be vitiated by instrumental or local circumstances, and how far the geometrical peculiarity of the observation itself may render it unfit for our present method of investigation.

I do not propose this course without being fully sensible of the difficulty and labour it entails upon me; but at the same time I feel perfectly assured that any

assistance which it is in the power of men of science to afford will be freely offered me; and especially in furnishing such observations as they themselves place most reliance on, together with the circumstances under which the observations were made.

XIV.—*On the Magnetic Curve.*



Let $F_{\prime}$ and $F_{\prime\prime}$ denote the intensities of the forces situated in the two poles T, U; and β the angle which the needle, subjected to the action of those forces and situated in a given point N $(x\,y)$, would make with the axis of (x) the magnet itself. Let also

$$r_{\prime}^2 = y^2 + (x+a)^2$$
$$r_{\prime\prime}^2 = y^2 + (x-a)^2.$$

Then the usual considerations give us

$$\frac{\frac{F_{\prime}y}{r_{\prime}^3} + \frac{F_{\prime\prime}y}{r_{\prime\prime}^3}}{\frac{F_{\prime}(x+a)}{r_{\prime}^3} + \frac{F_{\prime\prime}(x-a)}{r_{\prime\prime}^3}} = \tan\beta. \quad (37.)$$

But if we represent $\tan\beta$ by $\frac{dy}{dx}$, we shall have the differential equation of the curve to which the needle will be a tangent, and which passes through that point, $x\,y$. To find the equation of the curve itself it only remains then to integrate

$$\frac{\frac{F_{\prime}y}{r_{\prime}^3} + \frac{F_{\prime\prime}y}{r_{\prime\prime}^3}}{\frac{F_{\prime}(x+a)}{r_{\prime}^3} + \frac{F_{\prime\prime}(x-a)}{r_{\prime\prime}^3}} = \frac{dy}{dx}. \quad (38.)$$

Multiply the numerators of all the terms of the first side by y, and also multiply out the denominators; then there will result

$$\frac{F_{\prime}y^2\,dx - F_{\prime}y\,(x+a)\,dy}{r_{\prime}^3} + \frac{F_{\prime\prime}y^2\,dx - F_{\prime\prime}y\,(x-a)\,dy}{r_{\prime\prime}^3} = 0.$$

In the former of these numerators add and subtract $F_{\prime}(x+a)^2\,dx$, and in the latter $F_{\prime\prime}(x-a)^2\,dx$; then we shall obtain another form, which is immediately integrable. It is

$$\frac{F_{\prime}\{y^2+(x+a)^2\}\,dx - F_{\prime}(x+a)\,\{y\,dy+(x+a)^2\,dx\}}{r_{\prime}^3}$$
$$+\frac{F_{\prime\prime}\{y^2+(x-a)^2\}\,dx - F_{\prime\prime}(x-a)\,\{y\,dy+(x-a)\,dx\}}{r_{\prime\prime}^3} = 0;$$

the integral of which is

$$\frac{F_{\prime}(x+a)}{r_{\prime}} + \frac{F_{\prime\prime}(x-a)}{r_{\prime\prime}} = \frac{c}{a}, \quad (39.)$$

in which $\frac{c}{a}$ is the arbitrary constant which particularizes the individual curve we

may have occasion to consider. It is determinable from any one condition; as passing through a given point, touching a given line, &c.

But $\frac{x+a}{r_{\prime}}$ and $\frac{x-a}{r_{\prime\prime}}$ are the cosines of the angles which the directions of the component forces make with the axis of x, that is, with the magnetic axis; and hence, denoting these by $\theta_{\prime}$, $\theta_{\prime\prime}$, (being estimated from the same branch of the axis,) we shall have

$$F_{\prime}\cos\theta_{\prime} + F_{\prime\prime}\cos\theta_{\prime\prime} = 2\cos\beta', \quad \ldots \ldots \quad (40.)$$

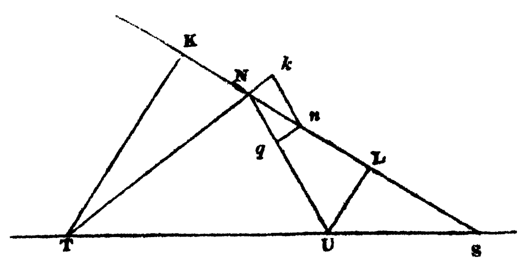
where $\beta' = \cos^{-1}\frac{c}{2a}$.

This equation having the values of $r_{\prime}$, $r_{\prime\prime}$ and β restored, becomes

$$\frac{F_{\prime}(x+a)}{\sqrt{y^2+(x+a)^2}} + \frac{F_{\prime\prime}(x-a)}{\sqrt{y^2+(x-a)^2}} = \frac{c}{2a}, \quad \ldots \ldots \quad (41.)$$

which, when deprived of its radicals and denominators, is of the *eighth degree*.

We might, however, obtain this property in a different manner; and as it also facilitates the investigation of one or two other theorems which we shall require hereafter, it may with propriety be added here.



Let T and U be the poles, N the centre of the needle. Let, as before, the forces be $F_{\prime}$ and $F_{\prime\prime}$; let T N S and U N S, the angles made by each of the component forces and the resultant one, be called $\Delta_{\prime}$ and $\Delta_{\prime\prime}$ respectively. Let $r_{\prime}$ and $r_{\prime\prime}$ be the distances T N, N U, and $\theta_{\prime}$, $\theta_{\prime\prime}$ the angles N T S, N U S; and take N k : N q :: $\frac{F_{\prime}}{r_{\prime}^2} : \frac{F_{\prime\prime}}{r_{\prime\prime}^2}$, and complete the parallelogram N $k n q$. Then N n is the position of the needle. Produce it to meet the axis T U in S; and draw the perpendiculars T K and U L upon N S. Then by the composition of forces

$$\frac{\sin\Delta_{\prime}}{\sin\Delta_{\prime\prime}}\left[=\frac{\sin p\,\mathrm{N}\,n}{\sin q\,\mathrm{N}\,n}\right] = \frac{\mathrm{N}p}{\mathrm{N}q} = \frac{\frac{F_{\prime\prime}}{r_{\prime\prime}^2}}{\frac{F_{\prime}}{r_{\prime}^2}} = \frac{F_{\prime\prime}}{F_{\prime}}\cdot\frac{r_{\prime}^2}{r_{\prime\prime}^2}. \quad \ldots \ldots \quad (42.)$$

Denote now the angle N S T by Σ, and T S, U S by t and u respectively. Then

$$\frac{\sin\Delta_{\prime}}{\sin\Delta_{\prime\prime}} = \frac{\frac{t}{r_{\prime}}\sin\Sigma}{\frac{u}{r_{\prime\prime}}\sin\Sigma} = \frac{t\,r_{\prime\prime}}{u\,r_{\prime}}; \quad \ldots \ldots \quad (43.)$$

or by comparing (42.) and (43.), we obtain,

$$\frac{t}{u} = \frac{F_{\prime\prime}}{F_{\prime}}\cdot\frac{r_{\prime}^3}{r_{\prime\prime}^3}, \quad \ldots \ldots \quad (44.)$$

a relation which, when $F_{\prime} \pm F_{\prime\prime} = 0$, is already known*.

* Vide LESLIE's Geometrical Analysis, art. "Magnetic Curves"; or Mr. BARLOW's Treatise on Magnetism in the Encyclopædia Metropolitana, p. 794.

Represent now the perpendiculars by $p_{,}$ and $p_{,,}$, and then, by the common formula for the inclination of the tangent to the radius-vector, we have

$$p_{,} = \frac{r_{,}^{2} d\theta_{,}}{ds} \text{ and } p_{,,} = \frac{r_{,,}^{2} d\theta_{,,}}{ds}.$$

Hence

$$\frac{p_{,}}{p_{,,}} = -\frac{r_{,}^{2} d\theta_{,}}{r_{,,}^{2} d\theta_{,,}} = \frac{t}{u} = \frac{F_{,,} r_{,}^{3}}{F_{,} r_{,,}^{3}},$$

or

$$F_{,} r_{,,} d\theta_{,} + F_{,,} r_{,} d\theta_{,,} = 0;$$

or again, since $\frac{r_{,}}{r_{,,}} = \frac{\sin\theta_{,}}{\sin\theta_{,,}}$, it becomes finally

$$F_{,} \sin\theta_{,} d\theta_{,} + F_{,,} \sin\theta_{,,} d\theta_{,,} = 0,$$

the integral of which, as before, is

$$F_{,} \cos\theta_{,} + F_{,,} \cos\theta_{,,} = 2\cos\beta'.$$

If now we take $F_{,} + F_{,,} = 0$, or $F_{,} = -F_{,,}$, we shall have our formula simplified, at the same time that we adopt the hypothesis which seems best to accord with all we yet know of the disposition of the forces in the artificial and also in a natural magnet; and hence it is the most appropriate assumption we can make respecting the constitution of the terrestrial magnet itself. We shall thus have

$$\cos\theta_{,} - \cos\theta_{,,} = 2\cos\beta,$$

where $\cos\beta = F_{,} \cos\beta$; or, since one of these angles is *external* to the triangle N T U formed by the magnet and its polar distances from N, we may substitute instead of $\theta_{,,}$ its supplement, and then the last equation will take the form

$$\cos\theta_{,} + \cos\theta_{,,} = 2\cos\beta. \quad \ldots\ldots\ldots\ldots \quad (45.)$$

This property was originally given by Professor PLAYFAIR in Professor ROBISON'S article "Magnetism," published in the First Supplement to the Encyclopædia Britannica. See also ROBISON'S Mechanical Philosophy, vol. iv. p. 350. Professor ROBISON, from one or two passages in his writings, seems to have entertained some idea that these curves could be rendered available to an explanation of the phenomena of terrestrial magnetism; but I do not recollect that either he or any one else has suggested *how* this was to be accomplished, nor, much less, attempted to actually accomplish it by such means.

XV.—If, still considering the axis of the magnet as the axis of x, we conceive the magnetic curve to revolve about that axis so as to describe a surface of revolution, its equation obviously is obtained by putting $y^2 + z^2$ instead of y^2 in the equation of the generating curve. That is, the magnetic surface is expressed by the equation

$$\frac{x+a}{\sqrt{y^2 + z^2 + (x+a)^2}} - \frac{x-a}{\sqrt{y^2 + z^2 + (x-a)^2}} = \frac{c}{a}. \quad \ldots\ldots \quad (46.)$$

In this equation of the magnetic surface, a, or half the length of the magnet, is constant, and c is arbitrary, giving different curves according as the value of that parameter is varied.

Again, since we can refer this surface, or any one of its meridional sections, to any new system of coordinates, we may conceive the last equation (46.) to be so transformed as to represent the same geometrical surface that it now does whilst it is referred to the centre of the earth as the origin, and to the mutual intersections of the equator and two rectangular meridians as axes of coordinates. This transformation, however, by the usual processes, would be extremely difficult—perhaps impracticable—on account of the labour it would require; but this labour may be almost wholly avoided by means of the property (PLAYFAIR's) of the curve referred to the polar angles expressed in equation (45.).

Let the coordinates of the poles T and U referred to the above-named axes be $a_{\prime} b_{\prime} c_{\prime}$ and $a_{\prime\prime} b_{\prime\prime} c_{\prime\prime}$ respectively; and the individual curve defined by the parameter β. Then viewing $\theta_{\prime}$ and $\theta_{\prime\prime}$ as the two *internal* angles, we shall have

$$\cos\theta_{\prime}=\frac{(x-a_{\prime})^2+(y-b_{\prime})^2+(z-c_{\prime})^2+(a_{\prime}-a_{\prime\prime})^2+(b_{\prime}-b_{\prime\prime})^2+(c_{\prime}-c_{\prime\prime})^2-(x-a_{\prime\prime})^2-(y-b_{\prime\prime})^2-(z-c_{\prime\prime})^2}{2\sqrt{\{(a_{\prime}-a_{\prime\prime})^2+(b_{\prime}-b_{\prime\prime})^2+(c_{\prime}-c_{\prime\prime})^2\}\times\{(x-a_{\prime})^2+(y-b_{\prime})^2+(z-c_{\prime})^2\}}}$$

$$=\frac{(a_{\prime}-x)(a_{\prime}-a_{\prime\prime})+(b_{\prime}-y)(b_{\prime}-b_{\prime\prime})+(c_{\prime}-z)(c_{\prime}-c_{\prime\prime})}{\sqrt{(a_{\prime}-a_{\prime\prime})^2+(b_{\prime}-b_{\prime\prime})^2+(c_{\prime}-c_{\prime\prime})^2}\times\sqrt{(x-a_{\prime})^2+(y-b_{\prime})^2+(z-c_{\prime})^2}} \quad . \; . \; . \; . \; . \quad (47.)$$

and

$$\cos\theta_{\prime\prime}=-\frac{(a_{\prime\prime}-x)(a_{\prime}-a_{\prime\prime})+(b_{\prime\prime}-y)(b_{\prime}-b_{\prime\prime})+(c_{\prime\prime}-z)(c_{\prime}-c_{\prime\prime})}{\sqrt{(a_{\prime}-a_{\prime\prime})^2+(b_{\prime}-b_{\prime\prime})^2+(c_{\prime}-c_{\prime\prime})^2}\times\sqrt{(x-a_{\prime\prime})^2+(y-b_{\prime\prime})^2+(z-c_{\prime\prime})^2}}. \quad . \; . \; . \quad (48.)$$

From (4.), (12.), and (13.), we have at once the equation of the surface, viz.

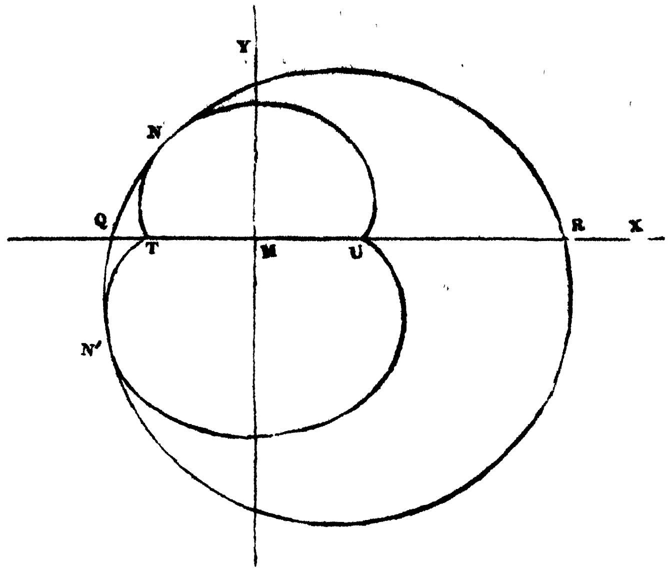
$$\left.\begin{aligned}&\frac{(x-a_{\prime})(a_{\prime}-a_{\prime\prime})+(y-b_{\prime})(b_{\prime}-b_{\prime\prime})+(z-c_{\prime})(c_{\prime}-c_{\prime\prime})}{\sqrt{(x-a_{\prime})^2+(y-b_{\prime})^2+(z-c_{\prime})^2}}\\&-\frac{(x-a_{\prime\prime})(a_{\prime}-a_{\prime\prime})+(y-b_{\prime\prime})(b_{\prime}-b_{\prime\prime})+(z-c_{\prime\prime})(c_{\prime}-c_{\prime\prime})}{\sqrt{(x-a_{\prime\prime})^2+(y-b_{\prime\prime})^2+(z-c_{\prime\prime})^2}}\\&=\pm 2\cos\beta\sqrt{(a_{\prime}-a_{\prime\prime})^2+(b_{\prime}-b_{\prime\prime})^2+(c_{\prime}-c_{\prime\prime})^2}=\pm 4a\cos\beta\end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (49.)$$

This equation, deprived of its radicals and denominators, like the equation of the generating curve, is of the *eighth order*.

Now by varying the parameter β (which defines the particular surface upon which the point we are considering is situated,) by minute increments, we shall have a series of thin strata, each of which is isolated and independent, and which, collectively, extend through all space. If, therefore, we conceive these strata to be infinitesimally thin, we may consider the magnetic influence extended over a series of surfaces, each of which has the property of being *touched* by a small needle placed at any point in that surface. If, moreover, we conceive the sphere and each of these surfaces to be cut by a series of meridian planes passing through the magnetic axis, we shall always find some one magnetic curve on each side of the magnetic axis which will touch the two segments into which the magnetic axis (prolonged if necessary) divides the circle lying in that magnetic meridian plane. At these points the needle will touch the sphere, or, which is the same thing, it will be horizontal to the earth; and

as there will be such points in each of the consecutive meridional sections, there will be a series of consecutive points on the surface of the earth at which the needle will be horizontal. These points lie in a continuous curve, which has been called the *magnetic equator*; and we proceed to inquire into the character of that curve.

XVI.—*On the Magnetic Equator.*



Let T U, as before, be the poles of the terrestrial magnet, and TNU that one of the magnetic curves which touches the corresponding magnetic meridian of the earth QNR in N.

Put for the moment the equation of the circle QNR under the general form

$$(x - c)^2 + (y - b)^2 = r^2; \quad . \; . \quad (50.)$$

and denoting by $g_{\prime}0$ and $g_{\prime\prime}0$ the coordinates of Q and R respectively, we shall find

$$c = \frac{g_{\prime} + g_{\prime\prime}}{2}, \text{ and } r^2 = b^2 + \left(\frac{g_{\prime} - g_{\prime\prime}}{2}\right)^2. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (51.)$$

Inserting (51.) in (50.), and reducing the equation, the circle is finally expressed by

$$x^2 - \overline{g_{\prime} + g_{\prime\prime}}\, x + g_{\prime} g_{\prime\prime} + y^2 - 2\,b\,y = 0. \quad . \; . \; . \; . \; . \; . \; . \quad (52.)$$

This involves the arbitrary quantity b, which is the parameter upon which the identical circle depends.

Since this circle is to *touch* the magnetic curve at some point N, the values of $\frac{dy}{dx}$ derived from the equations of the circle and magnetic curve at their common point $(x\,y)$ must be equal; and as the arbitrary constant in the equation of the magnetic curve vanishes by differentiation, we shall have three equations between which to eliminate the indeterminate quantities $\frac{dy}{dx}$ and b. This elimination will leave one equation between x and y, which will designate the locus of the point N.

By differentiating (52.) we obtain

$$\frac{dy}{dx} = -\frac{2\,x - \overline{g_{\prime} + g_{\prime\prime}}}{2\,(y - b)}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (53.)$$

And the differential equation of the magnetic curve is, from (38.) and $F_{\prime} + F_{\prime\prime} = 0$,

$$\frac{dy}{dx} = \frac{\frac{y}{r_{\prime}^3} - \frac{y}{r_{\prime\prime}^3}}{\frac{x+a}{r_{\prime}^3} - \frac{x-a}{r_{\prime\prime}^3}}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (54.)$$

Also, from (52.),

$$2(y-b)=\frac{y^2-x^2+\overline{g_{\prime}+g_{\prime\prime}}\,x-g_{\prime}g_{\prime\prime}}{y} \quad \ldots \ldots \quad (55.)$$

Insert (55.) in (53.); then there results, after combining (53.) and (54.) and slightly reducing

$$\frac{(r_{\prime\prime}^3-r_{\prime}^3)\,y}{(x+a)\,r_{\prime\prime}^3-(x-a)\,r_{\prime}^3}=\frac{(\overline{g_{\prime}+g_{\prime\prime}}-2\,x)\,y}{y^2-x^2+\overline{g_{\prime}+g_{\prime\prime}}\,x-g_{\prime}g_{\prime\prime}}, \quad \ldots \quad (56.)$$

which is a rectangular equation to the curve which is traced out by N. It separates at once into the two components

$$y=0 \quad \ldots \ldots \quad (57.)$$

$$\frac{r_{\prime\prime}^3-r_{\prime}^3}{(x+a)\,r_{\prime\prime}^3-(x-a)\,r_{\prime}^3}=\frac{\overline{y_{\prime}+g_{\prime\prime}}-2\,x}{y^2-x^2+\overline{g_{\prime}+g_{\prime\prime}}\,x-g_{\prime}g_{\prime\prime}}, \quad \ldots \quad (58.)$$

which last is readily reduced to

$$\{y^2+(x+a)^2-\overline{a+g_{\prime}}\,.\,\overline{a+g_{\prime}}\}\,r_{\prime\prime}^3=\{y^2+(x-a)^2-\overline{a-g_{\prime}}\,.\,\overline{a-g_{\prime\prime}}\}\,r_{\prime}^3. \quad (59.)$$

Squaring both sides, and restoring the values of $r_{\prime}$ and $r_{\prime\prime}$, it becomes an equation of the ninth order; in which, arranging according to powers of x or y, the coefficients become very complex, and altogether unmanageable by any of the usual methods. It is of the form

$$(x-\overline{g_{\prime}+g_{\prime\prime}})\,y^8+\mathrm{A}\,y^6+\mathrm{B}\,y^4+\mathrm{C}\,y^2+\mathrm{D}=0, \quad \ldots \quad (60.)$$

where A, B, C, D are functions of x, which in all cases render the terms not higher than of the ninth degree.

XVII.—Though we cannot completely discuss the course of the curve and the character of its singular points by means of this equation, we may yet learn some particulars of its general features with considerable facility; and as they will be of great use to us in our future inquiries, we shall insert them here.

1. Since y appears only in *even* powers, the curve is composed of pairs of branches, such that the branches in each pair are equal and symmetrically disposed with respect to the axis of x.

2. Since x appears of an *odd* degree, there will be at least one real value of x for every value of y, whether y be positive or negative. There will at least be one pair of equal and symmetrical branches, and these branches will be infinite ones.

3. Since (y^2) appears of the *fourth* degree, there may possibly be four values of y^2 for every specific value of x; but there cannot possibly be an odd number. Of these four possible roots any number may be *minus*, and the corresponding values of y itself be still impossible or imaginary. But by art. 2. there must be at least one pair of real values of y, there must be at least *two* real values of y^2, one of which must be +, for every value whatever of x.

4. Also four is the greatest number of pairs of symmetrical branches that can exist.

5. If the system be made to revolve round the axis of symmetry (that of x), it will

generate as many sheets of surface as there are branches in the curve; but as the symmetrical branches of the curve generate superposed sheets of surface, or sheets which are geometrically identical, the number actually described is only half as great as the number of branches. There may hence be one, two, three, or four sheets of surface generated, according as one, two, three, or four of the values of y^2 are real and positive; and upon any point in one of these a minute needle being placed, and a circle described through that point and the points Q R, the needle will find its repose in that plane, and be a tangent to the circle at that point.

6. The intersection of these four sheets of surface with the earth's surface would give four separate and continuous lines upon the surface of our globe, upon any point of which the needle being placed, it would be horizontal. In other words, there may be four such lines as that which has been denominated the magnetic equator.

7. Observation, however, seems to indicate only one single branch of this intersection; though it must be confessed that the greater part of the observations, and the mode of determination of the position of the equator, are far from satisfactory. The great difficulty of procuring good instruments, and the almost equally great difficulty in making a correct observation with any instrument whatever, at places which would give results free from suspicion of foreign and local sources of error; the extremely small number of observations actually attempted, and the very hypothetical character of the formula by which the equator is determined from observations made on either side at the distance of a few degrees; all these reasons, and others, render the delineation of this line, as laid down by M. Morlet, very far from satisfactory. The four branches may indeed be easily conceived to lie so near to one another, that of points which have been actually observed or inferred from observation and theoretical reductions, some might be in one and others in other branches of the fourfold system of lines; and hence that the spherical polygon traced through these might not be in reality composed of chords of any one single branch of the system.

8. It therefore becomes necessary to examine the curve more minutely as to the number and circumstances of the branches of which it is actually composed on any hypothesis which is consistent with the other phenomena to be accounted for. Since, however, the general equation in terms of x and y is altogether unfitted for our purpose, and the equation between the radius vector and polar angle offers no simplification in the form of the expression, I was led to attempt it by examining the relation which subsists between the angles made by radiants from the poles to points in the curve, with the line joining the poles. This also proved almost equally useless in respect to the purpose I had in view; but upon trying the radiants themselves the object was completely attained.

Resuming equation (59.), and recollecting that

$$y^2 + (x + a)^2 = r_{,}^2$$

$$y^2 + (x - a)^2 = r_{,,}^2,$$

and putting also for abbreviation (and at the same time retaining the homogeneity of the equations into which these terms enter,)

$$\overline{a+g_{,}} \,.\, \overline{a+g_{,,}} = k^2$$
$$\overline{a-g_{,}} \,.\, \overline{a-g_{,,}} = h^2,$$

we have (59.) converted into

$$(r_{,}^2 - h^2)\, r_{,,}^3 - (r_{,,}^2 - k^2)\, r_{,}^3 = 0\,; \qquad (61.)$$

or, arranging them in reference to $r_{,,}$, it is

$$r_{,,}^3 - \frac{r_{,}^3}{r_{,}^2 - k^2} \,.\, r_{,,}^2 = - \frac{k^2\, r_{,}^3}{r_{,}^2 - h^2}. \qquad (62.)$$

For $r_{,,}$ write $r + \frac{r_{,}^3}{3\,(r_{,}^2 - k^2)}$, and the equation (62.) becomes

$$r^3 - \frac{r_{,}^6 \,.\, r}{3\,(r_{,}^2 - k^2)^2} = \frac{2\, r_{,}^9}{27\,(r_{,}^2 - k^2)^3} - \frac{k^2\, r_{,}^3}{r_{,}^2 - h^2}, \qquad (63.)$$

which is a cubic equation wanting the second term, and which for accommodation to the usual notation may be written for the moment thus:

$$r^3 - 3\,c\,r = 2\,d.$$

Then we have

$$c^3 + d^2 = \frac{27\,k^2\, r_{,}^6 \left\{-4\, r_{,}^6 + 27\, k^2\, (r_{,}^2 - h^2)^2\right\}}{4 \times 27^2\, (r_{,}^2 - h^2)^4}. \qquad (64.)$$

Now in the case before us, putting $g_{,} = -(a + a_{,})$ and $g_{,,} = (a + a_{,,})$, which, since the poles of the terrestrial magnet being either *within* or *upon the surface* of the earth is always the case in nature, we have

$$\left.\begin{array}{l} h^2 = \overline{a-g_{,}} \,.\, \overline{a-g_{,,}} = -\, a_{,,}\,(2\,a + a_{,}) \\ k^2 = \overline{a+g_{,}} \,.\, \overline{a+g_{,,}} = -\, a_{,}\,(2\,a + a_{,,}) \end{array}\right\}, \qquad (65.)$$

and which equations, since a, $a_{,}$, and $a_{,,}$ are essentially $+$, are themselves essentially $-$. These values of h^2 and k^2 inserted in (30.) render the whole value of $c^3 + d^2$ essentially $+$, whatever the value of $r_{,}$ may be. There is hence one pair of symmetrical branches indicated by this method also, as in the former. But in addition to this we learn at once that there is *only one* such pair; since when the root is given by CARDAN's formula, (which is the case here,) that root is the only real one*.

9. The conclusion is now established, that there is only one sheet of the tangential surface compatible with the actual condition of the terrestrial magnet, and hence only one line of intersection between it and the earth's surface; or, in other words, *the magnetic equator is one isolated and continuous line on the surface of the earth.*

10. Also, since from other considerations it can be shown that the two poles are

* LAGRANGE's test might have been employed instead of this; but as that appears to be something more laborious, I have preferred the present one.

not in the same diameter of the earth, nor equally distant from the centre in any chord of the earth, the two axes of revolution of the sphere and tangential surface do not coincide; and hence their common intersection is not a plane, nor its trace on the sphere, that is, the magnetic equator, a circle of any magnitude whatever. It is therefore a curve of double curvature. The conclusion, therefore, deduced by BIOT from HUMBOLDT'S observations, and the conclusion deduced by MORLET from the discussion of all the observations he could collect from authentic sources, are quite consistent in this respect with the hypothesis of the duality of the poles.

11. The discussion of any further cases of this problem need not be given here. In a geometrical point of view the discussion would be interesting, and under that aspect this paper would be incomplete without them; but as they have no bearing upon the main object of the present research, and are moreover so perfectly analogous to those we have just given as to offer not the slightest difficulty by the same method which has been here employed, any further notice of them would be altogether superfluous, and irrelevant to the purpose we have in view.

12. It is to be remarked, however, that these results are true only on the hypothesis of the forces in the poles being related by the equation $F_{,} \pm F_{,,} = 0$. Under any other condition than this the highest power of x would not disappear from equation (59.); and hence the equation would be of an *even* order, and hence the branches of the magnetic equator would be *two* at least, and always an *even* number. The *apparent singleness* of branches furnished by observation is a strong argument in favour of the duality of the poles and equality of their intensities; but as the method by which the magnetic equator has been laid down is far from satisfactory, too much reliance should not be placed on this argument, decisive as it otherwise would certainly be.

13. The equation of the tangential surface and the equation of the sphere would completely define that line considered in reference to rectangular coordinates; that is, in the usual manner of considering the equations of lines situated in space. In the form of the equation of the locus of N on a meridian plane marked (60.), we have only to write $y^2 + z^2$ instead of y^2, and the result is the equation of the surface referred to the axis of x and any other two axes at right angles to it and to one another. The coordinates of the centre of the sphere referred to these axes, and the radius of the sphere, being also given, we have its equation in the usual form, viz. $r^2 = (x - a_{,})^2 + (y - b_{,})^2 + (z - c_{,})^2$. By the transformation of coordinates we can change the axes of reference to any given axes, as, for instance, to the polar axis and the intersection of the equator by two rectangular meridians. In the next place, to adapt the expression to the usual mode of denoting spherical position, (latitude and longitude,) we must transform this into a polar equation, and put the radius vector of the resulting equation constant and equal to the terrestrial radius. The equation thus obtained will be one between the latitude and longitude of the points which constitute the magnetic equator.

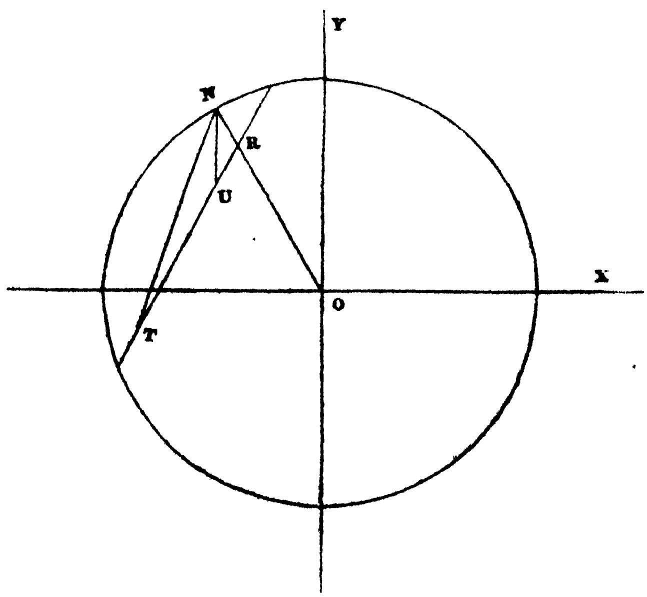
The state of the physical problem is not at present such as to render any further mathematical details respecting this curve necessary in this place.

XVIII.—*On the Points of the Earth's Surface at which the Needle takes a position vertical to the Horizon.*

As our hypothesis is that of two poles, or resultant centres of force, the freely-suspended magnetic needle will always lie in the plane which passes through its own centre and the centres of magnetic force. The dipping-needle lies, therefore, wholly in the plane passing through the place of observation and the true magnetic poles. But when the needle is vertical to the horizon, it passes through the centre of the earth; and hence the plane of the magnetic meridian also passes through the centre, and makes with the sphere a section, which is a great circle. Also, as this plane then passes through three points not in a right line, it is unique; or, in other words, there is only one circle of the sphere in which the needle can be placed to be capable of taking a vertical direction, and that is a great circle.

It is also obvious, from the expressions already given for the inclination of the tangent to the radiants from the poles to points in the magnetic curve, that there are only isolated points in that circumference in which the phenomenon of verticity can take place; and it is our business in this section to inquire into their possible number, and the method of determining their actual number and their respective positions.

XIX.—Let O be the centre of the great circle in which we have just shown all the possible vertical needles must lie, and T U the magnetic poles, and N one of these points. Take the centre of the circle as origin of coordinates.



Let T U be denoted by the coordinates $a_{\prime}\ b_{\prime}$ and $a_{\prime\prime}\ b_{\prime\prime}$ respectively; then the equation of the magnetic axis is

$$(x-a_{\prime})(b_{\prime\prime}-b_{\prime})=(y-b_{\prime})(a_{\prime\prime}-a_{\prime}) \quad (66.)$$

Also the equation of O N is

$$xy'-x'y=0, \quad . \quad . \quad . \quad (67.)$$

where $x'\,y'$ are the coordinates of N.

The intersection of these gives the coordinates of R, the point where the tangent to the magnetic curve at N, and whose poles are T and U, intersects the magnetic axis.

From (66.) we have

$$y = \frac{b_{\prime\prime} - b_\prime}{a_{\prime\prime} - a_\prime} x - \frac{b_{\prime\prime} a_\prime - b_\prime a_{\prime\prime}}{a_{\prime\prime} - a_\prime}, \quad \ldots \quad (68.)$$

and from (69.) we have

$$y = \frac{y'}{x_\prime} x, \quad \ldots \quad (69.)$$

which two equations, (68.) (69.), equated, give, after simple reduction,

$$\left.\begin{aligned} x &= \frac{(a_\prime b_{\prime\prime} - a_{\prime\prime} b_\prime) x_\prime}{(b_{\prime\prime} - b_\prime) x' - (a_{\prime\prime} - a_\prime) y'}, \\ \text{and hence also} \quad y &= \frac{(a_\prime b_{\prime\prime} - a_{\prime\prime} b_\prime) y'}{(b_{\prime\prime} - b_\prime) x' - (a_{\prime\prime} - a_\prime) y'}. \end{aligned}\right\} \quad \ldots \quad (70.)$$

Again, we have,

$$\left.\begin{aligned} \mathrm{T\,R}^2 &= \left\{ a_\prime - \frac{(a_\prime b_{\prime\prime} - a_{\prime\prime} b_\prime) x'}{(b_{\prime\prime} - b_\prime) x' - (a_{\prime\prime} - a_\prime) y'} \right\}^2 + \left\{ b_\prime - \frac{(a_\prime b_{\prime\prime} - a_{\prime\prime} b_\prime) y'}{(b_{\prime\prime} - b_\prime) x' - (a_{\prime\prime} - a_\prime) y'} \right\}^2 \\ &= \frac{\{(a_{\prime\prime} - a_\prime)^2 + (b_{\prime\prime} - b_\prime)^2\} (b_\prime x' - a_\prime y')^2}{\{(b_{\prime\prime} - b_\prime) x' - (a_{\prime\prime} - a_\prime) y'\}^2}. \end{aligned}\right\} \quad (71.)$$

And in the same manner we obtain

$$\mathrm{R\,U}^2 = \frac{\{(a_{\prime\prime} - a_\prime)^2 + (b_{\prime\prime} - b_\prime)^2\} (b_{\prime\prime} x' - a_{\prime\prime} y')}{\{(b_{\prime\prime} - b_\prime) x' - (a_{\prime\prime} - a_\prime) y'\}^2} \quad \ldots \quad (72.)$$

Also

$$\mathrm{T\,N}^2 = (a_\prime - x')^2 + (b_\prime - y')^2 \quad \ldots \quad (73.)$$

$$\mathrm{U\,N}^2 = (a_{\prime\prime} - x')^2 + (b_{\prime\prime} - y')^2. \quad \ldots \quad (74.)$$

But by the property of the magnetic curve, expressed in equation (44.), we have

$$\frac{\mathrm{R\,U}}{\mathrm{R\,T}} = \frac{\mathrm{U\,N}^3}{\mathrm{T\,N}^3},$$

or

$$\frac{b_\prime x' - a_\prime y'}{b_{\prime\prime} x' - a_{\prime\prime} y'} = \left\{ \frac{(a_\prime - x')^2 + (b_\prime - y')^2}{(a_{\prime\prime} - x')^2 + (b_{\prime\prime} - y')^2} \right\}^{\frac{3}{2}} \quad \ldots \quad (75.)$$

This is the equation of the curve of contact of the tangent from O to the magnetic curve, whose poles are T and U, with the curve; and in rectangular coordinates is, like the magnetic curve itself, of the eighth order when freed from fractions and radicals.

Having now eliminated $x\,y$ from the preceding equations in which they appeared, we may drop the distinguishing accents from $x'\,y'$ in (75.) and reduce the fractions and radicals. We thus obtain

$$(b_\prime x - a_\prime y)^2 \{(a_{\prime\prime} - x)^2 + (b_{\prime\prime} - y)^2\}^3 = (b_{\prime\prime} x - a_{\prime\prime} y)^2 \{(a_\prime - x)^2 + (b_\prime - y)^2\}^3 \ . \ (76.)$$

And, as the intersection of this curve with the circle gives the points concerning which this inquiry is instituted, we may write the circle at once, its radius being r,

$$x^2 + y^2 = r^2. \quad \ldots \quad (77.)$$

The determination of x and y from these two equations will require the solution of an equation of the tenth degree. For putting (76.) under the form

$$(b_{,}x - a_{,}y)^2 [a_{,,}^2 + b_{,,}^2 + x^2 + y^2 - 2(a_{,,}x + b_{,,}y]^3$$
$$= (b_{,,}x - a_{,,}y) [a_{,}^2 + b_{,}^2 + x^2 + y^2 - 2(a_{,}x + b_{,}y)]^3,$$

we have it converted at once, by means of (77.), into

$$\left.\begin{aligned}&(b_{,}x - a_{,}y)^2 [a_{,,}^2 + b_{,,}^2 + r^2 - 2(a_{,,}x + b_{,,}y)]^3\\ &= (b_{,,}x - a_{,,}y) [a_{,}^2 + b_{,}^2 + r^2 - 2(a_{,}x + b_{,}y)]^3.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (78.)$$

Hence by means of (77.), which is of the second, and (78.), which is of the fifth, degree, we obtain an equation of the tenth, from which to determine x or y, and hence to find the points at which the needle will be vertical. Still the reduction is extremely laborious, and hence, also, our means of determining how many of the roots are real, and how many are imaginary; that is to say, how many of its roots are compatible, *simultaneously or separately*, with the values to which $a_{,}$ $b_{,}$ and $a_{,,}$ $b_{,,}$ are limited by the physical which must be appended to the algebraical conditions from which the equations (77.) and (78.) were formed. Those conditions are $r^2 > a_{,}^2 + b_{,}^2$, and $r^2 > a_{,,}^2 + b_{,,}^2$, that is, of the poles being *within* the earth; but whether the perpendicular from the centre of the earth upon the magnetic axis intersects that axis *between* the poles or not, cannot be, *à priori*, nor yet from any knowledge furnished by experiment, at present determined. As, however, all the cases that can arise from all possible positions of the two poles are included in the above formulæ, and as they evidently cannot *simultaneously* exist, we are entitled to infer that all the roots are not simultaneously real. In the absence, however, of these considerations, we learn that there are not more than ten points on the surface of the earth at which the needle can be vertical; and that whatever may be the number of them, it is at all events even, viz. 2, 4, 6, 8, or 10. We also learn that how many soever of these be real, they are all in one plane passing through the centre of the earth; and with respect to the great circle in which it cuts the terrestrial surface, taken as the axis of spherical coordinates, *all magnetic phenomena on the surface of the earth are symmetrically disposed.* How far this is verified, within the limits of errors of observation and of local interference with the full development of the effects of the magnetic force, has not yet been inquired into. Indeed, till this plane has been determined it would be impossible to conduct the inquiry in a direct manner; and as only one of these points is yet actually assigned with any close degree of approximation, we are not yet in a condition to enter upon the inquiry. Still, these facts combined with observations relative to other phenomena, especially respecting dip and intensity, (the variation for obvious geometrical reasons included,) accurately made, might furnish important aid in a tentative determination of the plane itself; the method of proceeding in which must be sufficiently obvious to those inquirers to whose minds the geometry of coordinates is familiar.

In conclusion of the present paper I shall, though I have not been able to decompose the equation which results from (77.) and (78.) into factors, yet hazard the conjecture of its being even in its *literal* form capable of such a resolution; so that the component equations are, when viewed simultaneously, the one essentially imaginary with the values which render the other real. Of course these must be into factors of even degrees,—probably two of the fourth and one of the second. Several instances of this kind are well known to geometers; a very remarkable one of which is the expression given by M. Bret for the determination of the foci of a line of the second order, in Gergonne's *Annales des Mathématiques*, tom. viii. I offer it, however, only as a conjecture, which future researches may show, after all, to be too hastily made.

Royal Military Academy, Woolwich,
December 16, 1834.

XII. *Researches towards establishing a Theory of the Dispersion of Light. By the Rev.* Baden Powell, *M.A. F.R.S. Savilian Professor of Geometry in the University of Oxford.*

Received February 19,—Read March 12, 1835.

Introductory Remarks.

THE phenomena of prismatic dispersion, as originally discovered by Newton, and since examined throughout a vast range of transparent bodies by succeeding philosophers, especially Sir David Brewster, were principally considered with reference only to successive parts, or spaces, of the coloured spectrum, designated *generally* as the red, or violet, or mean rays.

The increasing precision of modern science has been evinced in the elaborate and justly celebrated researches of M. Fraunhofer, who, availing himself of the dark and bright lines to mark and designate distinct points of the spectrum, by prismatic observations for ten different media, solid and liquid, has determined in each the refractive indices for seven principal rays, thus always absolutely identifiable. We will use the term "definite rays" to signify the specific parts of the spectrum thus defined. As to the *law* of the phenomena, the first notion of a simple proportionality was soon disproved. The refrangibility was seen to vary considerably and irregularly for each ray and each medium; and when Fraunhofer had assigned serieses of numbers as the accurate expressions of the varying refractive powers throughout the several spectra, the apparent absence of any law connecting these numbers was only rendered more palpable. All that could be said was, that the numbers *increased* from the red to the blue end of the scale, and in a different way in each medium.

The first object of inquiry in the search after such a law, would be some other characteristic of the same definite rays, equally well determined; between which and the refractive index some connexion might possibly be found to subsist.

The only such characteristic, perhaps, is the *length of the interval* for each ray, the Newtonian fit, or the undulatory wave, which (by whatever *name* it be called,) has demonstrably a real existence in the nature of light; and the value of which, for each of the definite rays, has also been determined by Fraunhofer, with his usual accuracy, from phenomena totally independent of refraction, viz., his very remarkable experiments, in which a spectrum absolutely pure and perfect is obtained without the intervention of any prism, from the *interferences* produced by a fine grating of parallel wires covering an object-glass. The positions assumed by the successive

rays, here depend on nothing but the lengths of their periods or waves simply as such; and the intervals between them are precisely proportional to the differences of these lengths. These lengths decrease from the red to the blue end of the spectrum.

We might search for some empirical law which should connect these two serieses of data, the one being some inverse function of the other; but it would be more satisfactory should such a formula be supplied by any theory of light.

I shall not I trust be considered as assuming a controversial tone, if I observe that no researches directly suggesting any such formula *have been published* except those of M. Cauchy, on the hypothesis of undulations. In these, indeed, such a formula is not actually developed. But in a paper in the London and Edinburgh Journal of Science*, in the former part of which I have offered a brief abstract of M. Cauchy's peculiar theory of undulations, some remarks upon it are given, including the deduction of a formula in which the relation between the *length of a wave* and *the velocity of its propagation* is precisely expressed; this last quantity being in fact the same as the reciprocal of *the refractive index*.

Without entering any further into theoretical considerations, it will be admitted that such a formula, (from whatever source derived,) if found to supply anything like a representation of the law of nature, or a clue to guide us through the seeming disorder which prevails among the experimental results, would be entitled to attention.

It has therefore been my object, without reference to the support of a theory, to examine by means of this formula *the relation between the index of refraction and the length of the period or wave for each definite ray* throughout the whole series of numerical results which we at present possess. And it will become a matter of increasing interest to pursue observations on the indices of definite rays for a greater range of transparent media.

The present paper will be occupied with the discussion of the data already known; and before proceeding to that discussion I will merely add, that whatever degree of interest may attach to the inquiry, the merit is due to Professor Airy, in whose suggestion it originated.

General Observations on the Formula.

In the investigations in the paper above referred to, on substituting for the velocity of a wave expressed by $\frac{s}{k}$ its equivalent $\frac{1}{\mu}$, the formula at once presents the relation between the index μ and the length of a wave λ. H, r, and n are quantities dependent on the nature of the medium; r, by hypothesis, always being a sensible fraction of λ: thus the formula becomes

$$\frac{1}{\mu} = \mathrm{H}\left\{\frac{\sin\left(\frac{\pi r n}{\lambda}\right)}{\left(\frac{\pi r n}{\lambda}\right)}\right\}$$

* Nos. 31 et seq.

Here the value of μ will evidently vary with a change in the value of λ, or from one *ray* to another; it will also vary with a change in the constants H, r, or n, that is, from one *medium* to another.

The mere inspection of the formula will suffice to show that it exhibits at least a general accordance with the obvious constitution of the prismatic spectrum in the greater dispersion of the blue end.

For, in general, as λ is diminished, the arc $\left(\frac{\pi r n}{\lambda}\right)$ is increased, and consequently the ratio of the arc to its sine increases, or μ increases. And the variation in the value of this ratio, and consequently in that of μ, for a given variation in λ, is greater when the arc is greater, that is, when λ is less.

Thus, towards the blue end of the spectrum, where λ is least, the dispersion or expansion of the rays is greatest.

But we must proceed from these very general remarks to the more precise comparison of numerical values.

Comparison of Numerical Results.

In proceeding to apply the formula to actual calculation, we are met by several difficulties arising out of the peculiar form of the function. The process is, in fact, reduced to finding *arcs* which shall fulfill the *twofold* condition of being themselves in the ratio of the values of λ, while they are to their sines in the ratio of the values of μ. For this I have not been able to make any direct method available.

By indirect and tentative methods, however, and the assumption of arcs which were seen (from a table of the lengths of arcs,) to be nearly in the required ratio to their sines, I advanced by successive trials of greater or less arcs to more exact values. Those for the two extreme rays were usually assumed in the first instance, and their ratios to their sines compared with the ratios of the refractive indices; and these once brought to a sufficiently near accordance, a fundamental arc was obtained, from which those for the other rays were deduced on dividing by the corresponding value of λ; and the product of a constant coefficient multiplying the ratio of the arc and sine, which in theory ought to give the value of the refractive index, was compared with the index deduced from observation. This will sufficiently explain the meaning of the several columns in the tabular statement of the results.

It must be borne in mind that the values finally adopted are still only approximative, and are open to further correction by repeating the process; so that in all the cases here considered a still closer coincidence might probably be obtained were it thought desirable.

The fundamental data of these comparisons are (as already said) those very precise determinations of the value of λ for the several definite rays named by the letters B, C, D, &c., obtained by FRAUNHOFER from the interference-spectrum; and which

Sir J. HERSCHEL has justly characterized as data of the utmost value in the theory of light*. These values are as follows:

Ray.	Value of λ.
B	·00002541
C	·00002422
D	·00002175
E	·00001945
F	·00001794
G	·00001587
H	·00001464

With these values I have gone through every case of a refractive index for a definite ray at present known, that is, for every one of these seven definite rays in each of the ten substances whose refractive energy for the different rays was examined by FRAUNHOFER.

The following tabular statement gives the comparison between the refractive index for each ray in each medium, as given by FRAUNHOFER's observations in the first column, and as resulting from the formula of theory (adopting his independent determinations of the values of λ,) in the last; whilst in the intermediate columns the elements of the calculation are exhibited.

Flint Glass, No. 13. FRAUNHOFER.

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated values of $\mu = \text{const} \times \left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·6277	16° 10′	1·0134	1·6275
C	1·6297	16 41	1·0143	1·6299
D	1·6350	18 35	1·0178	1·6355
E	1·6420	20 44	1·0222	1·6426
F	1·6483	22 31	1·0261	1·6486
G	1·6603	25 29	1·0336	1·6609
H	1·6711	27 39	1·0399	1·6711
			const = 1·607	

Flint Glass, No. 23. FRAUNHOFER.

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated values of $\mu = \text{const} \times \left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·6265	16° 0′	1·0131	1·6269
C	1·6285	16 17	1·0135	1·6278
D	1·6337	18 15	1·0172	1·6335
E	1·6405	20 22	1·0214	1·6403
F	1·6467	22 8	1·0252	1·6464
G	1·6588	25 2	1·0325	1·6582
H	1·6697	27 9	1·0393	1·6697
			const = 1·606	

* See Treatise on Light, art. 751. 756.

Flint glass, No. 30. Fraunhofer.

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ = const. × $\left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·6236	16° 0′	1·0131	1·6239
C	1·6255	16 17	1·0135	1·6246
D	1·6306	18 15	1·0172	1·6305
E	1·6373	20 22	1·0214	1·6373
F	1·6435	22 8	1·0252	1·6434
G	1·6554	25 2	1·0325	1·6551
H	1·6660	27 9	1·0393	1·6660
			const. = 1·6033	

Flint glass, No. 3. Fraunhofer.

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ = const. × $\left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·6020	15° 20′	1·0120	1·6000
C	1·6038	16 5	1·0133	1·6039
D	1·6085	17 55	1·0164	1·6079
E	1·6145	19 59	1·0206	1·6145
F	1·6200	21 42	1·0243	1·6204
G	1·6308	24 33	1·0312	1·6313
H	1·6404	26 39	1·0369	1·6404
			const. = 1·582	

Crown glass, M. Fraunhofer.

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ = const. × $\left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·5548	12° 19′	1·0077	1·5548
C	1·5559	12 55	1·0085	1·5561
D	1·5591	14 23	1·0106	1·5593
E	1·5632	16 5	1·0133	1·5634
F	1·5667	17 26	1·0156	1·5671
G	1·5735	19 42	1·0199	1·5738
H	1·5795	21 22	1·0235	1·5792
			const. = 1·543	

Crown glass, No. 13. Fraunhofer.

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ = const. × $\left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·5243	11° 18′	1·0065	1·5243
C	1·5253	11 51	1·0071	1·5252
D	1·5280	13 12	1·0089	1·5279
E	1·5314	14 46	1·0112	1·5314
F	1·5343	16 0	1·0131	1·5343
G	1·5399	18 5	1·0168	1·5399
H	1·5447	19 37	1·0198	1·5444
			const. = 1·5145	

Crown glass, No. 9. Fraunhofer.

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated value of μ = const. × $\left(\frac{\text{arc}}{\text{sine}}\right)$.
B	1·5258	11° 18′	1·0065	1·5259
C	1·5269	11 51	1·0071	1·5269
D	1·5296	13 12	1·0089	1·5296
E	1·5330	14 46	1·0112	1·5332
F	1·5360	16 0	1·0131	1·5360
G	1·5416	18 5	1·0168	1·5416
H	1·5466	19 37	1·0198	1·5462
			const. = 1·5162	

Ray.	Observed values of μ.	Assumed values of $\frac{\pi r n}{\lambda}$.	Ratio $\left(\frac{\text{arc}}{\text{sine}}\right)$.	Calculated values of μ $=\text{const.}\times\left(\frac{\text{arc}}{\text{sine}}\right)$.
Oil of turpentine. Fraunhofer.				
B	1·4705	12° 25′	1·0078	1·4703
C	1·4715	13 1	1·0086	1·4715
D	1·4744	14 30	1·0107	1·4746
E	1·4783	16 13	1·0135	1·4786
F	1·4817	17 35	1·0159	1·4821
G	1·4882	19 51	1·0203	1·4886
H	1·4939	21 32	1·0239	1·4938
			const. = 1·459	
Solution of potash. Fraunhofer.				
B	1·3996	10 34	1·0056	1·3999
C	1·4005	11 5	1·0062	1·4008
D	1·4028	12 20	1·0077	1·4029
E	1·4056	13 10	1·0088	1·4044
F	1·4081	14 57	1·0114	1·4080
G	1·4126	16 55	1·0147	1·4126
H	1·4164	18 20	1·0173	1·4162
			const. = 1·3922	
Water. Fraunhofer (two experiments).				
B	1·3309	9 54	1·0050	1·3309
C	1·3317	10 25	1·0055	1·3315
D	1·3336	11 36	1·0068	1·3333
E	1·3358	12 57	1·0085	1·3355
F	1·3378	14 3	1·0101	1·3376
G	1·3413	15 51	1·0129	1·3413
H	1·3442	17 11	1·0151	1·3443
			const. = 1·3243	

Conclusion.

Upon comparing the numbers above given as resulting from theory and from observation, and bearing in mind that the assumptions of the constants on which the calculation depends are but tentative and approximative, and open to further correction, it will, I think, be allowed that the coincidences are quite sufficient to permit us to regard the formula as a very close representation of the law of nature.

We are thus, I think, justified in concluding, that for all the substances examined by Fraunhofer, viz. for four kinds of flint glass, three of crown glass, for water, solution of potash, and oil of turpentine, *the refractive indices* observed for each of the seven definite rays *are related to the lengths of waves for the same rays, as nearly as possible according to the formula above deduced from* M. Cauchy's *theory.*

Thus, then, for all the media as yet accurately examined, *the theory of undulations* (as modified by that distinguished analyst,) *supplies at once both the law and the explanation of the phenomena of dispersion.*

Oxford,
February 17, 1835.

METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY,

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF

THE PRESIDENT AND COUNCIL.

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge.............=83 feet $2\frac{1}{2}$ in.

———————————— above the mean level of the Sea (presumed about) ....=95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House...............=79 feet.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR JULY AND AUGUST, 1834.

1834.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	External Thermometer. Fahrenheit. 3 P.M.	External Thermometer. Self-registering. Lowest.	External Thermometer. Self-registering. Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
JULY												
T 1	30.319	71.2	30.220	69.8	55	63.2	67.0	51.4	67.6		NE	Lightly cloudy and overcast.—Evening, lowering.
W 2	30.080	65.3	30.033	69.2	56	61.7	71.2	55.8	71.8		NNE	Light wind.—A.M. Slightly lowering. P.M. Fine—light clouds. Evening clear.
T 3	30.089	64.7	30.059	68.7	55	59.8	73.2	55.7	73.7		N	A.M. Overcast. P.M. Fine—light clouds.
F 4	30.172	65.4	30.132	69.3	56	62.7	75.8	55.4	76.4		NE	Light wind.—A.M. Lightly cloudy. P.M. Fine and cloudless. Evening, fine—light high clouds.
S 5	30.107	65.9	30.030	71.0	58	62.7	74.2	56.7	77.1		N	A.M. Overcast—light wind.—P.M. Fair—lightly cloudy. Night, heavy rain.
●☉ 6	29.988	68.2	29.983	70.7	63	65.2	66.3	62.7	70.1	.389	E	A.M. Overcast. Noon, continued thunder with light rain. P.M. Clear—lowering.
M 7	29.994	71.2	29.954	72.6	61	67.8	73.3	61.8	75.7	.083	WSW	Clear—cloudy.—Evening, light rain.
T 8	29.893	71.3	29.909	72.8	65	66.8	73.7	61.7	75.3	.011	SSE	A.M. Rain. P.M. Fine—light clouds.
W 9	30.134	71.7	30.156	71.8	55	64.2	68.5	57.8	71.2	.111	NW	A.M. Lowering—light haze. P.M. Lightly overcast. Evening, clear.
T 10	30.105	75.2	30.057	72.4	56	69.1	69.5	58.2	73.4		SSE	Lightly cloudy.—Evening, fine and clear—light breeze.
F 11	30.132	74.7	30.057	73.2	55	68.2	73.9	56.2	76.3		WSW	Fine—nearly cloudless.—A.M. Light clouds, and cloudiness. P.M. Clear.
S 12	29.887	77.4	29.823	73.6	60	73.9	75.8	56.7	78.2		SSW	Fine and clear—nearly cloudless.—Light wind, a.m.
☉ 13	29.920	75.8	29.908	74.4	55	68.2	74.2	62.4	75.8		WSW	A.M. Cloudy. P.M. Fine and clear—a few light clouds.
M 14	30.045	71.8	30.071	74.3	54	68.7	75.4	60.7	77.0		WSW	Fine.—A.M. Clear—cloudy. P.M. Light clouds. Evening, cloudless.
T 15	30.263	77.3	30.239	74.8	57	70.8	77.6	59.3	79.7		SSW	Clear and cloudless.
W 16	30.282	73.6	30.220	75.4	61	71.6	80.6	62.1	81.6		WSW	Fine and cloudless.—Evening, clear.
T 17	30.186	79.3	30.089	77.8	60	74.7	85.2	64.0	86.7		WSW	Fine and cloudless—haze.—Evening, clear. Night, light unsteady breeze.
F 18	29.809	78.4	29.724	74.8	67	74.7	68.0	64.8	85.2		E	Light brisk unsteady wind.—A.M. Fine—light soft clouds. P.M. Dark—continued rain: thunder-storm at 7½ h.
S 19	29.508	72.3	29.595	71.3	63	63.2	60.6	63.2	65.4	.944	WSW	Continued rain.
○☉ 20	29.747	73.5	29.748	72.7	59	62.9	66.2	56.4	68.7	.203	SW	Fine—showery.—A.M. Cloudy. P.M. Clear—broken clouds—light wind.
M 21	29.760	70.1	29.784	69.6	58	62.7	65.7	58.8	68.4	.056	SE	Lowering—light rain.—Evening, fine—cloudy: distant thunder.
T 22	29.944	67.8	29.994	71.3	61	65.5	66.7	57.0	73.7	.014	ESE	A.M. Fine—light clouds and haze. P.M. Cloudy and overcast—occasional light showers: distant thunder at 3½ h.
W 23	30.076	72.2	30.031	71.7	60	67.9	73.4	59.1	75.8		E	Fine.—A.M. Cloudless—haze. P.M. Lightly cloudy.
T 24	30.067	70.3	30.043	72.2	61	66.8	70.3	61.3	73.7		NNE	A.M. Fair—lightly cloudy. P.M. Fine and clear—light clouds.
F 25	30.059	68.6	30.031	71.9	60	63.8	73.8	59.8	74.7		W	A.M. Overcast. P.M. Fine and clear—light clouds.
S 26	29.936	71.4	29.816	72.9	57	65.4	70.3	58.7	74.5	.056	WSW	Lightly overcast.—Fine, a.m.
☉ 27	29.677	66.3	29.791	69.8	60	60.7	65.5	55.7	69.0		E	A.M. Overcast.—P.M. Fair—lightly cloudy. Evening, clear.
M 28	30.027	72.3	30.030	72.4	57	68.5	73.2	57.2	74.5		E	Fine.—A.M. Cloudless—streaked cloudiness. P.M. Light clouds. Midnight, thunder storm.
T 29	30.019	70.0	29.996	73.3	69	69.2	79.2	62.9	80.3	.339	E	Lowering—light wind.—1½ h. to 3 h. p.m. distant thunder.
W 30	29.871	70.4	29.907	74.5	65	65.7	73.5	63.7	75.2	.958	E	A.M. Continued rain. Noon, heavy clouds. P.M. Fine and very clear—light clouds and breeze.
T 31	29.829	69.4	29.819	70.6	63	64.4	67.0	62.0	69.6	.200	NNE	A.M. Broken clouds. light rain from 7 h. to 10 h. P.M. Overcast and foggy: rain from 12½ h. to 2 h.
MEANS..	29.998	71.4	29.976	72.3	59.4	66.5	71.9	59.3	74.7	Sum. 3.364	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.876 3 P.M. 29.853
AUGUST												
F 1	29.878	70.7	29.863	73.4	65	69.2	75.0	63.8	78.2	.319	WSW	Fine—lightly cloudy—light haze—Evening, clear.
S 2	29.911	71.4	29.899	74.2	63	67.7	74.9	63.8	75.6		NNE	A.M. Lightly cloudy—unsteady air. P.M. Fine and clear—light clouds and breeze. Evening, cloudless.
☉ 3	29.918	70.7	29.877	72.3	53	64.2	72.8	56.4	73.3		N	Fine.—A.M. Lightly cloudy. P.M. Cloudless—light cloudiness: at 8½ h. heavy clouds in the N.W. with frequent lightning.
M 4	29.933	70.0	29.938	73.2	64	65.1	73.2	61.6	74.3		WSW	A.M. Overcast. P.M. Fine and clear—cloudy—light breeze. Evening, cloudy.
●T 5	29.899	72.7	29.889	73.7	64	70.0	71.4	61.9	74.2		SSE var.	Fine and lowering, alternately—light breeze.—Light rain, early a.m.
W 6	29.894	73.7	29.879	73.2	61	67.7	71.6	62.2	73.8		W	Clear.—A.M. Heavy clouds: at 11½ h. heavy shower. P.M. Fine—cloudy—light breeze: shower about 6 h.
T 7	29.984	70.2	29.983	70.7	62	65.6	67.9	58.3	68.8	.211	S	Overcast—broken clouds—light wind—showery.—Evening, continued rain.
F 8	29.805	69.8	29.810	72.7	67	67.9	70.4	63.5	74.3	.181	SSW	A.M. Lowering—light brisk wind. P.M. Soft broken clouds: shower at 3 h.
S 9	30.114	70 3	30.138	71.7	60	65.7	70.3	55.7	72.2	.161	NNE	A.M. Lightly cloudy. P.M. Fine—high clouds.
☉ 10	30.181	71.7	30.130	72.6	59	67.3	76.8	58.3	78.2	.011	N	Fine.—A.M. Overcast. P.M. Fine and clear—light clouds.
M 11	30.067	71.7	30.068	72.8	62	69.6	75.2	60.3	78.6		S	A.M Fine and cloudless—light cloudiness. P.M. Fair—lightly cloudy. Evening, cloudless—sultry.
T 12	30.128	72.3	30.095	73.4	64	71.7	78.6	61.3	79 7		WSW	Fine.—A.M. Lightly cloudy. P.M. Clear and cloudless—light airs. Evening, very clear.
W 13	30.001	75.7	30.037	75.7	63	75.2	77.8	64.2	83.3		SSE	Fine.—A.M. Cloudless. P.M. Lightly cloudy. Evening, clear. Night, lightly cloudy.
T 14	30.104	70.8	30.081	73.9	60	65.3	70.2	59.7	72.7		N	A.M. Fine—light clouds—light unsteady air. P.M. Overcast.
F 15	30.119	68.2	30.122	72.8	62	63.8	73.5	58.2	73.7		N	Fair—light mingled clouds—light unsteady breeze.
S 16	30.213	70.7	30.172	73.0	65	66.7	74.6	61.8	76.3		NNE	A.M. Cloudy—light wind. P.M. Fine—light clouds.
☉ 17	30.188	73.0	30.095	73.3	63	66.9	74.6	58.8	76.2		NE	Fine—light clouds.—A.M. Light wind. P.M. clear.
M 18	29.978	70.9	29.924	73.2	63	67.8	74.8	62.5	76.0		E	Fine—light clouds and wind.
○T 19	29.980	70.6	29.938	73.6	62	66.9	72.7	60.0	73.7		NNE	Light wind.—A.M. Fine—light clouds. P.M. Lightly overcast.
W 20	29.812	70.2	29.728	72.7	62	64.8	73.0	62.0	74.7		SSW	Fine—light broken clouds.—Evening, light rain at 8½ h.
T 21	29.784	72.0	29.726	72.6	56	65.0	70.5	58.6	72.6		SW	Fine.—A.M. Clear—light clouds. P.M. Nearly cloudless.
F 22	29.821	71.0	29.772	71.4	52	64.2	67.7	53.7	70.8		S	A.M. Fine and cloudless. P.M. Cloudy—light brisk wind: light shower at 3½ h.
S 23	29.962	69.7	29.944	69.7	50	62.3	66.9	52.0	69.6		S	Fine.—A.M. Nearly cloudless—light wind. P.M. Light showers and clouds.
☉ 24	29.814	66.7	29.728	67.0	54	61.0	58.6	51.8	67.9	.063	E	Cloudy and overcast—light wind.—Light rain, p.m.
M 25	29.685	64.8	29.746	66.0	49	59.0	62.7	47.5	65.2	.152	SSW	Fine—light clouds.—A.M. Clear. P.M. Light wind: shower at 8½ h.
T 26	29.821	62.0	29.768	65.6	52	58.2	63.9	50.2	66.7		S	A.M. Fog and light drizzling rain. P.M. Cloudy—light wind: at 4 h. thunder-storm.
W 27	29.808	62.7	29.798	65.4	52	54.8	64.0	49.3	65.2	.122	S	Fine—light clouds.—Clear—light wind, a.m.
T 28	29.946	62.7	29.905	64.9	50	56.8	62.5	46.2	65.2		SW	Light brisk wind.—A.M. Fine—nearly cloudless. P.M. Overcast: heavy shower at 10½ h.
F 29	29.748	64.0	29.655	66.0	57	62.2	64.5	56.4	68.0	.205	ESE	Overcast.—Light shower at 1½ h. p.m.—light brisk wind.
S 30	29.677	66.7	29.695	67.0	57	63.7	65.9	57.0	67.9		SE var.	A.M. Overcast—light rain and wind. P.M. Fine and clear—light clouds. At night, heavy rain.
☉ 31	29.744	67.2	29.784	68.0	59	64.5	65.9	57.2	67.4	.263	SSW	Fine—light clouds and wind.—Showery, p.m.
MEANS..	29.933	69.5	29.909	71.2	59.1	65.2	70.4	57.9	72.8	Sum. 1.688	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.819 3 P.M. 29.789

METEOROLOGICAL JOURNAL FOR SEPTEMBER AND OCTOBER, 1834.

1834.	9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
						Fahrenheit.		Self-registering.				
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		9 A.M.	3 P.M.	Lowest.	Highest.			
SEPTEMBER												
M 1	29.942	64.5	29.950	67.0	56	63.2	63.5	56.9	67.7		S	A.M. Cloudy. P.M. Light continued rain.
T 2	30.041	65.2	30.061	66.0	53	61.3	66.2	55.6	67.5	.158	SWW	Fine and clear—light clouds.
● W 3	30.164	63.6	30.122	67.2	56	61.2	66.0	53.7	68.4		S var.	A.M. Fine—light clouds and haze. P.M. Lightly overcast—light wind.
T 4	30.095	66.2	29.998	68.0	58	67.7	71.8	59.9	72.8		S	Fine.—A.M. Light broken clouds. P.M. Cloudless.
F 5	29.930	66.6	29.942	70.3	63	65.4	69.8	60.9	72.6		SSW	A.M. Overcast. P.M. Fair—cloudy—light wind.
S 6	30.044	66.8	30.069	69.3	56	62.4	67.0	56.7	68.7		SW	A.M. Fair—cloudy—light wind. P.M. Fine and cloudless.
☉ 7	30.208	64.0	30.093	67 3	54	58.5	66.2	53.0	67.3		SW	A.M. Hazy. P.M. Fine—nearly cloudless.
M 8	29.685	62.2	29.584	66.0	57	57.4	64.4	53.6	66.2	.092	ENE	Overcast.—Light continued rain, a.m.
T 9	29.406	63.2	29.491	65.9	57	59.7	66.3	53.7	66 9	.144	SSW	Showery—light wind.—A.M. Overcast. P.M. Fine and clear—light broken clouds.
W 10	29.812	62.8	29.792	65.0	55	59.6	61.7	51.7	65.2		SW	Overcast—light showers.
T 11	29.808	62.4	29.806	65.7	60	60.6	65.2	58.0	66.8		S	Overcast—light showers.—Light brisk wind, p.m.
F 12	30.138	62.6	30.210	65.9	54	57.1	65.9	53.2	66.7		SW	Fine—light haze.
S 13	30.499	60.9	30.491	63.6	50	57.2	63.2	48.0	64.6		E	Fine—light brisk wind.—A.M. Cloudless. P.M. Light clouds.
☉ 14	30.536	59.6	30.479	62 3	49	55.9	62.7	46.8	63.6		ESE	Fine.—A.M. Light haze. P.M. Clear—light clouds.
M 15	30.380	58.3	30.267	61.7	52	53.7	63.4	46.2	64.2		E	A.M. Foggy. P.M. Fine and cloudless—light wind.
T 16	30.047	59.7	30.008	62.4	53	58.9	68.2	49.9	68.4		E	Fine and cloudless—light wind and haze.
○ W 17	30.037	62.8	30.063	66.5	59	65.6	72.0	58.5	73.3		E	A.M. Light fog. P.M. Fine—nearly cloudless.
T 18	30.218	65.7	30.225	67.9	60	64.6	70.2	60.7	71.6		SE	Lightly overcast—light rain.
F 19	30.261	66.2	30.243	68.7	60	62.6	72.0	59.0	72 7	.041	S	A.M. Foggy. P.M. Cloudy.
S 20	30.382	68.6	30.356	70.6	64	67.9	72.4	60.2	74.8		N	Fine and cloudless—light haze and wind.
☉ 21	30.362	66.2	30.305	67.5	62	60.2	68.0	58.9	68.6		NE	Lightly overcast and hazy—light wind.
M 22	30.245	64.9	30.190	67.8	59	60.0	64.6	57.5	67.3		NE	Heavy clouds—light wind.
T 23	30.235	61.2	30.208	63.9	48	54.3	59.8	47.2	61.8		ENE	Fine.—A.M. Cloudless—light haze and wind. P.M. Lightly cloudy.
W 24	30.204	59.6	30.111	62.5	47	55.3	60.3	50.3	62.4		N	Overcast and foggy.
T 25	30.212	59.0	30.162	62.0	50	55.9	62.9	48.8	63.4		ENE	Fine—light wind.—A.M. Light clouds. P.M. Cloudless.
F 26	30.095	59.9	30.029	64.4	55	58.8	65.0	54.7	65.3		E	Lightly overcast and cloudy.—Light rain and wind, a.m.
S 27	29.876	62.2	29.920	64.7	60	62.3	65.8	58 8	66.9	.055	SSW	Overcast.—Light rain, p.m.
☉ 28	30.093	61.9	30.130	65.2	57	58.2	66.2	54.0	67.0	.092	SW	Lightly cloudy.—Fine, p.m.
M 29	30.285	60.2	30.229	63.3	56	55.3	63.4	51.0	63.8		ENE	A.M. Foggy—light wind. P.M. Fine—nearly cloudless.
T 30	30.190	58.2	30.128	61.6	53	54.2	61.3	46.9	62.3		E	Fine and cloudless.
MEANS..	30.114	62.9	30.089	65.7	55.8	59.8	65.8	54.1	67.3	Sum. .582	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 30.026 — 3 P.M. 29.992
OCTOBER												
W 1	30.174	57.4	30.119	60.4	50	54.1	61.2	47.4	61.4		N	Fine and cloudless.—A.M. Light haze. P.M. Light wind.
● T 2	30.104	56.5	30.064	59.7	51	52.6	62.6	45.3	62.6		E	Fine and cloudless. A.M. Light haze and wind. P.M. Very clear.
F 3	30.190	55.3	30.181	58.3	47	48.6	60.6	44.3	60.7		E	A.M. Strong haze—cloudless. P.M. Fine—hazy.
S 4	30.219	55.6	30.185	60.2	52	52.3	66.7	46.7	66.7		E	A.M. Strong fog. P.M. Clear and cloudless.
☉ 5	30.192	57.7	30.180	61.7	51	54.4	68.4	49.3	70.2		SE	A.M. Strong haze—cloudless. P.M. Fine and cloudless—light cloudiness.
M 6	30.259	60.3	30.200	63.2	56	56.4	68.8	53.8	70.2		SSW	Fine and cloudless—hazy.
T 7	30.230	62.9	30.193	65.3	60	60.4	67.6	55.9	68.3		WNW	Fine and cloudless—light streaked cloudiness.—Evening, clear.
W 8	30.178	63.0	30.108	64.8	58	58.9	63.3	56.5	64.3		WSW	A.M. Lightly cloudy—light wind. P.M. Fine and clear—light breeze and clouds.
T 9	29.982	63.2	29.901	65.7	60	61.7	65.7	58.3	66.4		SSW	Light wind.—A.M. Lightly cloudy. P.M. Fine—light soft clouds.
F 10	29.999	61.4	30.010	62.0	49	55.9	57.7	54.8	59.4		N	Fine—light clouds and wind.—Clear, p.m.
S 11	30.116	55.3	30.069	58.6	45	47.4	57.6	42.4	57.7		N	Fine.—A.M. Cloudless. P.M. Clear—thin flocculous clouds, and light wind.
☉ 12	30.146	55.7	30.142	58.4	47	50.8	58.3	46.6	58.7		N	Hazy.—Fine—light cloudiness, p.m. Evening, local haze—clear.
M 13	30.120	57.2	30.003	59.2	54	54.9	60.3	49.5	61.6		SW	Fine and cloudless.—A few light clouds and cloudiness, a.m.
T 14	29.768	57.6	29.691	60.7	57	58.3	62.4	50.3	63.6		SSW	Fine—soft floating clouds.—Clear, p.m.—Showers, noon and evening.
W 15	29.770	56.7	29.723	59.2	48	51.7	55.9	49.3	56.7	.058	WSW	Fine—light flocculous clouds.—Clear—light wind, p.m.
T 16	29.707	54.4	29.430	57.6	49	50.0	57.9	42.3	58.3		S	Unsteady wind.—A.M. Fair & clear—light clds. P.M. Cloudy—lt. rain. Ev., fine & clear—light clds.—light unsteady breeze.
○ F 17	29.307	55.8	29.445	57.7	48	53.6	51.2	49.4	57.3	.025	WNW	Strong unsteady breeze.—A.M. Fine and clear. P.M. Heavy rain—dark.
S 18	29.590	52.3	29.715	54.4	43	47.6	51.3	42.2	51.7	.094	WNW	Fine—light clds. & cloudiness—light brisk wind.—Evening, clear.
☉ 19	30.036	49.7	29.938	53.3	42	45.4	54.2	38.3	56.6		WSW	Lightly cloudy.—A.M. Light haze. P.M. Light breeze and deposition.
M 20	29.774	53.8	29.749	55.7	53	56.8	59.0	44.6	59.4		S	Light clouds and breeze.—A.M. Lightly cloudy. Noon, showers. P.M. Fair and clear.
T 21	29.987	54.7	30.188	55.2	53	53.3	51.3	50.3	56.3		WSW	A.M. Fine and cloudless—light haze. P.M. Fair—lightly cloudy—light wind.
W 22	29.943	51.4	29.820	53.7	48	49.4	55.3	39.8	55.3		SW var.	Lightly overcast & cldy.—light unsteady breeze.—Ev. light rain.
T 23	29.615	53.9	29.576	55.5	52	53.8	54.3	48.7	55.7		WSW	Light brisk unsteady wind.—A.M. Lowering—deposition. P.M. Fair and clear—lightly cloudy.
F 24	29.757	46.3	29.796	48.2	26	38.5	43.4	34.4	43.4	.008	NW var.	Light fresh breeze.—A.M. Fine and cloudless. P.M. Lightly cloudy. Evening, clear.
S 25	30.080	45.7	30.131	48.6	38	42.3	47.5	37.4	47.7		NNW var	Fine—light thin clouds—light brisk wind.
☉ 26	30.344	43.7	30.412	47.3	38	41.1	48.2	35.9	48.2		N	Fine—nearly cloudless—light wind.
M 27	30.404	45.3	30.419	48.4	46	46.8	53.3	37.3	53.3		NW var.	Lightly overcast and cloudy—light haze.
T 28	30.485	49.0	30.523	50.7	49	51.6	52.5	45.7	53.3		N	Lightly overcast—light wind.
W 29	30.665	50.2	30.618	50.7	45	48.2	49.2	46.8	49.3		N	Overcast—haze.
T 30	30.480	49.8	30.371	52.7	47	47.5	52.2	45.7	52.3		SW	Lightly cloudy—light haze and wind.
F 31	30.215	50.1	30.151	53.4	49	49.8	56.3	43.3	56.6		WSW	Fine—lightly cloudy—light cloudiness, haze, and wind.
MEANS..	30.059	54.3	30.034	56.8	48.7	51.4	57.2	46.2	58.2	Sum. .185	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.996 — 3 P.M. 29.964

METEOROLOGICAL JOURNAL FOR NOVEMBER AND DECEMBER, 1834.

1834.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	External Thermometer. Fahrenheit. 3 P.M.	External Thermometer. Self-registering. Lowest.	External Thermometer. Self-registering. Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
NOVEMBER												
● S 1	30.117	53.3	30.101	55.3	51	52.6	55.7	48.7	56.3		WSW	Fine—thin flocculous clouds—light haze and wind.
☉ 2	30.130	53.4	30.082	55.4	50	50.9	54.6	48.3	55.3		WSW	Fair—lightly cloudy.
M 3	30.059	53.3	30.019	55.3	50	52.1	55.6	48.8	55.6		SW	Fine and cloudless—light cloudiness.
T 4	30.139	53.0	30.045	56.3	48	48.2	56.9	44.8	58.2		WSW	A.M. Fine—lightly cloudy—light fog and deposition. P.M. Fair—cloudy. Night, brisk wind.
W 5	29.721	56.9	29.617	58.8	58	58.4	60.4	47.3	60.7		SSW	Cloudy and lowering—strong deposition—brisk unsteady wind.
T 6	29.665	58.8	29.687	60.8	57	58.2	60.7	56.5	61.5	.008	SSW	Fair—lightly cloudy.—Evening, light rain. Night, brisk wind.
F 7	29.497	59.7	29.478	61.4	58	59.6	57.7	54.4	61.3	.019	SSE	Light brisk wind.—A.M. Broken clouds. P.M. Fine and clear. Evening, very light rain.
S 8	29.463	56.6	29.458	57.7	50	50.1	51.7	47.5	61.3	.008	SSW	Fine and clear.—A.M. Light clouds. P.M. Nearly cloudless.
☉ 9	29.454	55.3	29.531	56.3	51	51.8	52.3	47.4	53.6		ESE	Fog and light rain.
M 10	29.854	52.3	29.939	51.7	46	46.5	46.3	45.2	46.5		NNE	Overcast—light wind.—Light rain, a.m.
T 11	30.184	47.6	30.218	50.5	39	44.3	49.3	39.2	49.3		N	Fair—lightly cloudy—light unsteady wind.
W 12	30.303	46.3	30.264	46.4	35	42.1	42.7	40.5	42.7		NNE	Lightly cloudy and overcast—light unsteady wind.
T 13	30.272	43.8	30.275	45.3	35	39.2	42.0	37.7	42.3		NNE	Fair—lightly cloudy—light wind.
F 14	30.400	43.3	30.387	46.7	40	40.0	46.2	34.8	46.4		N	Fine—light clouds—light brisk wind.
S 15	30.409	43.4	30.386	46.7	41	41.5	47.2	35.6	47.2		NNE	Fair—lightly cloudy—light wind.
○☉ 16	30.372	46.3	30.318	47.8	43	44.2	47.7	40.4	47.7		WNW	Overcast—haze.
M 17	30.188	46.3	30.090	48.7	43	45.0	49.8	40.7	49.8		WSW	A.M. Overcast. P.M. Fair—light clouds.
T 18	30.146	48.3	30.162	49.7	45	45.3	48.1	43.8	48.1		N	Fair—lightly cloudy.—Light brisk wind, p.m.
W 19	30.275	46.3	30.228	46.3	38	39.5	41.3	37.5	41.4		NNE	Cloudless—light wind and haze.
T 20	29.998	40.7	29.839	43.0	35	36.7	40.2	33.2	40.5		NE	Fine and nearly cloudless—light wind.
F 21	29.768	41.3	29.738	43.3	34	37.6	41.8	34.6	43.6		E	Overcast.—Light rain and wind, p.m.
S 22	29.728	44.3	29.715	47.2	44	44.8	47.4	36.7	47.4		E	Overcast—light haze.—A.M. Deposition. P.M. Light wind.
☉ 23	30.008	46.3	30.071	46.7	39	44.7	45.6	36.7	45.6		NNE	Lightly overcast—light fog—light unsteady wind.
M 24	30.124	43.7	30.063	46.3	42	42.8	45.9	36.8	45.9		NE	Fair—soft clouds—light breeze.
T 25	29.903	44.7	29.849	44.7	38	41.7	40.7	40.3	41.7		E	Overcast.—A.M. Haze. P.M. Light breeze.
W 26	29.843	43.3	29.852	43.9	36	38.8	40.0	37.0	40.0		NNW	Overcast—light wind.
T 27	29.953	41.4	29.928	44.3	36	36.2	45.7	33.3	45.7		WSW	Fine and cloudless—haze and cloudiness.
F 28	29.751	45.4	29.588	46.7	43	46.0	42.9	34.8	46.2		SSW	Lightly cloudy.—A.M. Fine and clear. P.M. Light wind.—Evening, light rain.
S 29	29.187	45.4	29.253	47.7	42	42.3	48.8	39.6	48.8		SSW	A.M. Fine—thin streaked clouds—deposition. P.M. Cloudy—light wind.
●☉ 30	29.556	46.3	29.703	48.0	42	44.6	46.2	40.8	52.3		WNW	Fair—lightly cloudy—light haze.
MEANS..	29.949	48.2	29.929	50.0	43.6	45.5	48.4	41.4	49.4	Sum. .035	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.904 3 P.M. 29.879
DECEMBER												
M 1	29.268	47.7	29.383	49.4	48	50.3	49.3	40.7	49.7		WSW	A.M. Light rain—light brisk unsteady wind. P.M. Fine—light clouds.
T 2	29.572	47.7	29.744	49.8	44	47.3	51.3	42.8	51.3	.017	WSW	Fair—light clouds, haze, and wind.
W 3	30.115	47.5	30.131	49.7	44	44.9	49.2	38.7	49.2		WSW	Lightly cloudy—fog and deposition.
T 4	30.187	50.0	30.156	51.2	48	48.7	50.3	43.7	50.7		SSW	Light haze.—A.M. Lightly cloudy. P.M. Fine.
F 5	30.202	47.7	30.164	49.6	43	43.4	47.5	39.4	47.5		SSW	A.M. Fog—deposition. P.M. Clear and cloudless.
S 6	30.204	47.8	30.188	50.0	46	46.7	49.2	40.4	50.7		SSW	Lightly cloudy—light fog and deposition.
☉ 7	30.181	50.9	30.091	52.3	50	51.4	53.4	45.3	53.5		S	A.M. Fair—lightly cloudy. P.M. Rain. Evening, clear.
M 8	30.263	49.8	30.346	50.1	39	43.8	44.8	42.6	44.8		WNW	Fine and cloudless—light haze.
T 9	30.555	46.4	30.499	47.8	39	39.0	44.6	37.3	44.6		SW	Cloudless—haze.
W 10	30.294	47.6	30.356	47.3	42	43.1	44.3	37.7	44.3	.097	N	Cloudless—light wind and cloudiness.
T 11	30.602	43.3	30.620	43.7	37	37.3	38.2	34.8	38.2		N	Cloudless—haze and cloudiness.
F 12	30.503	42.5	30.427	43.7	36	36.8	42.3	34.8	43.3		WSW	Lightly cloudy—haze.
S 13	30.487	45.4	30.473	46 0	42	42.2	43.1	35.7	43.5		NE	Fair—lightly cloudy—light wind and haze.
☉ 14	30.519	41.8	30.530	42.8	37	37.7	40.7	35.3	40.7		N	Overcast—light haze.—Light wind, p.m. Evening, clear.
M 15	30.595	42.0	30.588	43.5	40	40.4	42.6	35.5	44.2		N	Overcast—light fog and deposition.
○ T 16	30.608	45.3	30.542	46.8	44	45.1	45.7	39.3	45.7		N	Lightly cloudy—light wind and haze.
W 17	30.150	45.3	30.134	46.4	42	43.9	44.7	40.8	44.7		NNE	Light wind.—A.M. Nearly cloudless. P.M. Cloudy—light rain.
T 18	30.297	44.2	30.342	45.7	40	40.8	43.6	39.3	43.7		N	Fair—light wind.—A.M. Cloudless—light fog and deposition. P.M. Light clouds.
F 19	30.433	44.4	30.412	45.5	42	42.4	44.7	36.5	44.7		N	Overcast.—A.M. Fog—deposition. P.M. Light wind.
S 20	30.391	44.2	30.358	45.9	42	42.3	43.7	40.4	43.7		NNE	Lightly overcast—light haze and wind.
☉ 21	30.382	45.5	30.373	46.7	39	42.3	46.3	39.8	46.3		W	Overcast—haze—light wind.
M 22	30.528	45.1	30.533	44.7	38	40.0	40.4	39.4	40.4		NNE	Light haze.—A.M. Light soft clouds—light wind. P.M. Fine and cloudless.
T 23	30.507	41.3	30.503	42.7	34	34.0	38.9	31.7	39.0		NW	Light haze.—A.M. Lightly overcast—light wind and hoar frost. P.M. Fair—cloudless.
W 24	30.503	39.2	30.434	40.3	33	33.2	36.2	29.2	39.7		W	A.M. Fog—hoar frost. P.M. Fair—cloudless—haze.
T 25	30.358	41.3	30.370	43.3	41	41.3	45.8	32.3	45.8		W	A.M. Overcast—haze. P.M. Fair—nearly cloudless—light wind.
F 26	30.539	42.6	30.562	43.8	39	39.8	43.7	39.0	43.7		N	A.M. Fair—lightly cloudy. P.M. Overcast—light wind.
S 27	30.584	43.5	30.554	43.6	39	39.8	39 2	37.8	43.0		E	Overcast—light fog.
☉ 28	30.460	41.7	30.372	42.3	36	36.3	39.0	33.8	39.0		E	Fine—nearly cloudless.
M 29	30.204	40.3	30.100	42.3	38	38.3	43.6	32.6	48.7		SE	Fair—lightly cloudy—light fog and hoar frost.
● T 30	29.980	45.3	29 950	47.3	46	49.4	50.7	36.5	53.2		S	Overcast—light fog and deposition.—Light wind, p.m.
W 31	29.841	49.8	29 816	51.6	53	53.4	53.3	48.3	54.5		SSW var.	Lowering—deposition—light fog.—Showery—light wind, p.m.
MEANS..	30.300	45.1	30.292	46.3	41.3	42.4	44.8	38.1	45.6	Sum. .114	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 30.265 3 P.M. 30.253

PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXXV.

PART II.

LONDON:

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MDCCCXXXV.

ROYAL MEDALS.

HIS MAJESTY KING WILLIAM THE FOURTH, in restoring the Foundation of the Royal Medals, graciously Commanded a Letter, of which the following is an extract, to be addressed to the Royal Society, through His Royal Highness the DUKE OF SUSSEX, K.G., President:

"Windsor Castle, March 25, 1833.

"It is HIS MAJESTY'S wish,—

"First, That the Two Gold Medals, value of Fifty Guineas each, shall "henceforth be awarded on the day of the Anniversary Meeting of the "Royal Society, on each ensuing year, for the most important discoveries "in any one principal subject or branch of knowledge.

"Secondly, That the subject matter of inquiry shall be previously settled "and propounded by the Council of the Royal Society, three years pre-"ceding the day of such award.

"Thirdly, That Literary Men of all nations shall be invited to afford the "aid of their talents and research. And,

"Fourthly, That for the ensuing three successive years, the said Two "Medals shall be awarded to such important discoveries, or series of in-"vestigations, as shall be sufficiently established, or completed to the "satisfaction of the Council, within the last five years of the days of award, "for the years 1834 and 1835, including the present year, and for which "the Author shall not have previously received an honorary reward.

(Signed) "H. TAYLOR."

The Royal Medals for the year 1833 were awarded to

SIR JOHN FREDERICK WILLIAM HERSCHEL, K.H. F.R.S.,

for his Paper on the Investigation of the Orbits of Revolving Double Stars; and to

PROFESSOR AUGUSTE PYRAME DE CANDOLLE, of Geneva, Foreign Member of the Royal Society,

for his Discoveries and Investigations in Vegetable Physiology.

Those for 1834 were awarded to

JOHN WILLIAM LUBBOCK, ESQ., V.P. & TREAS. R.S.,

for his Papers on the Tides published in the Philosophical Transactions; and to

CHARLES LYELL, ESQ.,

for his Work entitled "Principles of Geology."

Those for the present year have been awarded to

MICHAEL FARADAY, D.C.L.,

for his Investigations and Discoveries contained in the Series of Experimental Researches in Electricity, published in the Philosophical Transactions, and more particularly for the Seventh Series, relating to the definite nature of electro-chemical action; and to

SIR WILLIAM ROWAN HAMILTON, Andrews' Professor of Astronomy in the University of Dublin, and Royal Astronomer of Ireland,

for the Papers published by him in the 16th and 17th volumes of the Transactions of the Royal Irish Academy, entitled "Supplement to an Essay on the Theory of "Systems of Rays," and more particularly for those Investigations at the conclusion of the third and last Supplement, which relate to the discovery of Conical Refraction.

The Council propose to give one of the Royal Medals in the year 1836, to the most important unpublished paper in Astronomy, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council also propose to give one of the Royal Medals in the year 1836 to the

most important unpublished paper in Animal Physiology, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council propose to give one of the Royal Medals in the year 1837 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in their Transactions, after the present date (November 27th, 1834,) and prior to the month of June in that year.

The Council also propose to give one of the Royal Medals in the year 1837 to the author of the best paper, to be entitled "Contributions towards a System of Geo-"logical Chronology founded on an examination of fossil remains, and their attendant "phenomena," such paper to be communicated to the Royal Society after the present date (December 1st, 1834,) and prior to the month of June 1837:—but in case no paper is presented to the Society fulfilling the conditions implied by the above Resolution, or possessing sufficient merit, the Council propose to give one of the Royal Medals in the year 1837 to the author of the best paper in Geology and Mineralogy, communicated to the Royal Society for insertion in their Transactions after the present date and prior to the month of June in that year.

The Council propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper on Chemistry, communicated to the Royal Society for insertion in their Transactions, after the present date (November 12th, 1835,) and prior to the month of June 1838.

The Council also propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in the Philosophical Transactions, after the present date (November 19, 1835,) and prior to the month of June 1838.

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XX. *Discovery of the Metamorphosis in the second type of the* Cirripedes, *viz. the* Lepades, *completing the Natural History of these singular Animals, and confirming their affinity with the* Crustacea. *By* J. V. THOMPSON, *F.L.S. Deputy Inspector-General of Hospitals. Communicated by Sir* JAMES MACGRIGOR, *Bart. M.D. F.R.S.* 355

XXI. *On the Double Metamorphosis in the Decapodous* Crustacea, *exemplified in* Cancer Mænas, *LINN. By* J. V. THOMPSON, *F.L.S. Deputy Inspector-General of Hospitals. Communicated by Sir* JAMES MACGRIGOR, *Bart. M.D. F.R.S.* 359

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APPENDIX.

Meteorological Journal kept at the Apartments of the Royal Society, by order of the President and Council.

ERRATA.

Page 37, line 12. *For* allied to *N. clausa*, *read* allied to *N. clausa*, Zool. Journal.
Page 37, line 19. For *Murex Rumphius*, MONT., read *Murex Bamfius*, DONOV.
Page 37, line 21. For *Fusus corneus* read *Fusus antiquus*, LINN.

CORRIGENDA.

Page 358, line 1, *for* PLATE V. *read* PLATE VI.
Page 362, line 3, *for* PLATE VI. *read* PLATE V.

PHILOSOPHICAL TRANSACTIONS.

XIII. *Continuation of the Paper on the Relations between the Nerves of Motion and of Sensation, and the Brain; more particularly on the Structure of the Medulla oblongata and the Spinal Marrow. By Sir* CHARLES BELL, *F.R.S. &c. &c. &c.*

Received March 25,—Read April 30, 1835.

IN this paper it will be necessary to enter on minute details of the anatomy; but they regard a subject hitherto untouched, although essential to the comprehension of the nervous system, without which, indeed, it could not be said that we had a knowledge of the nerves as a system.

The author has advanced, by slow and laborious researches, from observing the general arrangement of the nerves as they lie in the body, to the investigation of particular nerves and their endowments; and, finally, to the examination of these parts in the centre of the system, the brain and spinal marrow, which enables him to assign the reason of that perfect symmetry which reigns through the whole.

The subjects of his last paper have been examined again and again by dissection, and reviewed in every aspect. They have been found correct in every particular. But they necessarily lead to further investigation: they point more especially to a minute inquiry into the structure of the spinal marrow, and its relations to the encephalon on the one hand, and to the origin of the nerves on the other.

It might be imagined that the author of this paper had, in these inquiries, followed his preconceived notions; but it has not been so. On the contrary, when in search of the explanation of certain phenomena, he discovered a fact in the structure which diverted him from his design, and carried him in a new course of inquiry *.

In an anatomical investigation of so much delicacy, it is necessary, in order to understand the descriptions, that he who follows it by dissection should have the parts presented exactly in the same aspect, and trace them in a prescribed manner. When

* The paper on the Voice was undertaken as a preface to the investigation of the accessory respiratory nerves. In following that subject, the author found it indispensable to deviate into this inquiry, which proves to be the more important of the two.

the anatomist has recognised the parts, and verified the descriptions of the author, he will of course vary his mode of proceeding to satisfy himself.

Lay a portion of the spinal marrow, of two or three inches, on the dissecting board, and pin it so as to look upon the posterior surface of that cord. Begin by making a clean transverse section of it near one extremity, and inspect the newly-divided surface (see Plate III. fig. 1.). The first thing which we distinguish is the cineritious matter in the centre of the medullary. If we introduce the curette into the softer cineritious matter, we can separate the medullary columns (as in figg. 2. & 3.), and we distinguish these parts: the posterior columns, deeply divided by their sulcus; the lateral columns; and the anterior columns.

In making these divisions, directed by the natural sulci and by the cineritious matter, we may soon satisfy ourselves that there is but one absolute bond of union by nervous matter. We find the anterior columns tied together by a sort of commissure, and to that commissure is attached the anterior portion of the posterior columns at two points (fig. 3. D.).

Having contemplated the section of the spinal marrow, we proceed to the dissection by splitting up these columns. We raise the posterior columns together, in one piece; to do which we must divide them at the point of union with the anterior columns (fig. 4. B.). But except at this angle, the whole tract is raised without the slightest breach of its proper surface. When the columns are thus separated, the surfaces are found to be covered with cineritious matter. We have split the cineritious substance, and some of it lies on the lower surface of the part raised, and some on the upper surface of that which is below.

If we now clear away the cineritious substance from the columns below, we shall first discover the two lateral tracts or columns. We see them in their course, regular as nerves. These columns or cords, in this aspect and condition, take a rounded form, although they are of a different shape when packed together in their natural state.

And now may be observed a structure which is not without interest. If we make a slight breach upon the surface of the columns when divested of their cineritious covering, and insinuate the point of the curette, we raise a thin pellicle, like a distinct coat, and which we may separate all round. Having done this, and the remaining surface being smooth, we may pierce it again, and in a similar manner separate a third and a fourth layer, which, smooth and delicate themselves, leave the part below as regular as the natural or exterior surface. It appears that the superficial layers furnish the roots of the higher nerves, and that the lower layers go off into the roots of the nerves as they successively arise.

If we now follow the sensitive or posterior roots of the spinal nerves towards their origins, we find them entering and dispersing in the substance of these lateral columns. Some authors describe these roots as derived from the cineritious matter. This is quite at variance with my dissections. The cineritious matter is not of a con-

sistence or structure into which nerves can be traced: and through the whole column of the spinal marrow, up to the fifth and portio mollis of the seventh nerves of the head, the cineritious matter is superimposed on the columns and nerves *.

Between the lateral columns, the cineritious matter lies deep. Upon raising it, the anterior or motor columns are seen (fig. 4. D, D.). In essential circumstances they resemble the lateral columns, and they are distinct from them. The cineritious matter occupies a portion of the space between them; and as to the remaining part, the line of separation is distinct, and the surfaces are unbroken.

By the manner in which the dissection has been made, the posterior portion of the spinal marrow being raised, as it were, out of the heart of the cord, the remaining parts fall flat, and the lateral and anterior columns separate.

Having distinguished the columns which form the spinal marrow, their natural sulci, their proper connexions, and the distribution of the cineritious substance between them, we have in the next place to observe how these columns are arranged, and what change they undergo in the upper portion of the cord, called medulla oblongata. We approach from below the same parts which we looked upon in their relations with the brain in the last paper.

We must now have before us a portion of the spinal marrow with the medulla oblongata attached to it, and proceed with the dissection.

The parts being presented in the same aspect as before, we raise the two posterior columns, separating them from the others at the intervening cineritious matter. At the back of the medulla oblongata we find the posterior columns diverging, and forming the triangular space of the fourth ventricle; this space is laid open on tearing up the pia mater, which connects the cerebellum with the medulla oblongata. Each of these columns is now seen to consist of two, the outermost the larger, and that towards the central line the smaller, and in shape pyramidal †. Following up these diverging columns, we recognise them to be the processus cerebelli ad medullam oblongatam. These great tracts, which form a large portion of the spinal marrow, are now seen to bear relation to the cerebellum.

The posterior tracts or columns being raised, we have only the lateral and anterior columns, which belong to the cerebrum, to attend to. And here is the interesting part of this communication.

Once more observing the layer of cineritious matter, we brush it off from the

* It is easy to trace the roots of the sensitive portion of the spinal nerve into the lateral column. It should be observed at the same time, that in raising the posterior columns, by insinuating an instrument into the cineritious intermediate substance, there is a more intimate attachment of the medullary substance of the posterior column at its outer edge and in the line of the origins of the nerves. It is not impossible, therefore, that the posterior column may be connected with the sensitive root of the spinal nerves, though hitherto I have not traced the fibres.

† This subdivision of what I have called the posterior column of the spinal marrow is to be traced in the whole length of the spinal marrow.

lateral columns. This grey matter may be traced into the fourth ventricle, extending over the parts to be presently described, and over part of the roots of the fifth pair of nerves. It constitutes one sheet of matter from the cauda equina to the roots of the auditory nerves, and forms a grand septum between the anterior and lateral part of the spinal marrow which belongs to the cerebrum, and the posterior columns which are related to the cerebellum.

Union of the lateral Columns in the Medulla Oblongata.

On brushing away the cineritious matter from the cerebral portion of the spinal marrow, we recognise the two lateral columns. Upwards, or towards the brain, each of these columns has a double termination; first, in the root of the fifth nerve; and secondly, in the union of the columns, or, in other words, in their decussation.

These columns lie separate in the spinal marrow; but having ascended to the medulla oblongata, they fall together, and form one round column something less than half an inch in length. On tracing this united column upwards they are disentangled, but do not separate, for they now constitute those processes of the cerebrum which, in a former paper, we traced down from the back of the crura cerebri (fig. 5. B, B; fig. 6. A, A.).

On observing the portion of the united columns, the appearance is very much that which is presented by the union of the optic nerves; that is, however, rather when the part is thoroughly hardened in spirit: when it is somewhat more pliant, we can trace the filaments of one side into the column on the other side*. The decussation is the most perfect of any to be demonstrated in the brain and nerves.

Reverting to the statement in the former paper, that a septum divides the right and left sensitive tracts where they are seen in the fourth ventricle, and that in tracing that septum downwards it terminates at the point of decussation of these tracts: I have now to add, that the septum does not absolutely terminate, that it splits to permit the oblique course and decussation of the filaments of these columns. Thus separated at the union of the columns, the septa unite again below, and may be followed downwards into that connexion which binds the posterior portion of the spinal marrow to the anterior columns.

It remains a desideratum to know what is the nature of those fibrous septa which intervene and divide the longitudinal tracts of nervous matter. But whatever may be determined on this point, it is obvious that they form a perfect link or bond of union and mechanical strength, extending from the pons to the cauda equina. Around the commissures the fibres of these bands are especially interwoven†.

* Much of the anatomy, as I have here described it, may be made out in the recent parts. But it will be easier and more satisfactory, when the parts are soft, to drop them into spirits, so that the surfaces as they are exposed may be hardened and prepared for further dissection on a succeeding day.

† The true distinctions between the columns in the spinal marrow may be made, as we did those of the medulla oblongata, by observing the splitting of the septa. From the circumstance of the columns scaling off

When the two tracts or columns which descend from the posterior portions of the crura cerebri are transversely divided, where they form the slit of the calamus scriptorius, and when they are dissected down (fig. 4. A, A, B.), we obtain a very interesting view of the back part of the anterior columns (fig. 4. D, D.), or rather of the pyramidal bodies, and their decussation. We see the union and decussation of these bodies before they separate and descend to form the motor columns of the spinal marrow. The motor and sensitive columns,—which were close together in the crura cerebri, and which in their descent were separated in the pons, and by the septum which is continued down from the posterior transverse septum of the pons,—come here again into contact at the point of union and decussation*. The motor columns approach the sensitive columns, but no union takes place; the columns keep their respective courses down the spinal marrow. When we dissect these parts carefully at the back of the medulla oblongata, we may feel, and with sharp eyes we may see, very minute and yet uncommonly strong filaments which run among these parts. We may consider such filaments as a further proof how carefully these textures are guarded against laceration.

When the dissection is carefully made, we have thus a view of the posterior part of the decussation of the pyramidal bodies; and after their decussation we see them separate and descend in the two anterior or motor columns.

Concluding view of the Sensitive and Motor System of Nerves.

If it could be said hitherto that the distribution of the nervous system, more than any other part of the animal structure, evinces design, the conclusion is irresistible, when we perceive that the parts which minister to sensation and motion are arranged with a symmetry beyond what we expect to see in architectural plans or ornaments, where every part is balanced, and each has its counterpart.

It could not be well imagined that sensation and motion belonged to parts separate and dissimilar. Formerly I believed that the nerves of sensation, that is to say, the posterior roots of the spinal nerves, came from the posterior columns of the spinal marrow, and consequently from the cerebellum. Whilst entertaining this belief I found my progress barred, for it appeared to me incomprehensible that motion could result from an organ like the cerebrum, and sensation from the cerebellum, for there was no agreement between them. They conformed neither in size, shape, nor subdivisions. Sensation and volition are necessarily combined in every action of the frame†. Although these influences, of whatever nature they be, are pro-

in regular pellicles, we may else be deceived. On separating, for example, the posterior and lateral columns at the true sulcus of separation, we shall see the minute transverse fibres: which appearance is produced by the splitting of the septum. See the former paper.

* The motor and sensitive columns do not mix or decussate, but only the motor columns with each other, and the sensitive columns with each other.

† This has been illustrated in former papers, and particularly in treating of the actions of the lips.

jected in different directions, and belong to distinct filaments*, they must be finally conjoined and in union. The anatomy conforms to this idea; the cords of communication between the seat of volition and the organs of the body proceed from a centre, run parallel, undergo similar changes, and are blended in their ultimate distribution, as in their central or cerebral relations.

It is pleasing to see that through the labours of members of this Society the principles which have directed the author in the investigation of the human anatomy are likely to be extended in their application, by a correspondence being observed in the arrangement of the nervous tracts through every class of animals possessing volition. It has long appeared to the author that the system does not differ, even in the different classes of animals, although there is much apparent variety in the distribution of the nerves.

When it became a question whether or not *Crustacea* possessed the organ of hearing, the celebrated Scarpa undertook the investigation. With this purpose he did not pry about to discover the external organ of the sense. He looked to the brain, or cerebral ganglion,—recognised the part from which the acoustic nerve should come, according to the analogy of other animals. He found the nerve, and traced it to its destination; that simple rather than imperfect organ, which, but for the circumstance of the auditory nerve in its cavity, might have been supposed too defective in its organization to be capable of receiving the impulse of sounds.

In this manner is the nervous system to be studied; for there is an internal change, in accordance with outward organization, whilst the system or great plan does not vary. There is an endowment in each particular column; it is one through its whole course. An animal, or a class of animals, may have a particular organ developed, and with the external apparatus there is a corresponding or an adjusted condition of the appropriated nerve. Another class may be deficient in the external organization, when we shall in vain look for the accompanying nerve; it is contracted, or hardly visible; but with all this the system is unchanged.

From a more cursory view of the comparative anatomy than others may have taken, this is my conclusion; but my time for such investigations has been given almost exclusively to the human anatomy; and in it I hope it will be granted that the system, as it regards sensation and motion, has been displayed so as to increase the interest of these pursuits, and to direct the studies of the pathologist to beneficial results; much advantage could hardly have been expected by dissection of the brain, even from the utmost ingenuity of research, whilst the very elements of the subject, as regards the natural anatomy, were unknown.

* See the paper on the Nervous Circle.

Fig 1

Fig 2

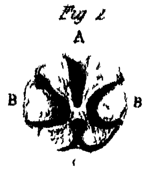


Fig 3

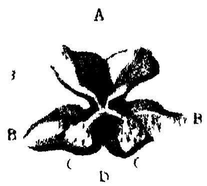


Fig 4

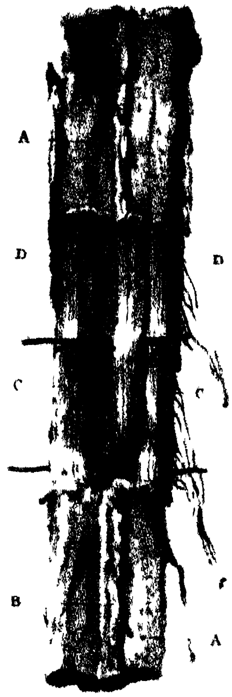


Fig 5

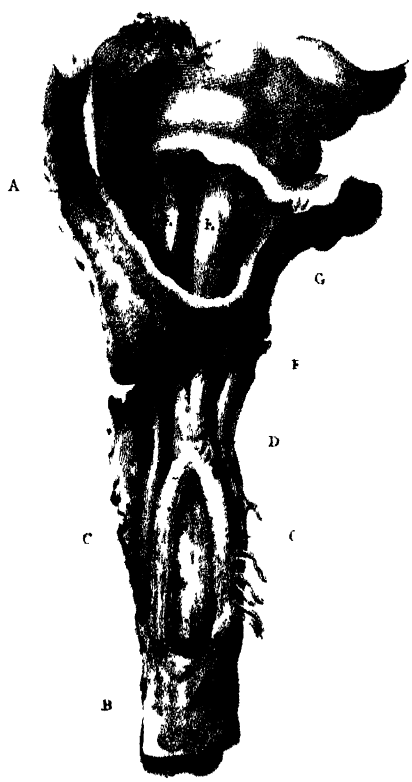
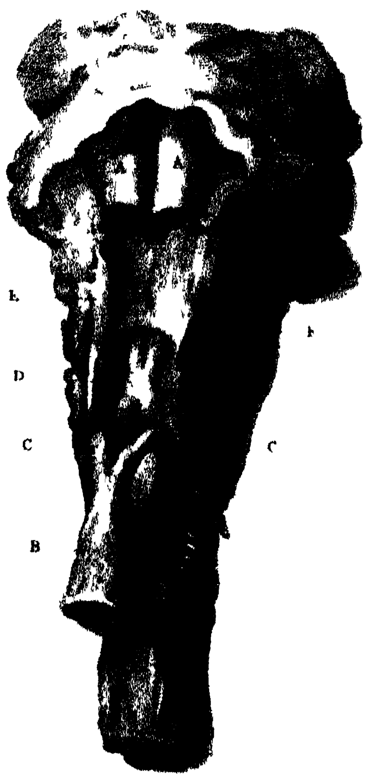


Fig 6



Drawn by Sir Chas Bell

Engraved by [illegible]

Explanation of the PLATE.

PLATE III.

Fig. 1. A transverse section of the spinal marrow, showing the distinctions of the medullary and cineritious substance.

Fig. 2. Shows the section with the medullary columns parted at their natural divisions, viz. by insinuating the curette into the cineritious substance, and opening the sulci.

A. The posterior column.

B, B. The lateral columns.

C. The anterior columns.

Fig. 3. The same parts still further separated, so as to exhibit the connexion between the posterior columns of the spinal marrow and the motor columns. The letters refer to the same parts as in the last figure.

D. The connexion between the posterior and anterior columns.

Fig. 4. In this view the posterior part of the spinal marrow, that which belongs to the cerebellum, is taken away, leaving those columns only which belong to the cerebrum. As the posterior portions (figg. 2. & 3. A.) enter deeply into the spinal marrow, when they are taken away the remaining columns fall flat on the board, and permit an easy separation.

A, A. The cineritious matter which intervenes between the columns belonging to the cerebrum, and those belonging to the cerebellum.

B. Projecting lines where the posterior columns of the spinal marrow were connected with the anterior. (See fig. 3. D.)

C, C. The lateral columns, or sensitive columns, after raising the cineritious substance. Into these the sensitive roots of the spinal nerves are traced.

D, D. A deeper dissection of the cineritious substance exposes here the posterior surface of the anterior or motor columns.

Fig. 5. This figure represents a posterior view of the upper part of the spinal marrow, and the medulla oblongata.

A. The two posterior columns of the spinal marrow being dissected up, they are here represented diverging towards the cerebellum at G.

B. The cineritious matter left on the remaining part of the spinal marrow, after raising the column (A.). The separation of the columns having been made at the intervening cineritious matter, both surfaces have that matter attached to them—both A and B.

C, C. The lateral columns of the spinal marrow (figg. 2. & 3. B, B.), displayed on their posterior surface. They are discovered on raising the cineritious matter B. Into these columns the posterior root of the spinal nerves are traced: they are the columns of sensation.

D. The short column formed by the union of the columns C, C. On dissecting this portion, the decussation of the columns will be seen.

E, E. The same columns which were lateral in the spinal marrow, now continued upwards, and visible in the fourth ventricle without dissection. They ascend under the valvula cerebri and under the corpora quadrigemina, and fall into the crura cerebri. So that, tracing them from above, each of these columns descends from that part of the crus cerebri which is posterior to the corpus nigrum.

F. The origin of the sensitive root of the fifth nerve of the encephalon.

G. The processus cerebelli ad medullam oblongatam.

Fig. 6. This figure represents the further dissection of the parts seen in fig. 5.

A, A. The columns marked E in the former plate. They are divided transversely, and the lower portion folded down, being separated from the parts below by a delicate dissection.

B. These columns folded down.

C, C. The lateral columns of the spinal marrow continued up into B.

D. The union of the anterior columns seen in their posterior aspect. The lateral or sensitive columns, and the anterior or motor columns, are held together at this point. But it appears more for security than reunion. A fine dissection exhibits them quite distinct; and the parts above continuous into the columns of the spinal marrow; each separately.

E, E. The sensitive roots of the fifth pair of nerves.

XIV. *Experimental Researches in Electricity.—Tenth Series.* By MICHAEL FARADAY, *D.C.L. F.R.S. Fullerian Prof. Chem. Royal Institution, Corr. Memb. Royal and Imp. Acadd. of Sciences, Paris, Petersburgh, Florence, Copenhagen, Berlin, &c. &c.*

Received June 16,—Read June 18, 1835.

§ 16. *On an improved form of the Voltaic Battery.* § 17. *Some practical results respecting the construction and use of the Voltaic Battery.*

1119. I HAVE lately had occasion to examine the voltaic trough practically, with a view to improvements in its construction and use; and though I do not pretend that the results have anything like the importance which attaches to the discovery of a new law or principle, I still think they are valuable, and may therefore, if briefly told, and in connexion with former papers, be worthy the approbation of the Royal Society.

§ 16. *On an improved form of the Voltaic Battery.*

1120. In a simple voltaic circuit (and the same is true of the battery) the chemical forces which, during their activity, give power to the instrument, are generally divided into two portions; the one of these is exerted locally, whilst the other is transferred round the circle (947. 996.); the latter constitutes the electric current of the instrument, whilst the former is altogether lost or wasted. The ratio of these two portions of power may be varied to a great extent by the influence of circumstances: thus, in a battery not closed, *all* the action is local; in one of the ordinary construction, *much* is in circulation when the extremities are in communication; and in the perfect one, which I have described (1001.), *all* the chemical power circulates and becomes electricity. By referring to the quantity of zinc dissolved from the plates (865. 1126.), and the quantity of decomposition effected in the volta-electrometer (711. 1126.) or elsewhere, the proportions of the local and transferred actions under any particular circumstances can be ascertained, and the efficacy of the voltaic arrangement, or the waste of chemical power at its zinc plates, be accurately determined.

1121. If a voltaic battery were constructed of zinc and platina, the latter metal surrounding the former, as in the double copper arrangement, and the whole being excited by dilute sulphuric acid, then no insulating divisions of glass, porcelain, or air would be required between the contiguous platina surfaces; and, provided these did not touch metallically, the same acid which, being between the zinc and platina, would excite the battery into powerful action, would, between the two surfaces of

platina, produce no discharge of the electricity, nor cause any diminution of the power of the trough. This is a necessary consequence of the resistance to the passage of the current which I have shown occurs at the place of decomposition (1007. 1011.); for that resistance is fully able to stop the current, and therefore act as insulation to the electricity of the contiguous plates, inasmuch as the current which tends to pass between them never has a higher intensity than that due to the action of a single pair.

1122. If the metal surrounding the zinc be copper (1045.), and if the acid be nitro-sulphuric acid (1020.), then a slight discharge between the two contiguous coppers does take place, provided there be no other channel open by which the forces may circulate; but when such a channel is permitted, the return discharge of which I speak is exceedingly diminished, in accordance with the principles laid down in the eighth series of these Researches.

1123. Guided by these principles I was led to the construction of a voltaic trough, in which the coppers, passing round both surfaces of the zincs, as in WOLLASTON'S construction, should not be separated from each other except by an intervening thickness of paper, or in some other way, so as to prevent metallic contact, and should thus constitute an instrument compact, powerful, economical, and easy of use. On examining, however, what had been done before, I found that the new trough was in all essential respects the same as that invented and described by Dr. HARE, Professor in the University of Pennsylvania, to whom I have great pleasure in referring it.

1124. Dr. HARE has fully described his trough*. In it the contiguous copper plates are separated by thin veneers of wood, and the acid is poured on to, or off, the plates by a quarter revolution of an axis, to which both the trough containing the plates, and another trough to collect and hold the liquid, are fixed. This arrangement I have found the most convenient of any, and have therefore adopted it. My zinc plates were cut from rolled metal, and when soldered to the copper plates had the form delineated, fig. 1. These were then bent over a gauge into the form fig. 2, and when packed in the wooden box constructed to receive them, were arranged as in fig. 3†, little plugs of cork being used to keep the zinc plates from touching the

Fig. 1.

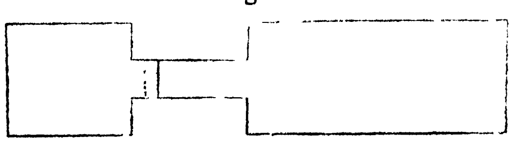


Fig. 2.

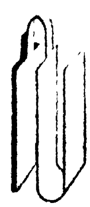
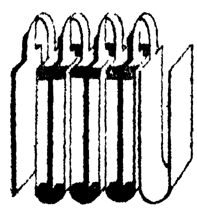


Fig. 3.



copper plates, and a single or double thickness of cartridge paper being interposed

* Philosophical Magazine, 1824, vol. lxiii. p. 241; or SILLIMAN'S Journal, vol. vii. See also a previous paper by Dr. HARE, Annals of Philosophy, 1821, vol. i. p. 329, in which he speaks of the non-necessity of insulation between the coppers.

† The papers between the coppers are, for the sake of distinctness, omitted in the figure.

between the contiguous surfaces of copper to prevent them from coming in contact. Such was the facility afforded by this arrangement, that a trough of forty pairs of plates could be unpacked in five minutes, and repacked again in half an hour; and the whole series was not more than fifteen inches in length.

1125. This trough, of forty pairs of plates three inches square, was compared, as to the ignition of a platina wire, the discharge between points of charcoal, the shock on the human frame, &c., with forty pairs of four-inch plates having double coppers, and used in porcelain troughs divided into insulating cells, the strength of the acid employed to excite both being the same. In all these effects the former appeared quite equal to the latter. On comparing a second trough of the new construction, containing twenty pairs of four-inch plates, with twenty pairs of four-inch plates in porcelain troughs, excited by acid of the same strength, the new trough appeared to surpass the old one in producing these effects, especially in the ignition of wire.

1126. In these experiments the new trough diminished in its energy much more rapidly than the one on the old construction, and this was a necessary consequence of the smaller quantity of acid used to excite it, which in the case of the forty pairs new construction was only one seventh part of that used for the forty pairs in the porcelain troughs. To compare, therefore, both forms of the voltaic trough in their decomposing powers, and to obtain accurate data as to their relative values, experiments of the following kind were made. The troughs were charged with a known quantity of acid of a known strength; the electric current was passed through a volta-electrometer (711.) having electrodes 4 inches long and 2·3 inches in width, so as to oppose as little obstruction as possible to the current; the gases evolved were collected and measured, and gave the quantity of water decomposed. Then the whole of the charge used was mixed together, and a known part of it analysed, by being precipitated and boiled with excess of carbonate of soda, and the precipitate well washed, dried, ignited, and weighed. In this way the quantity of metal oxidized and dissolved by the acid was ascertained; and the part removed from each zinc plate, or from all the plates, could be estimated and compared with the water decomposed in the volta-electrometer. To bring these to one standard of comparison, I have reduced the results so as to express the loss at the plates in equivalents of zinc for the equivalent of water decomposed at the volta-electrometer: I have taken the equivalent number of water as 9, and of zinc as 32·5, and have considered 100 cubic inches of the mixed oxygen and hydrogen, as they were collected over a pneumatic trough, to result from the decomposition of 12·68 grains of water.

1127. The acids used in these experiments were three,—sulphuric, nitric, and muriatic. The sulphuric acid was strong oil of vitriol; one cubical inch of it was equivalent to 486 grains of marble. The nitric acid was very nearly pure; one cubical inch dissolved 150 grains of marble. The muriatic acid was also nearly pure, and one cubical inch dissolved 108 grains of marble. These were always mixed with water by volumes, the standard of volume being a cubical inch.

1128. An acid was prepared consisting of 200 parts water, 4½ parts sulphuric acid, and 4 parts nitric acid; and with this both my trough, containing forty pairs of three-inch plates, and four porcelain troughs, arranged in succession, each containing ten pairs of plates with double coppers four inches square, were charged. These two batteries were then used in succession, and the action of each was allowed to continue for twenty or thirty minutes, until the charge was nearly exhausted, the connexion with the volta-electrometer being carefully preserved during the whole time, and the acid in the troughs occasionally mixed together. In this way the former trough acted so well, that for each equivalent of water decomposed in the volta-electrometer only from 2 to 2·5 equivalents of zinc were dissolved from each plate. In four experiments the average was 2·21 equivalents for each plate, or 88·4 for the whole battery. In the experiments with the porcelain troughs, the equivalents of consumption at each plate were 3·54, or 141·6 for the whole battery. In a perfect voltaic battery of forty pairs of plates (991. 1001.) the consumption would have been one equivalent for each zinc plate, or forty for the whole.

1129. Similar experiments were made with two voltaic batteries, one containing twenty pairs of four-inch plates, arranged as I have described (1124.), and the other twenty pairs of four-inch plates in porcelain troughs. The average of five experiments with the former was a consumption of 3·7 equivalents of zinc from each plate, or 74 from the whole: the average of three experiments with the latter was 5·5 equivalents from each plate, or 110 from the whole: to obtain this conclusion, two experiments were struck out, which were much against the porcelain troughs, and in which some unknown deteriorating influence was supposed to be accidentally active. In all the experiments, care was taken not to compare *new* and *old* plates together, as that would have introduced serious errors into the conclusions (1146.).

1130. When ten pairs of the new arrangement were used, the consumption of zinc at each plate was 6·76 equivalents, or 67·6 for the whole. With ten pairs of the common construction, in a porcelain trough, the zinc oxidized was, upon an average, 15·5 equivalents each plate, or 155 for the entire trough.

1131. No doubt, therefore, can remain of the equality or even the great superiority of this form of voltaic battery over the best previously in use, namely, that with double coppers, in which the cells are insulated. The insulation of the coppers may therefore be dispensed with; and it is that circumstance which principally permits of such other alterations in the construction of the trough as gives it its practical advantages.

1132. The advantages of this form of trough are very numerous and great. i. It is exceedingly compact, for 100 pairs of plates need not occupy a trough of more than three feet in length. ii. By Dr. HARE's plan of making the trough turn upon copper pivots which rest upon copper bearings, the latter afford *fixed* terminations; and these I have found it very convenient to connect with two cups of mercury, fastened in the front of the stand of the instrument. These fixed terminations give

the great advantage of arranging an apparatus to be used in connexion with the battery *before* the latter is put into action. iii. The trough is put into readiness for use in an instant, a single jug of dilute acid being sufficient for the charge of 100 pairs of four-inch plates. iv. On making the trough pass through a quarter of a revolution, it becomes active, and the great advantage is obtained of procuring for the experiment the effect of the *first contact* of the zinc and acid, which is twice or sometimes even thrice that which the battery can produce a minute or two after (1036. 1150.). v. When the experiment is completed, the acid can be at once poured from between the plates, so that the battery is never left to waste during an unconnected state of its extremities; the acid is not unnecessarily exhausted; the zinc is not uselessly consumed; and, besides avoiding these evils, the charge is mixed and rendered uniform, which produces a great and good result (1039.); and, upon proceeding to a second experiment, the important effect of *first contact* is again obtained. vi. The saving of zinc is very great. It is not merely that, whilst in action, the zinc performs more voltaic duty (1128. 1129.), but *all* the destruction which takes place with the ordinary forms of battery between the experiments is prevented. This saving is of such extent, that I estimate the zinc in the new form of battery to be thrice as effective as that in the ordinary form. vii. The importance of this saving of metal is not merely that the value of the zinc is saved, but that the battery is much lighter and more manageable; and also that the surfaces of the zinc and copper plates may be brought much nearer to each other when the battery is constructed, and remain so until it is worn out: the latter is a very important advantage (1148.). viii. Again, as, in consequence of the saving, thinner plates will perform the duty of thick ones, rolled zinc may be used; and I have found rolled zinc superior to cast zinc in action; a superiority which I incline to attribute to its greater purity (1144.). ix. Another advantage is obtained in the economy of the acid used, which is proportionate to the diminution of the zinc dissolved. x. The acid also is more easily exhausted, and is in such small quantity that there is never any occasion to return an old charge into use. Such old acid, whilst out of use, often dissolves portions of copper from the black flocculi usually mingled with it, which are derived from the zinc; now any portion of copper in solution in the charge does great harm, because, by the *local* action of the acid and zinc, it tends to precipitate upon the latter, and diminish its voltaic efficacy (1145.). xi. By using a due mixture of nitric and sulphuric acid for the charge (1139.), no gas is evolved from the troughs; so that a battery of several hundred pairs of plates may, without inconvenience, be close to the experimenter. xii. If, during a series of experiments, the acid becomes exhausted, it can be withdrawn, and replaced by other acid with the utmost facility; and after the experiments are concluded, the great advantage of easily washing the plates is at command. And it appears to me, that in place of making, under different circumstances, mutual sacrifices of comfort, power, and economy, to obtain a desired end, all are at once obtained by Dr. HARE's form of trough.

1133. But there are some disadvantages which I have not yet had time to overcome, though I trust they will finally be conquered. One is the extreme difficulty of making a wooden trough constantly water-tight under the alternations of wet and dry to which the voltaic instrument is subject. To remedy this evil, Mr. NEWMAN is now engaged in obtaining porcelain troughs. The other disadvantage is a precipitation of copper on the zinc plates. It appears to me to depend mainly on the circumstance that the papers between the coppers retain acid when the trough is emptied; and that this acid slowly acting on the copper, forms a salt, which gradually mingles with the next charge, and is reduced on the zinc plate by the local action (1120.): the power of the whole battery is then reduced. I expect that by using slips of glass to separate the coppers at their edges, their contact can be sufficiently prevented, and the space between them be left so open that the acid of a charge can be poured and washed out, and so be removed from *every part* of the trough when the experiments in which it is used are completed.

1134. The actual superiority of the troughs which I have constructed on this plan, I believe to depend, first and principally, on the closer approximation of the zinc and copper surfaces;—in my troughs they are only one tenth of an inch apart (1148.);—and, next, on the superior quality of the rolled zinc above the cast zinc used in the construction of the ordinary pile. It cannot be that insulation between the contiguous coppers is a disadvantage, but I do not find that it is any advantage; for when, with both the forty pairs of three-inch plates and the twenty pairs of four-inch plates, I used papers well imbibed with wax*, these being so large that when folded at the edges they wrapped over each other, so as to make cells as insulating as those of the porcelain troughs, still no sensible advantage in the chemical action was obtained.

1135. As, upon principle, there must be a discharge of part of the electricity from the edges of the zinc and copper plates at the sides of the trough, I should prefer, and intend having, troughs constructed with a plate or plates of crown glass at the sides of the trough: the bottom will need none, though to glaze that and the ends would be no disadvantage. The plates need not be fastened in, but only set in their places; nor need they be in large single pieces.

§ 17. *Some practical results respecting the construction and use of the Voltaic Battery.*

1136. The electro-chemical philosopher is well acquainted with some practical results obtained from the voltaic battery by MM. GAY-LUSSAC and THENARD, and given in the first forty-five pages of their 'Recherches Physico-Chimiques'. Although the following results are generally of the same nature, yet the advancement made in this branch of science of late years, the knowledge of the definite action of electricity,

* A single paper thus prepared could insulate the electricity of a trough of forty pairs of plates.

and the more accurate and philosophical mode of estimating the results by the equivalents of zinc consumed, will be their sufficient justification.

1137. *Nature and strength of the acid.*—My battery of forty pairs of three-inch plates was charged with acid consisting of 200 parts water and 9 oil of vitriol. Each plate lost, in the average of the experiments, 4·66 equivalents, or the whole battery 186·4 equivalents, of zinc, for the equivalent of water decomposed in the volta-electrometer. Being charged with a mixture of 200 water and 16 of the muriatic acid, each plate lost 3·8, or the whole battery 152, equivalents of zinc for the water decomposed. Being charged with a mixture of 200 water and 8 nitric acid, each plate lost 1·85, or the whole battery 74·16, equivalents of zinc for one equivalent of water decomposed. The sulphuric and muriatic acids evolved much hydrogen at the plates in the trough; the nitric acid no gas whatever. The relative strengths of the original acids have already been given (1127.); but a difference in that respect makes no important difference in the results when thus expressed by equivalents (1140.).

1138. Thus nitric acid proves to be the best for this purpose: its superiority appears to depend upon its favouring the electrolization of the liquid in the cells of the trough upon the principles already explained (905. 973. 1022.), and consequently favouring the transmission of the electricity, and therefore the production of transferable power (1120.).

1139. The addition of nitric acid might, consequently, be expected to improve sulphuric and muriatic acids. Accordingly, when the same trough was charged with a mixture of 200 water, 9 oil of vitriol, and 4 nitric acid, the consumption of zinc was at each plate 2·786, and for the whole battery 111·5, equivalents. When the charge was 200 water, 9 oil of vitriol, and 8 nitric acid, the loss per plate was 2·26, or for the whole battery 90·4, equivalents. When the trough was charged with a mixture of 200 water, 16 muriatic acid, and 6 nitric acid, the loss per plate was 2·11, or for the whole battery 84·4, equivalents. Similar results were obtained with my battery of twenty pairs of four-inch plates (1129.). Hence it is evident that the nitric acid was of great service when mingled with the sulphuric acid; and the charge generally used after this time for ordinary experiments consisted of 200 water, 4½ oil of vitriol, and 4 nitric acid.

1140. It is not to be supposed that the different strengths of the acids produced the differences above; for within certain limits I found the electrolytic effects to be nearly as the strengths of the acids, so as to leave the expression of force, when given in equivalents, nearly constant. Thus, when the trough was charged with a mixture of 200 water and 8 nitric acid, each plate lost 1·854 equivalent of zinc. When the charge was 200 water and 16 nitric acid, the loss per plate was 1·82 equivalent. When it was 200 water and 32 nitric acid, the loss was 2·1 equivalents. The differences here are not greater than happen from unavoidable irregularities, depending on other causes than the strength of acid.

1141. Again, when a charge consisting of 200 water, 4½ oil of vitriol, and 4 nitric

acid was used, each zinc plate lost 2·16 equivalents; when the charge with the same battery was 200 water, 9 oil of vitriol, and 8 nitric acid, each zinc plate lost 2·26 equivalents.

1142. I need hardly say that no copper is dissolved during the regular action of the voltaic trough. I have found that much ammonia is formed in the cells when nitric acid, either pure or mixed with sulphuric acid, is used. It is produced in part as a secondary result at the cathodes (663.) of the different portions of fluid constituting the necessary electrolyte, in the cells.

1143. *Uniformity of the charge.*—This is a most important point, as I have already shown experimentally (1042. &c.). Hence one great advantage of Dr. Hare's mechanical arrangement of his trough.

1144. *Purity of the zinc.*—If pure zinc could be obtained, it would be very advantageous in the construction of the voltaic apparatus (998.). Most zincs, when put into dilute sulphuric acid, leave more or less of an insoluble matter upon the surface in the form of a crust, which contains various metals, as copper, lead, zinc, iron, cadmium, &c., in the metallic state. Such particles, by discharging part of the transferable power, render it, as to the whole battery, local; and so diminish the effect. As an indication connected with the more or less perfect action of the battery, I may mention that no gas ought to rise from the zinc plates. The more gas which is generated upon these surfaces, the greater is the local action and the less the transferable force. The investing crust is also inconvenient, by preventing the displacement and renewal of the charge upon the surface of the zinc. Such zinc as, dissolving in the cleanest manner in a dilute acid, dissolves also the slowest, is the best; zinc which contains much copper should especially be avoided. I have generally found rolled Liege or Mosselman's zinc the purest; and to that circumstance attribute in part the advantage of the new battery (1134.).

1145. *Foulness of the zinc plates.*—After use, the plates of a battery should be cleaned from the metallic powder upon their surfaces, especially if they are employed to obtain the laws of action of the battery itself. This precaution was always attended to with the porcelain trough batteries in the experiments described (1125., &c.). If a few foul plates are mingled with many clean ones, they make the action in the different cells irregular, and the transferable power is accordingly diminished, whilst the local and wasted power is increased. No old charge containing copper should be used to excite a battery.

1146. *New and old plates.*—I have found voltaic batteries far more powerful when the plates were new than when they have been used two or three times; so that a new and a used battery cannot be compared together, or even a battery with itself on the first and after times of use. My trough of twenty pairs of four-inch plates, charged with acid consisting of 200 water, 4½ oil of vitriol, and 4 nitric acid, lost, upon the first time of being used, 2·32 equivalents per plate. When used after the fourth time with the same charge, the loss was from 3·26 to 4·47 equivalents per plate; the average

being 3·7 equivalents. The first time the forty pair of plates (1124.) were used, the loss at each plate was only 1·65 equivalent; but afterwards it became 2·16, 2·17, 2·52. The first time twenty pair of four-inch plates in porcelain troughs were used, they lost, per plate, only 3·7 equivalents; but after that, the loss was 5·25, 5·36, 5·9 equivalents. Yet in all these cases the zincs had been well cleaned from adhering copper, &c., before each trial of power.

1147. With the rolled zinc the fall in force soon appeared to become constant, i. e. to proceed no further. But with the cast zinc plates belonging to the porcelain troughs, it appeared to continue, until at last, with the same charge, each plate lost above twice as much zinc for a given amount of action as at first. These troughs were, however, so irregular that I could not always determine the circumstances affecting the amount of electrolytic action.

1148. *Vicinity of the copper and zinc.*—The importance of this point in the construction of voltaic arrangements, and the greater power, as to immediate action, which is obtained when the zinc and copper surfaces are near to each other than when removed further apart, are well known. I find that the power is not only greater on the instant, but also that the sum of transferable power, in relation to the whole sum of chemical action at the plates, is much increased. The cause of this gain is very evident. Whatever tends to retard the circulation of the transferable force, (i. e. the electricity,) diminishes the proportion of such force, and increases the proportion of that which is local (996. 1120.). Now the liquid in the cells possesses this retarding power, and therefore acts injuriously, in greater or less proportion, according to the quantity of it between the zinc and copper plates, i. e. according to the distances between their surfaces. A trough, therefore, in which the plates are only half the distance asunder at which they are placed in another, will produce more transferable, and less local, force than the latter; and thus, because the electrolyte in the cells can transmit the current more readily, both the intensity and quantity of electricity is increased for a given consumption of zinc. To this circumstance mainly I attribute the superiority of the trough I have described (1134.).

1149. The superiority of *double coppers* over single plates also depends in part upon diminishing the resistance offered by the electrolyte between the metals. For, in fact, with double coppers the sectional area of the interposed acid becomes nearly double that with single coppers, and therefore it more freely transfers the electricity. Double coppers are, however, effective, mainly because they virtually double the acting surface of the zinc, or nearly so; for in a trough with single copper plates and the usual construction of cells, that surface of zinc which is not opposed to a copper surface is thrown almost entirely out of voltaic action, yet the acid continues to act upon it and the metal is dissolved, producing very little more than local effect (947. 996.). But when by doubling the copper, that metal is opposed to the second surface of the zinc plate, then a great part of the action upon the latter is converted into transferable force, and thus the power of the trough as to quantity of electricity is highly exalted.

1150. *First immersion of the plates.*—The great effect produced at the first immersion of the plates, (apart from their being new or used (1146.),) I have attributed elsewhere to the unchanged condition of the acid in contact with the zinc plate (1003. 1037.): as the acid becomes neutralized, its exciting power is proportionably diminished. Hare's form of trough secures much advantage of this kind, by mingling the liquid, and bringing what may be considered as a fresh surface of acid against the plates every time it is used immediately after a rest.

1151. *Number of plates**.—The most advantageous number of plates in a battery used for chemical decomposition, depends almost entirely upon the resistance to be overcome at the place of action; but whatever that resistance may be, there is a certain number which is more economical than either a greater or a less. Ten pairs of four-inch plates in a porcelain trough of the ordinary construction, acting in the volta-electrometer (1126.) upon dilute sulphuric acid of spec. grav. 1·314, gave an average consumption of 15·4 equivalents per plate, or 154 equivalents on the whole. Twenty pairs of the same plates, with the same acid, gave only a consumption of 5·5 per plate, or 110 equivalents upon the whole. When forty pairs of the same plates were used, the consumption was 3·54 equivalents per plate, or 141·6 upon the whole battery. Thus the consumption of zinc arranged as twenty plates was more advantageous than if arranged either as ten or as forty.

1152. Again, ten pairs of my four-inch plates (1129.) lost 6·76 each, or the whole ten 67·6 equivalents of zinc, in effecting decomposition; whilst twenty pairs of the same plates, excited by the same acid, lost 3·7 equivalents each, or on the whole 74 equivalents. In other comparative experiments of numbers, ten pairs of the three-inch plates (1125.) lost 3·725, or 37·25 equivalents upon the whole; whilst twenty pairs lost 2·53 each, or 50·6 in all; and forty pairs lost on an average 2·21, or 88·4 altogether. In both these cases, therefore, increase of numbers had not been advantageous as to the effective production of *transferable chemical power* from the *whole quantity of chemical force* active at the surfaces of excitation (1120.).

1153. But if I had used a weaker acid or a worse conductor in the volta-electrometer, then the number of plates which would produce the most advantageous effect would have risen; or if I had used a better conductor than that really employed in the volta-electrometer, I might have reduced the number even to one; as, for instance, when a thick wire is used to complete the circuit (865., &c.). And the cause of these variations is very evident, when it is considered that each successive plate in the voltaic apparatus does not add anything to the *quantity* of transferable power or electricity which the first plate can put into motion, provided a good conductor be present, but tends only to exalt the *intensity* of that quantity, so as to make it more able to overcome the obstruction of bad conductors (994. 1158.).

1154. *Large or small plates*†.—The advantageous use of large or small plates for electrolyzations will evidently depend upon the facility with which the transferable

* Gay-Lussac and Thenard, Recherches Physico-Chimiques, tom. i. p. 29. † Ibid.

power or electricity can pass. If in a particular case the most effectual number of plates is known (1151.), then the addition of more zinc would be most advantageously made in increasing the *size* of the plates, and not their *number*. At the same time, large increase in the size of the plates would raise in a small degree the most favourable number.

1155. Large and small plates should not be used together in the same battery: the small ones occasion a loss of the power of the large ones, unless they be excited by an acid proportionably more powerful; for with a certain acid they cannot transmit the same portion of electricity in a given time which the same acid can evolve by action on the larger plates.

1156. *Simultaneous decompositions.*—When the number of plates in a battery much surpasses the most favourable proportion (1151—1153.), two or more decompositions may be effected simultaneously with advantage. Thus my forty pairs of plates (1124.) produced in one volta-electrometer 22·8 cubic inches of gas. Being recharged exactly in the same manner, they produced in each of two volta-electrometers 21 cubical inches. In the first experiment the whole consumption of zinc was 88·4 equivalents, and in the second only 48·28 equivalents, for the whole of the water decomposed in both volta-electrometers.

1157. But when the twenty pairs of four-inch plates (1129.) were tried in a similar manner, the results were in the opposite direction. With one volta-electrometer 52 cubic inches of gas were obtained; with two, only 14·6 cubic inches from each. The quantity of charge was not the same in both cases, though it was of the same strength; but on rendering the results comparative by reducing them to equivalents (1126.), it was found that the consumption of metal in the first case was 74, and in the second case 97, equivalents for the *whole* of the water decomposed. These results of course depend upon the same circumstances of retardation, &c., which have been referred to in speaking of the proper number of plates (1151.).

1158. That the *transferring*, or, as it is usually called, *conducting*, *power* of an electrolyte which is to be decomposed, or other interposed body, should be rendered as good as possible*, is very evident (1020. 1120.). With a perfectly good conductor and a good battery, nearly all the electricity is passed, i. e. *nearly all* the chemical power becomes transferable, even with a single pair of plates (867.). With an interposed non-conductor none of the chemical power becomes transferable. With an imperfect conductor more or less of the chemical power becomes transferable as the circumstances favouring the transfer of forces across the imperfect conductor are exalted or diminished: these circumstances are, actual increase or improvement of the conducting power, enlargement of the electrodes, approximation of the electrodes, and increased intensity of the passing current.

1159. The introduction of common spring water in place of one of the volta-electrometers used with twenty pairs of four-inch plates (1156.) caused such obstruction

* Gay-Lussac and Thenard, Recherches Physico-Chimiques, tom. i. pp. 13, 15, 22.

as not to allow one fifteenth of the transferable force to pass which would have circulated without it. Thus fourteen fifteenths of the available force of the battery were destroyed, being converted into local force, (which was rendered evident by the evolution of gas from the zincs,) and yet the platina electrodes in the water were three inches long, nearly an inch wide, and not a quarter of an inch apart.

1160. These points, i. e. the increase of conducting power, the enlargement of the electrodes, and their approximation, should be especially attended to in *volta-electrometers*. The principles upon which their utility depend are so evident that there can be no occasion for further development of them here.

Royal Institution,
October 11, 1834.

XV. *Discussion of Tide Observations made at Liverpool. By* JOHN WILLIAM LUBBOCK, *Esq. V.P. and Treas. R.S.*

Received and Read June 18, 1835.

BY permission of the British Association for the Advancement of Science, I am enabled to present to the Society a discussion by M. DESSIOU * of 13,327 observations of the tides made at Liverpool between the 1st of January 1774 and the 31st of December 1792. These observations, which were made by Mr. HUTCHINSON, Dockmaster at that place, belong to the Lyceum at Liverpool, and they were granted with the greatest readiness and liberality by the Committee of that Institution, upon the application of Mr. WHEWELL and myself, for the purposes of the present inquiry.

Mr. HUTCHINSON recorded the *solar time* of high water, and the height of the tide in feet and inches, at the Custom-House Dock gates, together with the direction and strength of the wind, and the state of the weather; also, during a great portion of the time, the height of the barometer and thermometer.

The following note is prefixed to the first book of these valuable observations: "These five years' observations upon the tides were made from solar time, and the winds from the true meridian, and their velocity judged according to Mr. SMEATON'S rule, our great storms going at the rate of sixty miles an hour; the thermometer kept in-doors, at the head of a staircase four stories high; by WILLIAM HUTCHINSON, at the Old Dock gates, Liverpool."

The following note is appended at the conclusion: "These observations, made from the beginning of 1768 to August 10, 1793, make twenty-five years, seven months and ten days, which I have given to our Library, exclusive of the 3000 observations given to Messrs. HOLDENS, to make their tide tables, as mentioned in their preface to them. I could not continue any longer to make observations, for want of the command of our dock gate men and gauge rod to take the night tides. Having resigned my place as Dockmaster, this journal ceases by me, WILLIAM HUTCHINSON."

These notes contain the only information with respect to the manner in which the observations were made which the books afford. The observations appear to have been carefully conducted, but no precautions are stated to have been taken to ensure the accuracy of the time; and it is difficult to fix whether by solar time is meant *apparent* solar or *mean* solar time: this point ought not to have been left in doubt. This point of uncertainty does not however, in any sensible degree, affect the Tables VI., VII., VIII., and IX., which have reference to the variations of the moon's parallax,

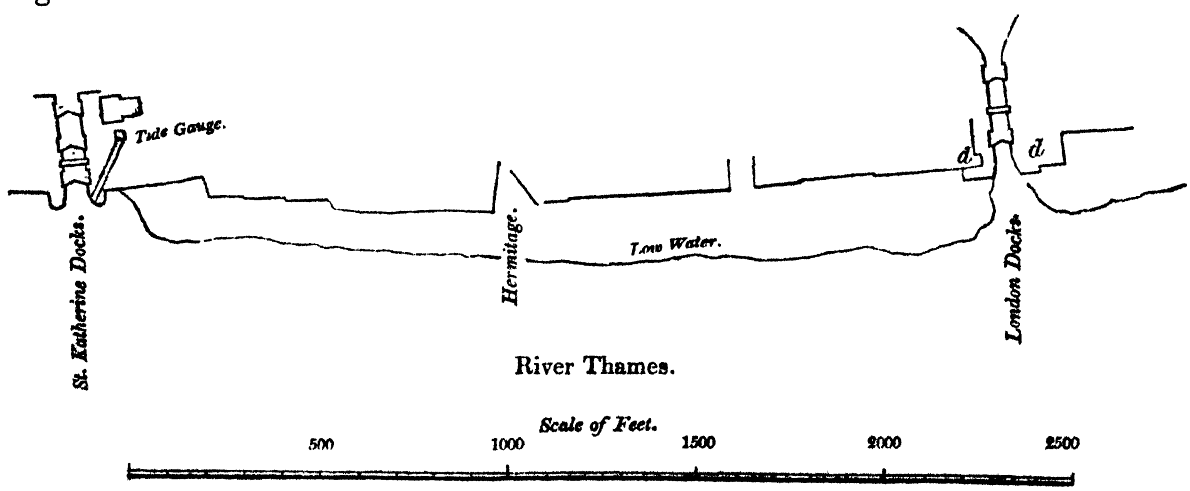
* M. DESSIOU has received from the British Association more than £100 for this work.

or those which have reference to the height of high water; but the equation of time is mixed up with the changes in the interval between the moon's transit and the time of high water due to changes in the moon's declination; and any uncertainty with respect to the manner in which the clock has been regulated is therefore much to be regretted.

The plan pursued is the same as that which I adopted with respect to the London Dock observations, and which seems to me to be the only one, in the present state of the subject, which can be resorted to with advantage. If the theory were complete, and the laws or analytical expressions of the phenomena had been made out satisfactorily, it would be possible to proceed at once to determine the constants, which might be done by means of fewer observations, and those might be selected which appeared entitled to the greatest confidence: at present, however, I do not think that this course can be safely pursued.

I trust that this laborious work, which M. Dessiou has accomplished at my instigation, and by the liberal support of the British Association, will not be without utility, and will afford data upon which mathematicians who may hereafter improve the theory of the tides may safely rest their conclusions.

The tides in this port continue to be observed at the London and St. Katherine Docks. These Docks are contiguous, so that the places at which the observations are made are not distant from each other more than 900 yards, as appears from the diagram underneath.



We may therefore, I think, safely conclude, that whatever cause affects the tide at one place will equally affect it at the other; and hence, if we find the difference in the registers of the times and heights of high water much greater than the average difference, suspicion arises that the observation at one or the other place must be erroneous. The observations at the London Docks are made (at the place of the letter *d*) by a person who notes the time when the water has begun to fall, that is, *has made its mark.* Those at the St. Katherine Docks are made by noting upon a slate (ruled for the purpose) the height of the water every minute for a short time

before high water is expected, all which is afterwards copied into a book ruled in the same manner, and the time of high water, with the height, is easily inferred. The height is ascertained by means of a rod or tide-gauge, connected with a float, which is placed in a chamber, into which the water enters through a culvert, so that the ripple or agitation of the water in the river is avoided as much as possible. A clock, carefully regulated, stands close at hand. This plan has been adopted at my suggestion, and, if the observer and the transcriber of the observation do their duty, it does not seem to me to be susceptible of any improvement.

I find by examining the registers of the observations at the London and the St. Katherine Docks, that the tide is on the average about five minutes later there than at the former place, and the difference in height between the lines or zero points, from which the rise is measured, is about five feet. Hence I formed Tables A. and B. by first adding five minutes to the time of high water, and five feet to the height of high water given in the London Dock books, and then comparing the times so altered with those of the St. Katherine Docks. The discrepancies exhibited by these Tables may be attributed to the carelessness of the observers at one or both places, or to the inevitable difficulty of measuring with precision the quantities sought. I next formed Tables C. and D. by adding five minutes to the time of high water given in the St. Katherine Docks register, and five feet to the predicted heights in the British Almanac, in order to compare the observations with the predictions given in the British Almanac, and which are founded upon the Tables, the construction of which I have explained in the Philosophical Transactions, excluding only those observations which, by discordance with the cotemporary observations at the London Docks and with the prediction, appear doubtful. It results from this comparison, that the average error of the predictions of the time of high water contained in the British Almanac is about ten minutes, that is, when the plus and minus errors are not allowed to counteract each other. The average error of Mr. Bulpit's Table is about the same. The tide predictions of Mr. Epps are evidently founded upon the same Tables and the same methods with those in White's Ephemeris, and do not agree quite so well with observation. Those of Mr. Innis are more inaccurate.

When the plus and minus errors are allowed to counteract each other, the average error of the predictions in the British Almanac being, from the observations of the first six months of this year, -9 minutes, leads me to suspect that a change in the establishment has taken place owing to the removal of the old London Bridge. If I am right in this conjecture, it is worthy of remark how sensibly the phenomena of the tides are affected by any slight alteration of local circumstances. Moreover, the height of high water appears to be less by 2 inches than formerly. If the predicted times be increased by 9 minutes and the heights be diminished by 2 inches, the predictions will then agree with observation, nearly as well as the observations at the London and St. Katherine Docks agree with each other.

Table A.

Showing a comparison of the observations of the Times of High Water made at the London Docks, increased by five minutes, and those at the St. Katherine Docks. The observations marked with an * appear doubtful.

Date. 1835.	January.			February.			March.			April.			May.			June.		
	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.	London Docks. +5 min.	St. Kath. Docks.	Difference.
	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m
1.	3 5	3 12	− 7	4 20	4 17	+ 3	3 15	3 6	+ 9	3 50	3 52	− 2	3 55	3 49	+ 6	4 45	4 42	+ 3
	3 35	3 47	−12	4 30	4 47	− 17	3 45	3 45	0	4 15	4 16	− 1	4 10	4 5	+ 5	4 10*	4 53	−43
2.	3 55	3 51	+ 4	4 55	4 57	− 2	3 50	3 41	+ 9	4 25	4 26	− 1	4 20	4 22	− 2	5 25	5 25	0
	4 25	4 32	− 7	5 20	5 28	− 8	4 15	4 22	− 7	4 45	4 42	+ 3	4 35	4 36	− 1	5 35	5 43	− 8
3.	4 45	4 38	+ 7	5 15	5 13	+ 2	4 20	4 24	− 4	4 45	4 44	+ 1	4 50	4 46	+ 4	6 25	6 20	+ 5
	5 10	5 9	+ 1	6 5	6 2	+ 3	4 25	4 28	− 3	5 5	5 3	+ 2	5 0	5 8	− 8	6 35	6 34	+ 1
4.	5 25	5 27	− 2	6 5	6 3	+ 2	4 50	4 51	− 1	5 20	5 18	+ 2	5 35	5 26	+ 9	7 5	7 17	−12
	6 5	5 56	+ 9	6 25	6 27	− 2	5 10	5 16	− 6	5 35	5 41	− 6	5 50	5 47	+ 3	7 25	......	......
5.	6 20	5 56	+24	6 45	6 37	+ 8	5 20	5 25	− 5	5 50	5 49	+ 1	6 15	6 6	+ 9	8 20	8 12	+ 8
	6 50	6 45	+ 5	7 15	7 17	− 2	5 35	5 37	− 2	6 10	6 12	− 2	6 35	6 37	− 2	8 55	8 57	− 2
6.	7 5	6 47	+18	7 25	7 23	+ 2	5 45	5 41	+ 4	6 35	6 32	+ 3	7 10	7 17	− 7	9 45	9 43	+ 2
	7 30	7 31	− 1	6 15*	8 6*	−111	6 15	6 16	− 1	7 5	7 2	+ 3	7 45	7 56	−11	10 5	10 6	− 1
7.	7 35	7 22	+13	7 50	7 42	+ 8	6 15	6 16	− 1	7 35	7 31	+ 4	8 25	8 20	+ 5	10 40	10 47	− 7
	8 20	8 12	+ 8	8 35	8 45	− 10	6 45	6 46	− 1	8 5	8 1	+ 4	9 5	9 10	− 5	11 20	11 3	+17
8.	8 20	8 11	+ 9	8 45	9 7	− 22	7 15	7 15	0	8 50	9 1	−11	10 10	10 7	+ 3	11 45	11 49	− 4
	9 10	9 8	+ 2	9 35	9 39	− 4	7 15	7 17	− 2	9 55	9 51	+ 4	10 45	10 44	+ 1	12 5	12 3	+ 2
9.	9 20	9 15	+ 5	10 15*	10 47	− 32	7 50	7 45	+ 5	10 35	10 42	− 7	11 15	11 16	− 1	......	......	......
	10 20	10 22	− 2	9 50*	10 25*	− 35	8 55	9 5	−10	11 25	11 28	− 3	11 45	11 50	− 5	12 45	12 43	+ 2
10.	9 15*	10 17	−62	......	......	......	9 5	9 0	+ 5	11 55	12 8	−13	......	......	......	12 55	12 58	− 3
	11 5	11 9	− 4	12 20*	10 42*	+ 98	9 35	10 0	−25	......	......	......	12 15	12 20	− 5	1 35	1 31	+ 4
11.	11 10	11 21	−11	12 35	12 27	+ 8	10 35	10 55	−20	12 20	12 18	+ 2	12 35	12 38	− 3	1 55	1 49	+ 6
	12 5	12 12	− 7	12 55	12 35	+ 20	12 0	11 58	+ 2	1 5	12 57	+ 8	12 55	1 3	− 8	2 15	2 19	− 4
12.	......	......	......	1 20	1 2	+ 18	12 0	12 6	− 6	1 20	1 12	+ 8	1 20	1 15	+ 5	2 35	2 43	− 8
	12 15	12 22	− 7	1 35	1 43	− 8	......	......	......	1 40	1 44	− 4	1 45	1 52	− 7	3 10	3 7	+ 3
13.	1 5	1 2	+ 3	1 55	2 2	− 7	12 50	12 57	− 7	1 55	1 45	+10	2 5	2 3	+ 3	3 25	3 33	− 8
	1 25	1 22	+ 3	2 30	2 28	+ 2	1 20	1 14	+ 6	2 20	2 25	− 5	2 35	2 43	− 8	3 45	3 51	− 6
14.	1 35	1 37	− 2	2 45	2 44	+ 1	1 45	1 47	− 2	2 40	2 35	+ 5	2 55	2 53	+ 2	4 20	4 18	+ 2
	1 55	2 2	7	3 15	3 17	− 2	2 5	2 7	− 2	3 5	3 7	− 2	3 20	3 27	− 7	4 35	4 37	− 2
15.	2 20	2 22	− 2	3 35	3 35	0	2 20	2 23	− 3	3 20	3 23	− 3	3 30	3 37	− 7	5 10	5 13	− 3
	2 35	2 36	− 1	3 50	3 48	+ 2	2 35	2 38	− 3	3 45	3 42	+ 3	4 0	4 2	− 2	5 25	5 23	+ 2
16.	2 55	3 1	− 6	4 0	4 3	− 3	2 55	2 57	− 2	4 5	3 57	+ 8	4 25	4 25	0	5 55	6 2	− 7
	3 0	3 1	− 1	4 25	4 33	− 8	3 25	3 27	− 2	4 25	4 22	+ 3	4 55	4 33	+22	6 10	6 8	+ 2
17.	3 30	3 36	− 6	4 45	4 53	− 8	3 45	3 31	+14	4 45	4 38	+ 7	5 15	5 18	− 3	6 50	6 54	− 4
	3 45	3 53	− 8	5 20	5 18	+ 2	4 5	4 16	−11	5 10	5 7	+ 3	5 35	5 32	+ 3	6 55	6 58	− 3
18.	4 10	4 2	+ 8	5 25	5 25	0	4 25	4 19	+ 6	5 25	5 27	− 2	6 10	6 12	− 2	7 50	7 42	+ 8
	4 40	4 46	− 6	6 0	6 12	− 12	4 50	4 56	− 6	5 50	5 57	− 7	6 20	6 27	− 7	7 55	7 55	0
19.	5 0	4 52	+ 8	6 10	6 7	+ 3	5 15	5 7	+ 8	6 10	6 17	− 7	7 10	7 12	− 2	8 50	8 50	0
	5 30	5 33	− 3	6 35	6 37	− 2	5 35	5 30	+ 5	6 40	6 43	− 3	7 20	7 17	+ 3	9 0	8 56	+ 4
20.	5 50	5 48	+ 2	7 5	6 47	+ 18	5 50	5 41	+ 9	7 25	7 22	+ 3	8 15	8 21	− 6	10 5	9 58	+ 7
	6 10	6 13	− 3	7 30	7 27	+ 3	6 20	6 17	+ 3	7 50	7 50	0	8 40	8 37	+ 3	10 10	10 7	+ 3
21.	6 35	6 33	+ 2	7 45	7 43	+ 2	6 35	6 31	+ 4	8 30	8 37	− 7	9 35	9 33	+ 2	10 40	10 52	−12
	7 10	7 11	− 1	8 25	8 30	− 5	7 5	7 37*	−32	8 55	9 7	−12	9 55	9 47	+ 8	11 5	10 57	+ 8
22.	7 25	7 25	0	8 55	8 52	+ 3	7 25	7 22	+ 3	10 5	10 13	− 8	10 45	10 44	+ 1	11 40	11 52	−12
	7 55	7 57	− 2	9 35	9 28	+ 7	8 10	8 16	− 6	10 30	10 27	+ 3	11 20	10 57	+23	11 55	11 54	+ 1
23.	8 25	8 16	+ 9	10 35	10 35	0	9 10	9 1	+ 9	11 35	11 37	− 2	11 50	11 45	+ 5	......	......	......
	9 5	9 7	− 2	11 5	11 2	+ 3	9 50	9 57	− 7	11 50	11 46	+ 4	11 50	11 59	− 9	12 20	12 22	− 2
24.	9 35	9 29	+ 6	11 25	11 23	+ 2	10 35	10 37	− 2	......	......	......	......	......	......	12 40	12 37	+ 3
	10 10	9 56	+14	......	......	......	11 20	11 22	− 2	12 25	12 26	− 1	12 35	12 36	− 1	1 10	1 3	+ 7
25.	10 35	10 41	− 6	12 15	12 15	0	12 0	12 7	− 7	12 45	12 37	+ 8	12 50	1 16	−26	1 25	1 12	+13
	11 25	11 27	− 2	12 50	12 52	− 2	......	......	......	1 15	......	......	1 15	1 33	−18	1 45	1 42	+ 3
26.	......	......	......	1 30	1 33	− 3	12 25	12 27	− 2	1 20	1 17	+ 3	1 20	1 12	+ 8	2 5	2 0	+ 5
	12 10	11 57	+13	1 35	1 32	+ 3	12 55	1 3	− 8	1 45	1 53	− 8	1 45	1 44	+ 1	2 20	2 15	+ 5
27.	12 45	12 42	+ 3	1 50	1 52	− 2	1 10	1 4	+ 6	1 55	1 54	+ 1	1 50	1 53	3	2 35	2 35	0
	1 5	1 7	− 2	2 25	2 28	− 3	1 45	1 47	− 2	2 15	2 22	− 7	2 15	2 17	− 2	2 55	2 53	+ 2
28.	1 40	1 31	+ 9	2 35	2 30	+ 5	2 10	1 52	+18	2 35	2 37	− 2	2 25	2 28	− 3	3 5	3 13	− 8
	2 0	2 10	−10	3 0	3 6	− 6	2 25	2 28	− 3	2 50	2 53	− 3	2 50	2 53	− 3	3 35	3 37	− 2
29.	2 25	2 25	0	......	......	......	2 30	2 27	+ 3	3 5	3 8	− 3	3 5	3 4	+ 1	3 55	3 57	− 2
	2 45	2 47	− 2	......	......	......	2 50	2 58	− 8	3 25	3 32	− 7	3 25	3 28	− 3	4 10	4 13	− 3
30.	3 10	3 2	+ 8	......	......	......	3 5	3 3	+ 2	3 30	3 26	+ 4	3 35	3 37	− 2	4 40	4 42	− 2
	3 25	3 32	− 7	......	......	......	3 20	3 22	− 2	3 40	3 35	+ 5	3 55	3 58	− 3	4 55	4 57	− 2
31.	3 45	3 44	+ 1	......	......	......	3 25	3 28	− 3	......	......	......	4 15	4 12	+ 3	......	......	......
	4 5	4 13	− 8	......	......	......	3 40	3 48	− 8	......	......	......	4 25	4 23	+ 2	......	......	......

Table B.

Showing a comparison of the observations of the Heights of High Water made at the London Docks, increased by five feet, and those at the St. Katherine Docks. The observations marked with an * appear doubtful.

Date. 1835.	January. London Docks. + 5 ft.	January. St. Kath. Docks.	January. Difference.	February. London Docks. + 5 ft.	February. St. Kath. Docks.	February. Difference.	March. London Docks. + 5 ft.	March. St. Kath. Docks.	March. Difference.	April. London Docks. + 5 ft.	April. St. Kath. Docks.	April. Difference.	May. London Docks. + 5 ft.	May. St. Kath. Docks.	May. Difference.	June. London Docks. + 5 ft.	June. St. Kath. Docks.	June. Difference.
	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in
1.	28 0	28 0	0	27 0	27 0	0	27 11	27 9	+ 2	26 9	26 10	− 1	27 4	27 2	+ 2	26 6	26 6	0
	29 6	29 6	0	26 4	26 3	+ 1	27 8	27 8	0	26 7	26 7	0	27 0	26 11	+ 1	25 10	25 10	0
2.	27 4	27 3	+ 1	25 10	25 11	− 1	28 4	28 2	+ 2	26 3	26 3	0	27 0	27 0	0	26 0	25 7	+ 5
	27 2	27 3	− 1	26 11	26 11	0	26 0	25 10	+ 2	26 9	26 3	+ 6	26 10	26 9	+ 1	25 5	25 4	+ 1
3.	26 9	26 9	0	26 8	26 8	0	27 0	26 11	+ 1	27 4	27 3	+ 1	26 9	26 8	+ 1	25 7	25 8	− 1
	26 11	26 10	+ 1	26 11	26 11	0	26 11	26 11	0	26 5	26 5	0	26 1	26 1	0	24 10	24 9	+ 1
4.	26 3	26 2	+ 1	25 9	25 10	− 1	29 1	29 1	0	26 6	26 4	+ 2	25 11	25 9	+ 2	25 5	25 4	+ 1
	26 0	26 0	0	25 11	25 11	0	25 7	25 6	+ 1	25 8	25 8	0	25 1	25 11	−10	24 6	......	......
5.	25 10	25 10	0	24 0	24 5	− 5	26 8	26 7	+ 1	25 7	25 7	0	25 3	25 3	0	25 4	23 0*	+28
	25 6	25 10	− 4	23 2*	23 1*	+ 1	26 3	26 4	− 1	24 7	24 4	+ 3	23 3	23 5	− 2	24 5	24 4	+ 1
6.	24 6	24 4	+ 2	24 8	24 9	− 1	22 5	22 3	+ 2	23 11	23 11	0	23 8	23 8	0	25 7	25 7	0
	24 7	24 8	− 1	24 6	24 6	0	25 3	25 3	0	23 3	23 3	0	23 6	23 6	0	25 0	25 1	− 1
7.	24 2	24 2	0	21 2	21 1	+ 1	24 5	24 4	+ 1	23 6	26 4*	−34	24 3	24 2	+ 1	26 0	26 2	− 2
	24 4	24 5	− 1	22 10	22 9	+ 1	22 8	22 7	+ 1	22 8	23 4	− 8	23 11	23 9	+ 2	25 10	25 11	− 1
8.	23 9	25 10*	−25	24 3	24 4	− 1	24 3	24 4	− 1	23 4	23 3	+ 1	24 10	24 10	0	27 0	26 11	+ 1
	23 7	23 8	− 1	23 0	23 0	0	23 2	23 2	0	23 8	23 8	0	24 7	24 9	− 2	26 10	26 6	+ 4
9.	22 6	22 6	0	25 1	25 1	0	21 9	21 7	+ 2	23 11	23 10	+ 1	25 11	26 0	− 1	......	......	......
	23 2	23 1	+ 1	21 5	21 0	+ 5	21 4	21 3	+ 1	24 9	24 9	0	25 6	25 5	+ 1	27 4	27 4	0
10.	25 9	25 8	+ 1	......	......	......	22 0	21 9	+ 3	27 3	27 4	− 1	......	......	......	27 5	27 6	− 1
	23 10	23 10	0	25 0	25 1	− 1	23 7	23 6	+ 1	......	......	......	26 8	26 8	0	27 10	27 11	− 1
11.	24 6	24 5	+ 1	23 7*	25 0	−17	21 2	21 1	+ 1	25 10	26 0	− 2	27 0	26 9	+ 3	28 0	27 10	+ 2
	25 3	25 3	0	24 2	23 8	+ 6	24 10	24 10	0	27 11	28 0	− 1	27 10	28 0	− 2	28 0	28 0	0
12.	......	......	......	24 3*	25 3	−12	25 7	25 8	− 1	26 8	26 7	+ 1	27 4	27 4	0	28 2	28 2	0
	25 4	25 3	+ 1	27 7	27 7	0	......	......	......	27 8	27 7	+ 1	28 5	28 6	− 1	28 1	28 0	+ 1
13.	24 6	24 7	− 1	26 3	26 8	− 5	26 8	26 8	0	27 5	27 1	+ 4	29 0	28 10	+ 2	28 4	28 4	0
	25 5	25 5	0	27 3	27 3	0	27 10	27 10	0	28 7	28 8	− 1	28 8	28 3	+ 5	27 11	27 10	+ 1
14.	26 3	26 3	0	27 6	27 7	− 1	26 8	26 9	− 1	28 3	28 3	0	30 2	30 3	− 1	28 2	28 2	0
	27 0	27 0	0	27 2	27 1	+ 1	26 3	26 3	0	28 9	28 9	0	29 10	29 10	0	27 3	27 3	0
15.	27 4	27 4	0	27 8	27 3	+ 5	28 3	28 4	− 1	29 1	29 1	0	29 6	29 6	0	27 6	27 7	− 1
	26 11	26 10	+ 1	27 7	27 7	0	28 11	29 0	− 1	29 9	29 8	+ 1	28 9	28 10	− 1	27 4	27 1	+ 3
16.	27 8	27 7	+ 1	27 4	27 4	0	28 5	28 6	− 1	29 11	29 10	+ 1	28 6	28 7	− 1	27 0	26 11	+ 1
	25 6	25 3	+ 3	28 3	27 4	+11	29 1	29 0	+ 1	29 0	29 1	− 1	27 3	28 1	−10	26 3	26 1	+ 2
17.	27 11	28 0	− 1	27 7	27 7	0	28 9	28 9	0	28 10	28 10	0	27 2	27 8	− 6	26 4	26 5	− 1
	28 3	28 0	+ 3	27 9	27 10	− 1	29 0	29 1	− 1	28 0	28 0	0	26 4	26 4	0	25 6	25 6	0
18.	27 11	27 10	+ 1	27 3	27 2	+ 1	29 0	29 0	0	27 4	27 5	− 1	26 11	27 0	− 1	25 4	25 3	+ 1
	27 3	27 4	− 1	25 8	25 7	+ 1	29 1	29 2	- 1	26 1	26 0	+ 1	26 0	25 11	+ 1	24 8	24 2	+ 6
19.	27 7	27 3	+ 4	27 4	27 4	0	28 4	28 4	0	25 7	27 6*	−23	26 1	26 0	+ 1	26 0*	23 11	+25
	27 5	27 6	− 1	25 10	25 9	+ 1	27 7	27 6	+ 1	25 1	25 1	0	24 6	24 5	+ 1	24 7	24 7	0
20.	29 0*	26 11	+25	24 3	24 4	− 1	27 7	27 7	0	25 0	25 1	− 1	25 3	25 2	+ 1	24 0	24 0	0
	27 9	27 6	+ 3	25 11	25 11	0	26 8	26 6	+ 2	24 2	24 1	+ 1	24 3	24 0	+ 3	24 9	24 8	+ 1
21.	25 5	25 5	0	26 0	26 1	− 1	26 7	26 6	+ 1	24 6	24 6	0	24 11	24 10	+ 1	25 0	24 11	+ 1
	25 6	25 7	− 1	25 3	25 3	0	25 7	25 7	0	23 3	23 2	+ 1	24 3	24 3	0	24 0	24 2	− 2
22.	25 10	26 0	- 2	25 0	24 11	+ 1	25 6	25 6	0	25 2	25 1	+ 1	25 1	25 0	+ 1	25 3	25 2	+ 1
	25 9	25 10	− 1	24 1	24 1	0	25 2	25 3	− 1	23 10	23 10	0	24 6	24 7	− 1	24 3*	26 1	−22
23.	24 10	25 0	− 2	20 10*	26 10	−72	24 2	24 4	− 2	23 8*	24 8	−12	25 8	25 9	− 1	......	......	......
	25 2	25 3	− 1	25 4	25 3	+ 1	23 8	23 7	+ 1	24 10	24 9	+ 1	25 3	25 1	+ 2	26 3	26 2	+ 1
24.	24 3	24 5	− 2	26 2	26 2	0	25 0	25 1	− 1	......	......	......	......	......	......	25 10	25 11	− 1
	26 6	26 6	0	......	......	......	23 9	23 9	0	25 9	25 10	− 1	26 3	26 2	+ 1	26 0	25 11	+ 1
25.	23 11	23 11	0	25 3	25 4	− 1	25 4	25 4	0	25 1	25 1	0	25 0	25 11	−11	24 0	23 9	+ 3
	26 2	26 3	− 1	24 3	24 3	0	......	......	......	26 6	......	......	25 10	25 8	+ 2	25 6	25 6	0
26.	......	......	......	25 11	24 11	+12	24 6	24 6	0	25 7	25 7	0	25 6	26 6	−12	28 0	27 10	+ 2
	25 4	25 4	0	26 7	27 7	−12	25 11	26 0	− 1	27 0	27 4	− 4	26 9	26 9	0	26 10	26 10	0
27.	26 3	26 4	− 1	26 3	26 3	0	26 3	26 3	0	26 6	26 5	+ 1	27 0	26 10	+ 2	28 6	28 6	0
	26 7	26 8	− 1	26 6	26 7	− 1	27 0	26 11	+ 1	26 8	26 8	0	27 5	27 6	− 1	26 9	27 9	−12
28.	26 7	26 7	0	26 11	26 9	+ 2	27 3	26 6	+ 9	26 7	26 7	0	27 3	27 2	+ 1	27 5	27 3	+ 2
	26 9	26 8	+ 1	28 8	28 9	− 1	27 0	27 2	− 2	27 3	27 2	+ 1	27 3	27 4	− 1	27 1	27 0	+ 1
29.	26 11	26 11	0	......	......	......	27 1	27 0	+ 1	28 0	27 11	+ 1	27 3	27 3	0	27 2	27 1	+ 1
	27 8	27 9	− 1	......	......	......	27 1	27 1	0	28 10	28 11	− 1	27 8	27 7	+ 1	26 9	26 10	− 1
30.	27 3	27 3	0	......	......	......	27 0	27 0	0	28 1	28 1	0	27 4	27 4	0	26 8	26 8	0
	27 10	27 10	0	......	......	......	27 6	27 6	0	27 4	27 5	− 1	27 1	27 1	0	26 6	26 7	− 1
31.	27 1	27 1	0	......	......	......	27 3	27 8	− 5	......	......	......	26 9	26 8	+ 1	......	......	......
	27 6	27 4	+ 2	......	......	......	27 5	27 5	0	......	......	......	26 3	26 3	0	......	......	......

Table C.

Showing a comparison of the observed Times of High Water at the St. Katherine Docks, increased by five minutes, with the predicted Times given in the British Almanac. The observations marked with an * appear doubtful.

Date. 1835.	January. British Alman.	January. St. Kath. Docks. +5 min.	January. Error of Prediction.	February. British Alman.	February. St. Kath. Docks. +5 min.	February. Error of Prediction.	March. British Alman.	March. St. Kath. Docks. +5 min.	March. Error of Prediction.	April. British Alman.	April. St. Kath. Docks. +5 min.	April. Error of Prediction.	May. British Alman.	May. St. Kath. Docks. +5 min.	May. Error of Prediction.	June. British Alman.	June. St. Kath. Docks. +5 min.	June. Error of Prediction.
	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m	h m	h m	m
1.	3 14	3 17	−3	4 20	4 22	−2	3 24	3 11	+13	3 56	3 57	−1	3 55	3 54	+1	4 41	4 47	−6
	3 33	3 52	−19	4 38	4 52	−14	3 42	3 50	−8	4 11	4 21	−10	4 11	4 10	+1	5 1	4 58	+3
2.	3 56	3 56	0	4 55	5 2	−7	3 58	3 46	+12	4 23	4 31	−8	4 25	4 27	−2	5 22	5 30	−8
	4 20	4 37	−17	5 12	5 33	−21	4 11	4 27	−16	4 36	4 47	−11	4 40	4 41	−1	5 46	5 48	−2
3.	4 40	4 43	−3	5 30	5 18	+12	4 26	4 29	−3	4 50	4 49	+1	4 56	4 51	+5	6 13	6 25	−12
	5 1	5 14	−13	5 45	6 7	−22	4 41	4 33	+8	5 3	5 8	−5	5 14	5 13	+1	6 40	6 39	+1
4.	5 20	5 32	−12	6 2	6 8	−6	4 56	4 56	0	5 20	5 23	−3	5 33	5 31	+2	7 6	7 22	−16
	5 43	6 1	−18	6 20	6 32	−12	5 9	5 21	−12	5 37	5 46	−9	5 55	5 52	+3	7 39	......	......
5.	6 2	6 1	+1	6 36	6 42	−6	5 25	5 30	−5	5 56	5 54	+2	6 20	6 11	+9	8 14	8 17	−3
	6 25	6 50	−25	6 55	7 22	−27	5 42	5 42	0	6 14	6 17	−3	6 46	6 42	+4	8 51	9 2	−11
6.	6 47	6 52	−5	7 14	7 28	−14	5 58	5 46	+12	6 36	6 37	−1	7 18	7 22	−4	9 27	9 48	−21
	7 11	7 36	−25	7 35	8 11*		6 13	6 21	−8	7 4	7 7	−3	7 55	8 1	−6	9 59	10 11	−12
7.	7 34	7 27	+7	8 0	7 47	+13	6 32	6 21	+11	7 33	7 36	−3	8 36	8 25	+11	10 30	10 52	−22
	7 55	8 17	−22	8 25	8 50	−25	6 50	6 51	−1	8 14	8 6	+8	9 18	9 15	+3	11 3	11 8	−5
8.	8 18	8 16	+2	9 3	9 12	−9	7 10	7 20	−10	9 0	9 6	−6	9 59	10 12	−13	11 34	11 54	−20
	8 46	9 13	−27	9 39	9 44	−5	7 36	7 22	+14	9 45	9 56	−11	10 40	10 49	−9	12 4	12 8	−4
9.	9 15	9 20	−5	10 17	10 52	−35	8 4	7 50	+14	10 30	10 47	−17	11 15	11 21	−6	......	......	......
	9 46	10 27	−41	11 1	10 30*		8 49	9 10	−21	11 12	11 33	−21	11 48	11 55	−7	12 31	12 48	−17
10.	10 19	10 22	−3	......	......	......	9 32	9 5	+27	11 45	12 13	−28	......	......	......	12 57	1 3	−6
	10 53	11 14	−21	11 36	10 47*		10 20	10 5	+15	......	......	......	12 15	12 25	−10	1 21	1 36	−15
11.	11 24	11 26	−2	12 8	12 32	−24	11 4	11 0	+4	12 15	12 23	−8	12 37	12 43	−6	1 46	1 54	−8
	11 50	12 17	−27	12 28	12 40	−12	11 39	12 3	−24	12 43	1 2	−19	12 59	1 8	−9	2 10	2 24	−14
12.	......		......	1 8	1 7	+1	12 4	12 11	−7	1 10	1 17	−7	1 20	1 20	0	2 36	2 48	−12
	12 18	12 27	−9	1 33	1 48	−15	......		......	1 35	1 49	−14	1 43	1 57	−14	3 2	3 12	−10
13.	12 42	1 7	−25	1 56	2 7	−11	12 43	1 2	−19	1 53	1 50	+3	2 5	2 8	−3	3 27	3 38	−11
	1 6	1 27	−21	2 21	2 33	−12	1 11	1 19	−8	2 11	2 30	−19	2 27	2 48	−21	3 53	3 56	−3
14.	1 29	1 42	−13	2 41	2 49	−8	1 34	1 52	−18	2 31	2 40	−9	2 51	2 58	−7	4 19	4 23	−4
	1 52	2 7	−15	3 1	3 22	−21	1 57	2 12	−15	2 53	3 12	−19	3 13	3 32	−19	4 37	4 42	−5
15.	2 13	2 27	−14	3 21	3 40	−19	2 18	2 28	−10	3 16	3 28	−12	3 37	3 42	−5	5 0	5 18	−18
	2 33	2 41	−8	3 40	3 53	−13	2 38	2 43	−5	3 38	3 47	−9	4 1	4 7	−6	5 23	5 28	−5
16.	2 54	3 6	−12	4 0	4 8	−8	3 0	3 2	−2	3 58	4 2	−4	4 24	4 30	−6	5 44	6 7	−23
	3 9	3 6	+3	4 22	4 38	−16	3 20	3 32	−12	4 20	4 27	−7	4 46	4 38	+8	6 9	6 13	−4
17.	3 33	3 41	−8	4 41	4 58	−17	3 39	3 36	+3	4 39	4 43	−4	5 11	5 23	−12	6 36	6 59	−23
	3 53	3 58	−5	5 1	5 23	−22	3 58	4 21	−23	5 1	5 12	−11	5 36	5 37	−1	7 3	7 3	0
18.	4 16	4 7	+9	5 24	5 30	−6	4 20	4 24	−4	5 22	5 32	−10	6 2	6 17	−15	7 28	7 47	−19
	4 38	4 51	−13	5 46	6 17	−31	4 39	5 1	−22	5 46	6 2	−16	6 27	6 32	−5	8 1	8 0	+1
19.	5 1	4 57	+4	6 7	6 12	−5	5 0	5 12	−12	6 8	6 22	−14	6 55	7 17	−22	8 32	8 55	−23
	5 24	5 38	−14	6 30	6 42	−12	5 22	5 35	−13	6 35	6 48	−13	7 28	7 22	+6	9 1	9 1	0
20.	5 51	5 53	−2	6 53	6 52	+1	5 44	5 46	−2	7 4	7 27	−23	8 1	8 26	−25	9 31	10 3	−32
	6 17	6 18	−1	7 15	7 32	−17	6 5	6 22	−17	7 38	7 55	−17	8 41	8 42	−1	10 1	10 12	−11
21.	6 39	6 38	+1	7 40	7 48	−8	6 28	6 36	−8	8 25	8 42	−17	9 20	9 38	−18	10 30	10 57	−27
	7 1	7 16	−15	8 13	8 35	−22	6 52	7 42*		9 12	9 12	0	9 59	9 52	+7	11 0	11 2	−2
22.	7 25	7 30	−5	8 54	8 57	−3	7 19	7 27	−8	9 58	10 18	−20	10 38	10 49	−11	11 31	11 57	−26
	7 48	8 2	−14	9 41	9 33	+8	7 52	8 21	−29	10 42	10 32	+10	11 15	11 2	+13	11 56	11 59	−3
23.	8 18	8 21	−3	10 28	10 40	−12	8 39	9 6	−27	11 21	11 42	−21	11 44	11 50	−6	......	......	......
	8 49	9 12	−23	11 11	11 7	+4	9 27	10 2	−35	11 55	11 51	+4	12 6	12 4	+2	12 18	12 27	−9
24.	9 23	9 34	−11	11 50	11 28	+22	10 17	10 42	−25	......	......	......	......	......	......	12 41	12 42	−1
	10 1	10 1	0	......		......	11 5	11 27	−22	12 22	12 31	−9	12 25	12 41	−16	1 3	1 8	−5
25.	10 43	10 46	−3	12 25	12 20	+5	11 45	12 12	−27	12 46	12 42	+4	12 44	1 21	−37	1 23	1 17	+6
	11 21	11 32	−11	12 57	12 57	0	......	......		1 8	......		1 1	1 38	−37	1 42	1 47	−5
26.	......	......	......	1 25	1 38	−13	12 16	12 32	−16	1 28	1 22	+6	1 20	1 17	+3	2 0	2 5	−5
	11 58	12 2	−4	1 52	1 37	+15	12 44	1 8	−24	1 44	1 58	−14	1 37	1 49	−12	2 18	2 20	−2
27.	12 31	12 47	−16	2 10	1 57	+13	1 10	1 9	+1	1 55	1 59	−4	1 52	1 58	−6	2 38	2 40	−2
	1 5	1 12	−7	2 31	2 33	−2	1 33	1 52	−19	2 10	2 27	−17	2 8	2 22	−14	2 57	2 58	−1
28.	1 31	1 36	−5	2 49	2 35	+14	1 50	1 57	−7	2 25	2 42	−17	2 23	2 33	−10	3 15	3 18	−3
	1 58	2 15	−17	3 7	3 11	−4	2 8	2 33	−25	2 40	2 58	−18	2 40	2 58	−18	3 33	3 42	−9
29.	2 19	2 30	−11	......	......	......	2 27	2 32	−5	2 54	3 13	−19	2 57	3 9	−12	3 52	4 2	−10
	2 42	2 52	−10	......	......	......	2 44	3 3	−19	3 8	3 37	−29	3 14	3 33	−19	4 12	4 18	−6
30.	3 4	3 7	−3	......	......	......	2 59	3 8	−9	3 24	3 31	−7	3 30	3 42	−12	4 32	4 47	−15
	3 25	3 37	−12	......	......	......	3 13	3 27	−14	3 39	3 40	−1	3 47	4 3	−16	4 53	5 2	−9
31.	3 43	3 49	−6	......	......	......	3 27	3 33	−6	......	......	......	4 7	4 17	−10	......	......	......
	4 2	4 18	−16	......	......	......	3 43	3 53	−10	......	......	......	4 26	4 28	−2	......	......	......

Table D.

Showing a comparison of the observed Heights of High Water at the St. Katherine Docks, with the predicted Heights given in the British Almanac, increased by five feet. The observations marked with an * appear doubtful.

Date.	January.			February.			March.		
1835.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.
	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.
1.	27 10	28 0	− 2	27 5	27 0	+ 5	27 6	27 9	− 3
	27 9	29 6	−21	27 3	26 3	+12	27 6	27 8	− 2
2.	27 8	27 3	+ 5	27 1	25 11	+14	27 6	28 2	− 8
	27 6	27 3	+ 3	26 10	26 11	− 1	27 5	25 10	+19
3.	27 3	26 9	+ 6	26 6	26 8	− 2	27 2	26 11	+ 3
	27 0	26 10	+ 2	26 3	26 11	− 8	27 0	26 11	+ 1
4	26 7	26 2	+ 5	25 11	25 9	+ 2	26 10	29 1	−27
	26 2	26 0	+ 2	25 7	25 11	− 4	26 8	25 6	+14
5.	25 11	25 10	+ 1	25 4	24 5	+11	26 5	26 7	− 2
	25 7	25 10	− 3	25 1	23 1*		26 2	26 4	− 2
6.	25 3	24 4	+11	24 9	24 9	0	25 10	22 3	+43
	25 0	24 8	+ 4	24 5	24 6	− 1	25 5	25 3	+ 2
7.	24 8	24 2	+ 6	24 2	21 1	+37	25 3	24 4	+11
	24 5	24 5	0	23 11	22 9	+14	24 11	22 7	+28
8.	24 2	25 10*		23 10	24 4	− 6	24 7	24 4	+ 3
	24 0	23 8	+ 4	23 10	23 0	+10	24 3	23 2	+13
9.	23 11	22 6	+17	24 1	25 1	−12	24 1	21 7	+30
	23 11	23 1	+10	24 7	21 0	+43	23 11	21 3	+32
10.	24 0	25 8	−20	......	......	......	24 1	21 9	+28
	24 3	23 10	+ 5	25 1	25 1	0	24 6	23 6	+12
11.	24 7	24 5	+ 2	25 6	25 0	+ 6	24 11	21 1	+46
	25 1	25 3	− 2	26 0	23 8	+28	25 6	24 10	+ 8
12.	......	......	......	26 7	25 3	+16	26 0	25 8	+ 4
	25 3	25 3	0	27 0	27 7	− 7	......	......	......
13.	25 10	24 7	+15	27 5	26 8	+ 9	26 8	26 8	0
	26 4	25 5	+11	27 9	27 3	+ 6	27 2	27 10	− 8
14.	26 8	26 3	+ 5	27 11	27 7	+ 4	27 6	26 9	+ 9
	27 0	27 0	0	28 1	27 1	+12	27 10	26 3	+19
15.	27 5	27 4	+ 1	28 3	27 3	+12	28 2	28 4	− 2
	27 7	26 10	+ 9	28 4	27 7	+ 9	28 4	29 0	− 8
16.	27 9	27 7	+ 2	28 5	27 4	+13	28 5	28 6	− 1
	27 10	25 3	+31	28 4	27 4	+12	28 6	29 0	− 6
17.	28 0	28 0	0	28 2	27 7	+ 7	28 6	28 9	− 3
	28 0	28 0	0	28 0	27 10	+ 2	28 6	29 1	− 7
18.	28 0	27 10	+ 2	27 9	27 2	+ 7	28 4	29 0	− 8
	27 11	27 4	+ 7	27 5	25 7	+22	28 2	29 2	−12
19.	27 9	27 3	+ 6	27 0	27 4	− 4	27 11	28 4	− 5
	27 6	27 6	0	26 7	25 9	+10	27 7	27 6	+ 1
20.	27 3	26 11	+ 4	26 0	24 4	+20	27 1	27 7	− 6
	27 0	27 6	− 6	25 5	25 11	− 6	26 6	26 6	0
21.	26 8	25 5	+15	25 0	26 1	−11	25 11	26 6	− 7
	26 3	25 7	+ 8	24 7	25 3	− 8	25 4	25 7	− 3
22.	25 10	26 0	− 2	24 4	24 11	− 7	24 10	25 6	− 8
	25 5	25 10	− 5	24 4	24 1	+ 3	24 5	25 3	−10
23.	25 1	25 0	+ 1	24 7	26 10	−27	24 2	24 4	− 2
	24 9	25 3	− 6	24 11	25 3	− 4	24 2	23 7	+ 7
24.	24 8	24 5	+ 3	25 4	26 2	−10	24 5	25 1	− 8
	24 9	26 6	−21	......	......	......	24 9	23 9	+12
25.	25 0	23 11	+13	25 9	25 4	+ 5	25 2	25 4	− 2
	25 4	26 3	−11	26 3	24 3	+24	......	......	.. ...
26.	......	......	......	26 7	24 11	+20	25 6	24 6	+12
	25 9	25 4	+ 5	26 10	27 7	− 9	25 10	26 0	− 2
27.	26 2	26 4	− 2	26 1	26 3	− 2	26 1	26 3	− 2
	27 7	26 8	+11	27 4	26 7	+ 9	26 7	26 11	− 4
28.	26 11	26 7	+ 4	27 6	26 9	+ 9	26 10	26 6	+ 4
	27 2	26 8	+ 6	27 6	28 9	−15	27 1	27 2	− 1
29.	27 5	26 11	+ 6	......	......	......	27 4	27 0	+ 4
	27 7	27 9	− 2	......	......	......	27 5	27 1	+ 4
30.	27 8	27 3	+ 5	......	......	......	27 5	27 0	+ 5
	27 8	27 10	− 2	......	......	......	27 5	27 6	− 1
31.	27 7	27 1	+ 6	......	......	......	27 4	27 8	− 4
	27 6	27 4	+ 2	......	......	......	27 3	27 5	− 2

Date.	April.			May.			June.		
1835.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.	British Alman. + 5 ft.	St. Kath. Docks.	Error of Prediction.
	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.	ft. in.	ft. in.	in.
1.	27 3	26 10	+ 5	27 2	27 2	0	26 11	26 6	+ 5
	27 2	26 7	+ 7	27 1	26 11	+ 2	26 9	25 10	+11
2.	27 0	26 3	+ 9	26 11	27 0	− 1	26 4	25 7	+ 9
	26 10	26 3	+ 7	26 9	26 9	0	26 2	25 4	+10
3.	26 8	27 3	− 7	26 6	26 8	− 2	25 11	25 8	+ 3
	26 6	26 5	+ 1	26 3	26 1	+ 2	25 8	24 9	+11
4	26 3	26 4	− 1	26 0	25 9	+ 3	25 5	25 4	+ 1
	25 11	25 8	+ 3	25 8	25 11	− 3	25 2		......
5.	25 7	25 7	0	25 4	25 3	+ 1	25 2	23 0*	
	25 3	24 4	+11	25 0	23 5	+19	25 3	24 4	+11
6.	24 11	23 11	+12	24 9	23 8	+13	25 5	25 7	− 2
	24 7	23 3	+16	24 8	23 6	+14	25 8	25 1	+ 7
7.	24 4	26 4*		24 8	24 2	+ 6	25 10	26 2	− 4
	24 3	23 4	+11	24 10	23 9	+13	26 1	25 11	+ 2
8.	24 3	23 3	+12	25 3	24 10	+ 5	26 5	26 11	− 6
	24 7	23 8	+11	25 9	24 9	+12	26 9	26 6	+ 3
9.	25 1	23 10	+15	26 3	26 0	+ 3	......	......	......
	25 8	24 9	+11	26 9	25 5	+16	27 0	27 4	− 4
10.	26 2	27 4	−14	......	......	......	27 4	27 6	− 2
	......	......	......	27 0	26 8	+ 4	27 7	27 11	− 4
11.	26 8	26 0	+ 8	27 4	26 9	+ 7	27 10	27 10	0
	27 2	28 0	−10	27 7	28 0	− 5	28 0	28 0	0
12.	27 8	26 7	+13	27 9	27 4	+ 5	28 1	28 2	− 1
	27 11	27 7	+ 4	28 0	28 6	− 6	28 2	28 0	+ 2
13.	28 1	27 1	+12	28 2	28 10	− 8	28 2	28 4	− 2
	28 3	28 8	− 5	28 3	28 3	0	28 1	27 10	+ 3
14.	28 4	28 3	+ 1	28 4	30 3	−23	27 10	28 2	− 4
	28 5	28 9	− 4	28 4	29 10	−18	27 7	27 3	+ 4
15.	28 6	29 1	− 7	28 4	29 6	−14	27 2	27 7	− 5
	28 6	29 8	−14	28 2	28 10	− 8	26 9	27 1	− 4
16.	28 5	29 10	−17	27 11	28 7	− 8	26 4	26 11	− 7
	28 3	29 1	−10	27 5	28 1	− 8	25 11	26 1	− 2
17.	27 11	28 10	−11	27 0	27 8	− 8	25 6	26 5	−11
	27 6	28 0	− 6	26 6	26 4	+ 2	25 1	25 6	− 5
18.	27 0	27 5	− 5	26 1	27 0	−11	24 9	25 3	− 6
	26 6	26 0	+ 6	25 6	25 11	− 5	24 6	24 2	+ 4
19.	25 11	27 6*		25 0	26 0	−12	24 3	23 11	+ 4
	25 3	25 1	+ 2	24 7	24 5	+ 2	24 2	24 7	− 5
20.	24 9	25 1	− 4	24 3	25 2	−11	24 1	24 0	+ 1
	24 4	24 1	+ 3	24 2	24 0	+ 2	24 1	24 8	− 7
21.	24 2	24 6	− 4	24 2	24 10	− 8	24 2	24 11	− 9
	24 1	23 2	+11	24 4	24 3	+ 1	24 4	24 2	+ 2
22.	24 3	25 1	−10	24 7	25 0	− 5	24 7	25 2	− 7
	24 6	23 10	+ 8	24 11	24 7	+ 4	24 10	26 1	−15
23.	24 11	24 8	+ 3	25 1	25 9	− 8	......	......	......
	25 3	24 9	+ 6	25 3	25 1	+ 2	25 2	26 2	−12
24.	......	......	......	......	......	......	25 5	25 11	− 6
	25 7	25 10	− 3	25 6	26 2	− 8	25 10	25 11	− 1
25.	25 11	25 1	+10	25 8	25 11	− 3	26 3	23 9	+30
	26 3	......	......	26 0	25 8	+ 4	26 6	25 6	+12
26.	26 7	25 7	+12	26 3	26 6	− 3	26 10	27 10	−12
	26 9	27 4	− 7	26 6	26 9	− 3	27 1	26 10	+ 3
27.	26 10	26 5	+ 5	26 8	26 10	− 2	27 4	28 6	−14
	26 11	26 8	+ 3	26 11	27 6	− 7	27 5	27 9	− 4
28.	27 1	26 7	+ 6	27 1	27 2	− 1	27 6	27 3	+ 3
	27 2	27 2	0	27 2	27 4	− 2	27 7	27 0	+ 7
29.	27 3	27 11	− 8	27 3	27 3	0	27 6	27 1	+ 5
	27 3	28 11	−20	27 4	27 7	− 3	27 6	26 10	+ 8
30.	27 3	28 1	−10	27 4	27 4	0	27 5	26 8	+ 9
	27 3	27 5	− 2	27 3	27 1	+ 2	27 3	26 7	+ 8
31.	......	......	......	27 2	26 8	+ 6	......	......	......
	......	......	......	27 1	26 3	+10	......	......	......

Table I.

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Old Docks (as recorded by Mr. Hutchinson), corresponding to the Apparent Solar Time of the Moon's Transit, in each month of the year. (If Mr. Hutchinson's clock was regulated according to *mean* solar time, the interval must be *diminished* by the equation of time given at foot of each month.)

Number of Observations.	Moon's Transit.	Interval.	Height of Tide.	Mean of Moon's Decl.	Number of Observations.	Moon's Transit.	Interval.	Height of Tide.	Mean of Moon's Decl.	Number of Observations.	Moon's Transit.	Interval.	Height of Tide.	Mean of Moon's Decl.
	January.					February.					March.			
	h m	h m	ft. in.	°		h m	h m	ft. in.	°		h m	h m	ft. in.	°
90	31·2	11 13·3	17 5·6	18	89	31·9	11 17·9	18 0·6	10	105	30·0	11 19·1	18 4·7	5
97	1 31·6	10 59·1	17 9·0	15	92	1 31·4	11 8·0	18 0·5	5	99	1 30·9	11 3·7	18 5·9	8
93	2 31·3	10 46·8	17 6·1	10	90	2 30·9	10 49·4	17 8·4	5	101	2 31·9	10 49·0	17 7·3	13
103	3 30·1	10 37·1	16 1·0	5	90	3 29·3	10 37·4	16 8·9	8	92	3 31·2	10 33·0	16 3·4	16
100	4 28·5	10 31·2	15 2·2	5	92	4 29·1	10 27·3	15 2·5	14	89	4 30·2	10 20·3	14 5·8	21
105	5 30·6	10 33·9	13 8·3	8	86	5 29·2	10 26·2	13 4·7	18	89	5 30·4	10 16·2	13 0·0	22
96	6 29·7	10 49·2	12 7·3	14	86	6 29·7	10 37·8	11 11.5	21	88	6 31·6	10 30·5	11 4·6	22
95	7 30·5	11 16·3	12 4·3	19	84	7 30·8	11 8·3	11 9·3	21	85	7 30·4	11 5·9	11 3·7	22
93	8 30·8	11 38·1	13 6·7	19	79	8 30·4	11 37·6	12 9·4	23	94	8 31·0	11 41·3	12 8·5	20
84	9 30·6	11 45·1	14 11·5	23	82	9 29·8	11 51·1	14 5·7	22	98	9 31·8	11 50·6	14 8·1	15
89	10 31·1	11 38·4	16 0·0	22	79	10 31·1	11 43·8	15 11·6	19	97	10 31·2	11 44·8	16 6·9	10
84	11 31·6	11 34·1	16 8·9	22	84	11 30·2	11 31·5	17 4·8	15	92	11 29·7	11 33·5	17 8·1	6
Equat. of Time *to be added to app. time*...		+10	...	...	...	...	+15	...	...	...	...	+9	...	...
	April.					May.					June.			
90	29·9	11 19·4	18 0·1	12	86	29·1	11 16·5	17 4·1	20	79	30·3	11 13·4	16 8·8	23
91	1 30·5	11 1·0	17 10·6	17	89	1 30·1	10 57·9	17 1·9	22	85	1 30·0	10 55·5	16 10·1	22
87	2 30·0	10 43·4	17 0·1	20	85	2 31·4	10 38·3	16 6·6	23	85	2 30·0	10 39·8	16 7·9	20
87	3 30·9	10 30·2	15 11·6	22	82	3 30·3	10 25·1	15 9·1	22	92	3 30·0	10 29·6	15 9·8	16
87	4 31·0	10 14·0	14 5·1	23	93	4 29·2	10 16·0	14 6·0	20	98	4 30·3	10 24·9	14 8·3	11
85	5 30·7	10 12·9	12 10·6	22	97	5 30·4	10 18·8	13 3·9	16	96	5 29·6	10 31·5	13 9·0	6
90	6 30·5	10 31·9	11 5·2	20	99	6 29·3	10 41·6	12 4·2	12	107	6 29·6	10 52·8	13 0·8	5
92	7 30·4	11 14·5	11 8·6	16	105	7 29·7	11 20·6	12 7·0	7	98	7 30·4	11 24·7	13 1·1	8
93	8 29·6	11 43·9	13 3·2	11	103	8 30·0	11 46·9	13 9·0	5	97	8 30·2	11 44·0	13 10·4	12
96	9 28·4	11 54·2	15 0·3	6	102	9 30·5	11 52·6	15 11·2	7	92	9 31·1	11 49·1	14 11·4	17
104	10 29·1	11 48·3	16 7·2	5	97	10 31·2	11 46·8	16 3·2	12	85	10 30·3	11 41·8	15 11·0	20
94	11 29·1	11 36·1	17 7·2	7	89	11 30·0	11 31·2	17 0·1	17	83	11 31·1	11 28·7	16 7·0	22
Equat. of Time *to be added to app. time*...		0	....	...	...	...	−4	...	...	...	...	0	...	...
	July.					August.					September.			
88	32·0	11 13·1	17 1·2	19	95	31·2	11 17·2	17 8·4	11	96	31·1	11 20·1	18 3·8	4
89	1 31.1	10 57·6	17 1·8	16	99	1 29·5	11 4·6	17 11·6	6	96	1 31·3	11 2·6	18 4·8	7
100	2 30·6	10 45·2	16 10·1	11	105	2 29·0	10 51·8	17 4·6	5	97	2 30·4	10 48·7	17 10·8	12
101	3 31·5	10 37·4	16 3·4	6	102	3 29·3	10 35·5	16 5·5	7	93	3 30·3	10 34·9	16 6·0	17
101	4 31·1	10 33·1	14 11·3	5	97	4 29·6	10 30·2	15 0·4	13	88	4 30·2	10 21·3	14 11·3	20
99	5 29·7	10 34·2	13 10·0	8	96	5 30·0	10 28·7	13 5·6	17	82	5 29·0	10 16·9	13 2·7	22
101	6 29·5	10 51·5	12 10·6	13	89	6 29·7	10 38·3	12 0·0	21	92	6 29·7	10 32·1	11 7·5	23
96	7 30·4	11 12·3	12 8·8	18	85	7 29·2	11 8·2	11 9·5	22	82	7 30·7	11 5·2	11 9·3	22
91	8 30·7	11 39·2	13 4·9	21	90	8 29·8	11 37·4	12 9·9	25	89	8 30·0	11 39·3	12 10·3	20
86	9 29·7	11 48·5	14 7·8	22	86	9 30·8	11 47·0	14 4·3	22	92	9 31·4	11 50·5	14 9·9	16
91	10 30·5	11 39·5	15 7·3	23	91	10 30·9	11 42·3	15 10·3	19	91	10 30·7	11 46·3	16 4·9	11
89	11 32·6	11 26·7	16 6·5	22	91	11 30·4	11 30·1	17 0·0	16	101	11 32·2	11 34·8	17 8·5	6
Equat. of Time *to be added to app. time*...		+ 5	...	...	....	...	+4	...	...	...	...	−5	...	...
	October.					November.					December.			
93	29·1	11 19·3	18 6·3	11	85	32·6	11 17·8	17 11·6	20	80	31·3	11 12·6	17 3·1	23
95	1 29·3	11 1·9	18 3·5	16	81	1 31·5	10 58·2	17 8·8	22	85	1 30·4	10 54·7	17 3·4	21
89	2 28·1	10 42·9	17 7·7	20	83	2 30·5	10 39·0	17 0·0	23	86	2 29·9	10 40·8	16 9·6	19
93	3 29·1	10 26·9	16 6·0	22	84	3 29·6	10 24·2	16 0·0	22	97	3 29·3	10 28·8	15 10·9	16
95	4 30·8	10 14·6	14 9·0	23	94	4 30·0	10 14·7	14 6·0	20	102	4 31·1	10 23·9	14 11·3	11
86	5 31·3	10 12·2	13 1·3	22	89	5 32·1	10 17·3	13 3·6	16	103	5 30·9	10 30·7	13 9·7	8
91	6 31·2	10 29·4	11 8·2	20	99	6 30·0	10 40·4	12 3·3	13	109	6 31·5	10 53·0	13 0·6	5
97	7 31·2	11 11·8	11 11·6	17	99	7 30·1	11 21·3	12 7·5	7	100	7 31·2	11 24·0	13 2·7	7
97	8 31·6	11 46·0	13 4·9	13	99	8 29·4	11 47·6	13 11·3	5	100	8 29·1	11 45·5	14 0·8	12
97	9 30·9	11 54·1	15 2·0	7	99	9 29·5	11 53·8	15 7·3	6	90	9 28·5	11 48·3	15 3·9	17
98	10 28·7	11 48·9	16 9·7	4	93	10 29·0	11 46·8	16 8·6	12	95	·10 28·9	11 42·5	16 4·6	20
96	11 29·8	11 35·3	17 11·1	7	96	11 30·8	11 32·7	17 6·1	17	84	11 30·6	11 27·6	17 1·7	21
Equat. of Time *to be added to app. time*...		−14	...	...	...	...	−15	...	...	...	...	−4	...	...

The argument of all the Tables is the time of the Moon's transit at Greenwich, taken immediately from the Nautical Almanac, which only amounts to neglecting the Moon's proper motion during twelve minutes.

TABLE II. (Interpolated from Table I.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at Liverpool Old Dock, for each month in the year.

Moon's Transit.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Mean.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
30	11 13·7	11 18·3	11 19·1	11 19·4	11 16·3	11 13·4	11 13·5	11 17·5	11 20·4	11 19·1	11 18·4	11 12·9	11 17·1
1 30	10 59·5	11 8·2	11 3·9	11 1·1	10 57·9	10 55·5	10 57·8	11 4·5	11 3·0	11 1·7	10 58·7	10 54·8	11 0·7
2 30	10 47·0	10 49·7	10 49·4	10 43·4	10 38·6	10 39·8	10 45·2	10 51·6	10 48·8	10 42·3	10 39·1	10 40·8	10 43·9
3 30	10 37·1	10 37·3	10 33·3	10 30·4	10 25·1	10 29·6	10 37·6	10 35·4	10 34·0	10 26·7	10 24·1	10 28·7	10 31·5
4 30	10 31·1	10 27·3	10 20·3	10 14·2	10 16·0	10 24·9	10 33·1	10 30·2	10 21·3	10 14·7	10 14·7	10 23·9	10 22·7
5 30	10 33·8	10 26·3	10 16·2	10 12·8	10 18·8	10 31·6	10 34·2	10 28·7	10 16·9	10 12·2	10 17·1	10 30·6	10 23·3
6 30	10 49·3	10 37·9	10 30·2	10 31·8	10 41·9	10 52·9	10 51·6	10 38·4	10 32·1	10 29·1	10 40·4	10 52·5	10 41·0
7 30	11 16·1	11 8·0	11 5·8	11 8·5	11 20·6	11 24·6	11 12·2	11 8·6	11 4·8	11 11·0	11 21·3	11 23·4	11 14·4
8 30	11 37·8	11 37·4	11 40·7	11 44·0	11 46·9	11 44·0	11 38·9	11 37·4	11 39·3	11 45·1	11 47·9	11 45·7	11 42·2
9 30	11 45·0	11 51·0	11 50·3	11 54·3	11 52·5	11 49·2	11 48·5	11 46·9	11 50·3	11 54·0	11 53·8	11 48·2	11 50·4
10 30	11 38·5	11 44·0	11 44·9	11 48·2	11 46·9	11 41·8	11 39·6	11 42·4	11 46·4	11 48·7	11 46·7	11 42·2	11 44·2
11 30	11 34·2	11 31·5	11 33·4	11 36·0	11 31·2	11 28·9	11 27·2	11 30·1	11 35·2	11 35·3	11 32·9	11 27·7	11 32·0

TABLE III. (Interpolated from Table I.)

Showing the Height of High Water at Liverpool Old Docks, corresponding to the Apparent Solar Time of the Moon's Transit, in each month of the year.

Moon's Transit.	January	February.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Mean.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
30	17·47	18·05	18·38	18·01	17·33	16·73	17·08	17·68	18·30	18·52	17·95	17·25	17·73
1 30	17·74	18·04	18·49	17·88	17·15	16·84	17·14	17·96	18·39	18·28	17·73	17·28	17·78
2 30	17·52	17·69	17·63	17·01	16·56	16·74	16·84	17·37	17·89	17·62	16·99	16·80	17·22
3 30	16·08	16·73	16·31	15·98	15·75	15·81	16·29	16·46	16·50	16·48	16·01	15·89	16·19
4 30	15·14	15·20	14·47	14·44	14·48	14·69	14·96	15·03	14·94	14·76	14·50	14·95	14·96
5 30	13·69	13·39	13·01	12·89	13·33	13·75	13·83	13·46	13·19	13·15	13·33	13·82	13·40
6 30	12·61	11·96	11·42	11·44	12·35	13·07	12·87	12·00	11·63	11·70	12·28	13·06	12·20
7 30	12·38	11·78	11·31	11·71	12·58	13·09	12·72	11·80	11·78	11·95	12·62	13·21	12·24
8 30	13·56	12·79	12·68	13·28	13·75	13·86	13·39	12·82	12·86	13·37	13·95	14·07	13·36
9 30	14·95	14·47	14·62	15·07	15·91	14·93	14·64	14·34	14·78	15·13	15·61	15·35	14·98
10 30	15·98	15·96	16·54	16·61	16·28	15·91	15·61	15·84	16·38	16·83	16·73	16·38	16·25
11 30	16·73	17·39	17·67	17·60	17·01	16·57	16·52	16·99	17·69	17·92	17·50	17·14	17·23

Table IV.

Showing the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Mean Interval, for every month in the year.

Moon's Transit.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
h m	m	m	m	m	m	m	m	m	m	m	m	m
30	− 3·4	+ 1·2	+ 2·0	+ 2·3	− 0·8	− 3·7	− 3·6	+ 0·4	+ 3·3	+ 2·0	+ 1·3	− 4·2
1 30	− 1·2	+ 8·2	+ 3·2	+ 1·6	− 2·8	− 5·2	− 2·9	+ 3·8	+ 2·3	+ 1·0	− 2·0	− 5·9
2 30	+ 3·1	+ 5·8	+ 5·5	− 0·5	− 5·3	− 4·1	+ 1·3	+ 7·7	+ 4·9	− 1·6	− 4·8	− 3·1
3 30	+ 5·6	+ 5·8	+ 1·8	− 1·1	− 6·4	− 1·9	+ 6·1	+ 3·9	+ 2·5	− 4·8	− 7·4	− 8·4
4 30	+ 8·4	+ 4·6	− 2·4	− 8·5	− 6·7	+ 2·2	+10·4	+ 7·5	− 1·4	− 8·0	− 8·0	− 7·2
5 30	+10·5	+ 3·0	− 7·1	−10·5	− 4·5	+ 8·3	+10·9	+ 5·4	− 6·4	−11·1	− 6·2	+ 7·3
6 30	+ 8·3	− 3·1	−10·8	− 9·2	+ 0·9	+11·9	+10·6	− 2·6	− 8·9	−11·9	− 0·6	+11·5
7 30	+ 1·7	− 6·4	− 8·6	− 5·9	+ 6·2	+10·2	− 3·9	− 5·8	− 9·3	+ 3·4	+ 6·9	+ 7·3
8 30	− 4·4	− 4·8	− 1·5	+ 1·8	+ 4·7	+ 1·8	− 3·3	− 0·4	− 1·5	+ 7·3	+ 5·7	+ 7·9
9 30	− 5·4	+ 0·6	− 0·1	+ 3·9	+ 2·1	− 1·2	− 1·9	+ 1·9	+ 5·3	+ 3·6	+ 8·8	− 2·2
10 30	− 5·7	− 0·2	+ 0·7	+ 4·0	+ 2·7	− 2·4	− 4·6	+ 3·9	+ 2·2	+ 4·5	+ 2·5	− 2·0
11 30	+ 2·2	− 0·5	+ 1·4	+ 4·0	− 0·8	− 5·3	− 4·8	− 1·9	+ 3·2	+ 3·3	+ 0·9	− 4·3

Table V.

Showing the Difference in the Height of High Water and the Mean Height for every month in the year.

Moon's Transit.	January.	February.	March.	April.	May.	June.	July	August.	Sept.	October.	Nov.	Dec.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
30	−·26	+·32	+·65	+·28	−·40	−1·00	−·65	−·05	+·57	+·79	+·22	−·48
1 30	−·04	+·26	+·71	+·10	−·63	− ·94	−·64	+·18	+·61	+·50	−·05	−·50
2 30	+·30	+·47	+·41	−·21	−·66	− ·48	−·38	+·15	+·67	+·40	−·23	−·42
3 30	−·11	+·54	+·12	−·21	−·44	− ·38	+·10	+·27	+·31	+·29	−·18	−·30
4 30	+·18	+·24	−·49	−·52	−·48	− ·27	00	+·07	−·02	−·20	−·46	−·01
5 30	+·29	−·01	−·39	−·51	−·07	+ ·35	+·43	+·06	−·21	−·25	−·07	+·42
6 30	+·41	−·24	−·78	−·76	+·15	+ ·87	+·67	−·20	−·57	−·50	+·08	+·86
7 30	+·14	−·46	−·93	−·53	+·34	+ ·85	+·48	−·44	−·46	−·29	+·38	+·97
8 30	+·20	−·57	−·68	−·08	+·19	+ ·50	+·03	−·54	−·50	+·01	+·59	+·71
9 30	−·03	−·51	−·36	+·09	+·93	− ·05	−·34	−·64	−·20	+·15	+·63	+·37
10 30	−·27	−·29	+·29	+·36	+·03	− ·34	−·64	−·41	+·13	+·58	+·48	+·13
11 30	−·50	+·16	+·44	+·37	−·22	− ·66	−·71	−·24	+·46	+·69	+·27	−·09

The quantities in this and the preceding Table are chiefly owing to the correction for the Moon's Declination.

Table VI.

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water, at the Liverpool Old Docks, corresponding to the Apparent Solar Time of the Moon's Transit, for every minute of her Horizontal Parallax.

Number of Observations.	Hor. Par. 54′.				Number of Observations.	Hor. Par. 55′.			
	Moon's Transit.	Interval.	Height of Tide.	Moon's Declination.		Moon's Transit.	Interval.	Height of Tide.	Moon's Declination.
	h m	h m	ft. in.	°		h m	h m	ft. in.	°
199	30·3	11 26·4	16 7·0	15	156	30·2	11 23·1	16 10·6	14
194	1 30·6	11 7·2	16 5·8	15	169	1 31·1	11 5·3	16 7·8	15
189	2 29·9	10 49·5	15 9·9	15	181	2 31·1	10 48·9	16 0·2	15
165	3 29·5	10 32·1	14 8·4	15	199	3 30·0	10 34·8	15 1·3	15
149	4 29·1	10 23·6	13 7·3	16	234	4 31·0	10 23·5	13 10·0	16
119	5 29·5	10 24·8	12 3·4	16	246	5 30·6	10 25·8	12 5·8	15
124	6 31·5	10 46·5	11 0·5	17	255	6 29·8	10 44·9	11 4·3	15
140	7 30·4	11 27·5	11 4·8	15	223	7 29·7	11 24·6	11 5·4	15
181	8 30·4	11 57·7	12 8·3	15	204	8 29·5	11 53·0	12 9·4	15
180	9 30·3	12 3·6	14 1·8	15	181	9 30·3	12 0·9	14 4·1	15
197	10 29·7	11 55·3	15 4·5	15	173	10 30·0	11 53·3	15 6·9	15
187	11 29·5	11 41·4	16 3·6	15	164	11 29·5	11 39·8	16 4·6	14
	Hor. Par. 56′.					Hor. Par. 57′.			
119	30·7	11 16·0	17 3·5	14	95	31·9	11 17·1	17 6·4	14
119	1 31·0	11 1·3	17 1·1	14	109	1 31·5	11 1·9	17 7·6	14
113	2 30·0	10 46·3	16 8·1	14	108	2 30·3	10 45·6	17 0·4	14
145	3 29·8	10 33·7	15 7·0	15	117	3 30·6	10 33·3	16 2·0	14
144	4 31·3	10 24·0	14 2·1	15	131	4 30·7	10 24·6	14 10·6	14
139	5 30·9	10 24·7	12 10·8	15	148	5 29·8	10 23·3	13 4·8	15
162	6 29·3	10 43·2	11 8·5	15	136	6 30·4	10 41·1	12 6·3	15
143	7 29·7	11 19·9	12 1·2	15	132	7 30·3	11 16·0	12 4·0	14
140	8 30·3	11 41·1	13 1·9	14	115	8 29·7	11 43·7	13 4·2	15
129	9 30·7	11 56·5	14 8·1	14	111	9 29·9	11 51·5	14 9·5	14
121	10 31·0	11 49·4	15 11·9	14	108	10 29·7	11 46·7	16 0·3	14
114	11 32·7	11 36·3	16 9·9	15	105	11 30·2	11 33·1	17 0·0	14
	Hor. Par. 58′.					Hor. Par. 59′.			
91	30·4	11 15·7	17 11·6	13	99	30·1	11 13·2	18 4·5	14
97	1 28·4	10 59·2	18 1·4	14	99	1 32·1	10 56·5	18 5·6	14
105	2 30·2	10 45·2	17 6·4	14	111	2 30·8	10 42·5	18 0·4	15
128	3 31·2	10 31·2	16 7·0	14	136	3 30·6	10 30·8	17 2·3	14
131	4 32·2	10 23·0	15 4·3	15	200	4 32·4	10 21·8	15 10·5	15
148	5 31·4	10 24·5	14 0·0	15	290	5 31·4	10 21·8	14 5·9	15
150	6 30·6	10 38·5	12 10·0	14	299	6 29·0	10 34·8	13 3·4	15
138	7 30·5	11 10·7	12 8·0	15	194	7 28·8	11 0·8	12 11·6	15
119	8 29·0	11 37·2	13 9·1	14	140	8 29·4	11 31·0	13 11·2	15
107	9 29·4	11 46·7	15 1·0	14	108	9 29·2	11 41·8	15 5·1	15
101	10 30·6	11 42·2	16 5·2	13	100	10 29·0	11 38·5	16 7·4	14
88	11 29·9	11 29·3	17 3·1	14	103	11 30·3	11 25·9	17 8·0	14
	Hor. Par. 60′.					Hor. Par. 61′.			
105	32·6	11 10·6	18 7·7	15	212	30·6	11 6·0	19 2·2	15
111	1 32·3	10 54·9	19 0·9	15	200	1 29·3	10 52·0	19 4·7	15
161	2 34·1	10 40·1	18 8·8	15	133	2 25·8	10 39·3	18 10·9	16
226	3 29·7	10 28·2	17 8·7	16					
147	4 27·0	10 20·5	16 3·4	16					
23	5 16·7	10 14·7	15 0·4	15					
21	6 45·7	10 43·1	13 0·8	16					
148	7 24·4	11 4·6	13 3·4	16					
226	8 32·0	11 25·8	14 1·7	16					
163	9 28·0	11 37·9	15 7·6	15	125	9 35·5	11 33·7	15 11·4	15
114	10 30·7	11 34·3	17 0·6	15	196	10 30·9	11 30·1	17 3·9	15
110	11 31·3	11 23·1	18 1·1	14	212	11 30·7	11 19·7	18 5·6	15

Mean of the preceding.

Moon's Transit.	Interval.	Height of Tide.
h m	h m	ft. in.
0 30·8	11 16·0	17 9·7
1 30·8	11 0·0	17 10·2
2 30·3	10 44·7	17 4·0
3 30·2	10 32·0	16 2·0
4 30·5	10 23·0	14 10·3
5 27·6	10 22·8	13 6·2
6 32·3	10 41·7	12 3·0
7 29·1	11 14·9	12 3·8
8 30·0	11 41·3	13 5·0
9 30·4	11 49·0	15 0·0
10 30·2	11 43·7	16 3·4
11 30·5	11 31·0	17 3·0

Table VII. (Interpolated from Table VI.)

	H. P. 54′.		H. P. 55′.		H. P. 56′.		H. P. 57′.		H. P. 58′.		H. P. 59′.		H. P. 60′.		H. P. 61′.	
Moon's Transit.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.
h m	h m	ft.	h m	ft.	h m	ft.	h m	ft.	h m	ft.	h m	ft.	h m	ft.	h m	ft.
30	11 26·4	16·58	11 23·1	16·88	11 16·3	17·29	11 18·1	17·53	11 15·9	17·96	11 13·2	18·37	11 11·8	18·62	11 6·3	19·17
1 30	11 7·5	16·46	11 4·7	16·64	11 1·8	17·09	11 2·6	17·63	10 58·4	18·12	10 57·5	18·45	10 56·0	19·09	10 51·7	19·39
2 30	10 49·5	15·82	10 49·5	16·02	10 46·3	16·67	10 45·7	17·04	10 45·1	17·54	10 42·8	18·04	10 41·9	18·75	10 38·4	18·87
3 30	10 31·3	14·69	10 34·8	15·11	10 33·6	15·58	10 33·5	16·17	10 31·6	16·58	10 31·0	17·19	10 28·1	17·70		
4 30	10 23·7	13·59	10 23·7	13·86	10 24·3	14·20	10 24·7	14·89	10 23·3	15·36	10 21·9	15·87	10 19·6	17·72		
5 30	10 25·0	12·28	10 25·8	12·49	10 24·7	12·91	10 23·3	13·39	10 24·3	14·01	10 21·8	14·49				
6 30	10 45·3	11·07	10 45·1	11·36	10 43·8	11·70	10 40·8	12·53	10 38·1	12·84	10 35·5	13·29				
7 30	11 27·0	11·39	11 24·9	11·45	11 20·2	12·10	11 15·7	12·33	11 10·2	12·66	11 1·9	12·96	11 8·5	13·29		
8 30	11 57·5	12·68	11 53·3	12·79	11 48·4	13·15	11 43·9	13·36	11 37·8	13·77	11 31·4	13·93	11 24·7	14·10		
9 30	12 3·6	14·14	12 0·8	14·33	11 56·2	14·66	11 51·5	14·79	11 47·8	15·09	11 41·9	15·42	11 38·3	15·68	11 33·5	16·06
10 30	11 55·2	15·38	11 53·3	15·57	11 49·6	15·97	11 46·6	16·04	11 42·3	16·43	11 38·3	16·62	11 34·6	17·04	11 30·3	17·31
11 30	11 41·2	16·30	11 39·6	16·39	11 34·9	16·79	11 32·2	17·00	11 29·3	17·26	11 26·0	17·66	11 22·9	18·08	11 19·9	18·45

Table VIII.

Showing the Difference in the Interval between the Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to fifty-seven minutes of the Moon's Horizontal Parallax.

Moon's Transit.	H. P. 54′.	H. P. 55′.	H. P. 56′.	H. P. 57′.	H. P. 58′.	H. P. 59′.	H. P. 60′.	H. P. 61′.
h m	m	m	m	m	m	m	m	m
30	+ 8·3	+5·0	−1·8	0	−2·2	− 4·9	− 6·3	−11·8
1 30	+ 4·9	+2·1	+0·8	0	−4·2	− 5·1	− 6·6	−10·9
2 30	+ 3·8	+3·8	+0·6	0	−0·6	− 2·9	− 3·8	− 7·3
3 30	− 2·2	+1·3	+0·1	0	−1·9	− 2·5	− 5·4	
4 30	− 1·0	−1·0	−0·4	0	−1·4	− 2·8	− 4·9	
5 30	+ 1·7	+2·5	+1·4	0	+1·0	− 1·5		
6 30	+ 4·5	+4·3	+3·0	0	−2·7	− 5·3		
7 30	+11·3	+9·2	+4·5	0	−5·5	−13·8	− 7·2	
8 30	+13·6	+9·4	+4·5	0	−6·1	−12·5	−19·2	
9 30	+12·1	+9·3	+4·7	0	−3·7	− 9·6	−13·2	−18·0
10 30	+ 8·6	+6·7	+3·0	0	−4·3	− 8·3	−12·0	−16·3
11 30	+ 8·0	+6·4	+1·7	0	−3·9	− 7·2	−10·3	−13·3

Table IX.

Showing the Difference between the Height of High Water and the Height corresponding to fifty-seven minutes of the Moon's Horizontal Parallax.

Moon's Transit.	H. P. 54′.	H. P. 55′.	H. P. 56′.	H. P. 57′.	H. P. 58′.	H. P. 59′.	H. P. 60′.	H. P. 61′.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
30	− ·95	− ·65	−·24	0	+·46	+ ·84	+1·09	+1·64
1 30	−1·17	− ·99	−·54	0	+·49	+ ·82	+1·46	+1·76
2 30	−1·22	−1·02	−·37	0	+·50	+1·00	+1·71	+1·83
3 30	−1·48	−1·06	−·59	0	+·41	+1·02	+1·53	
4 30	−1·30	−1·03	−·69	0	+·47	+ ·98	+2·83	
5 30	−1·11	− ·90	−·48	0	+·62	+1·10		
6 30	−1·46	−1·17	−·83	0	+·31	+ ·76		
7 30	− ·94	− ·88	−·23	0	+·33	+ ·63	+ ·96	
8 30	− ·68	− ·57	−·21	0	+·41	+ ·57	+ ·74	
9 30	− ·65	− ·46	−·13	0	+·30	+ ·63	+ ·89	+1·27
10 30	− ·66	− ·47	−·07	0	+·39	+ ·58	+1·00	+1·27
11 30	− ·70	− ·61	−·21	0	+·26	+ ·66	+1·08	+1·45

Table X.

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water, at the Liverpool Old Docks, corresponding to the Apparent Solar Time of the Moon's Transit, for every three degrees of her Declination north and south. The Equation of Time to be added to Apparent Time.

1° 30′ N. to 1° 30′ South Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
48	29·3	+ 2·5	11 22·2	18 4·0	57·0
45	1 29·5	+ 6·0	11 5·0	18 3·3	57·4
52	2 28·7	+ 9·2	10 51·6	17 6·0	57·0
51	3 28·2	+ 8·8	10 39·3	16 9·5	57·1
46	4 31·3	+ 6·1	10 31·4	15 0·1	56·8
44	5 30·9	+ 2·8	10 34·5	14 1·3	57·0
39	6 30·6	− 2·3	10 54·8	13 0·9	56·8
46	7 29·7	− 5·8	11 23·3	12 11·0	56·8
47	8 28·5	− 9·1	11 46·8	14 1·3	57·0
46	9 28·6	− 9·3	11 53·8	15 1·6	57·1
55	10 33·3	− 6·5	11 47·8	16 10·5	57·3
40	11 34·1	− 2·2	11 33·5	16 10·3	57·9

1° 30′ to 4° 30′ North Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
54	27·3	+ 2·2	11 21·1	18 3·5	57·2
52	1 30·0	+ 6·3	11 5·7	18 3·7	57·1
45	2 29·2	+ 8·2	10 51·4	17 1·1	57·5
54	3 33·2	+ 8·6	10 37·5	16 4·5	57·0
50	4 30·2	+ 6·1	10 28·1	15 2·1	56·9
42	5 31·0	+ 2·6	10 35·4	13 7·4	56·7
49	6 29·5	+ 2·1	10 54·0	13 1·6	56·9
43	7 29·0	− 6·6	11 23·6	13 2·9	57·1
47	8 27·3	− 7·9	11 47·0	13 10·0	57·0
43	9 27·6	− 8·9	11 50·1	15 5·6	57·4
50	10 27·5	− 6·4	11 44·2	16 8·4	57·0
45	11 25·3	− 2·7	11 37·4	17 7·6	57·3

4° 30′ to 7° 30′ North Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
55	31·3	+ 1·6	11 19·6	18 2·0	57·3
47	1 30·0	+ 6·5	11 5·7	18 5·9	57·2
48	2 30·9	+ 6·8	10 50·4	17 8·5	57·1
48	3 29·1	+ 7·8	10 38·6	16 4·7	56·8
46	4 31·8	+ 5·5	10 31·0	15 0·0	57·0
42	5 31·3	+ 2·3	10 42·4	13 9·1	56·7
45	6 31·8	− 2·8	10 52·6	12 8·2	57·3
45	7 30·0	− 5·2	11 24·2	13 2·4	58·2
44	8 31·1	− 7·2	11 49·9	13 5·9	56·4
53	9 28·2	− 7·7	11 53·8	15 2·0	57·1
46	10 33·8	− 5·2	11 46·7	16 8·6	57·3
50	11 31·0	− 1·7	11 34·3	17 2·9	57·5

1° 30′ to 4° 30′ South Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
52	30·4	+ 3·0	11 18·3	18 6·6	57·7
41	1 30·5	+ 7·0	11 3·6	18 3·5	58·0
48	2 30·9	+ 8·1	10 49·8	17 7·1	57·5
41	3 31·9	+ 8·3	10 39·9	16 6·2	56·7
41	4 29·3	+ 6·6	10 31·6	15 5·3	57·2
45	5 29·0	+ 2·3	10 34·5	13 9·1	56·6
49	6 30·1	+ 1·1	10 53·5	12 11·4	57·0
44	7 33·1	− 6·5	11 27·1	13 0·1	57·0
45	8 32·4	− 8·1	11 41·3	13 1·6	57·3
54	9 30·0	− 8·6	11 54·4	15 3·8	56·7
47	10 28·5	− 6·0	11 41·8	16 8·0	57·3
54	11 31·1	− 4·6	11 34·8	17 9·0	57·4

4° 30′ to 7° 30′ South Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
53	29·8	+ 2·5	11 20·4	18 0·0	57·3
43	1 31·3	+ 6·0	11 5·2	18 6·0	57·6
46	2 31·3	+ 8·0	10 50·4	17 8·7	57·6
44	2 32·0	+ 7·9	10 38·3	16 6·4	57·3
45	4 30·0	+ 5·0	10 29·6	15 5·2	56·8
47	5 27·0	+ 3·1	10 32·7	14 1·4	57·0
51	6 29·4	− 2·1	10 51·0	12 10·3	57·0
50	7 30·4	− 5·1	11 21·6	12 11·5	57·0
40	8 29·0	− 6·8	11 47·0	13 10·0	56·9
48	9 31·9	− 7·5	11 54·5	15 6·3	57·2
46	10 30·3	− 5·6	11 44·9	16 11·5	57·8
52	11 30·2	− 1·9	11 32·0	18 0·0	57·7

7° 30′ to 10° 30′ North Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
52	30·1	+ 1·3	11 20·3	18 1·1	57·1
54	1 30·7	+ 4·3	11 4·7	18 2·1	57·2
50	2 28·7	+ 6·0	10 49·2	17 5·1	57·3
55	3 27·8	+ 5·9	10 41·5	16 5·2	56·6
54	4 29·7	+ 5·1	10 30·3	14 11·3	56·7
45	5 28·9	+ 1·2	10 32·1	13 9·4	56·4
51	6 27·5	− 1·5	10 49·7	12 8·3	56·6
48	7 27·8	− 4·8	11 22·5	12 7·2	56·5
48	8 31·7	− 6·6	11 45·2	14 1·7	57·1
58	9 30·2	− 6·2	11 54·8	15 1·3	56·7
51	10 33·3	− 4·5	11 46·4	16 6·7	57·3
47	11 30·8	− 1·1	11 35·6	17 7·9	56·9

10° 30′ to 13° 20′ North Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
54	30·8	+ 1·2	11 19·3	18 0·1	57·3
64	1 32·3	+ 2·3	11 3·4	17 10·2	56·2
45	2 31·7	+ 3·8	10 48·8	17 4·5	57·0
53	3 29·9	+ 3·9	10 35·3	16 0·4	56·8
51	4 30·7	+ 2·6	10 28·0	14 10·4	56·7
57	5 30·4	+ 1·3	10 27·4	13 6·3	56·7
54	6 30·4	− 1·2	10 48·0	12 4·6	56·5
60	7 27·4	− 2·3	11 20·1	12 6·5	56·5
59	8 30·0	− 4·2	11 48·5	13 7·7	56·5
59	9 30·5	− 2·4	11 52·0	14 11·2	57·0
60	10 29·1	− 2·6	11 48·5	16 2·6	56·9
61	11 27·0	− 1·4	11 36·6	17 2·5	56·8

13° 30′ to 16° 30′ North Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
58	31·6	+ 0·1	11 18·8	17 9·6	56·7
54	1 28·0	0·0	11 3·1	17 7·8	56·7
66	2 28·9	+ 1·5	10 47·3	16 10·8	55·8
67	3 27·6	+ 0·1	10 33·2	16 1·7	56·5
71	4 28·8	+ 0·1	10 25·2	14 8·1	56·4
75	5 29·0	+ 1·0	10 27·1	13 2·1	56·2
72	6 30·6	0·0	10 44·7	12 3·1	56·4
69	7 32·2	− 1·0	11 20·8	12 1·6	56·3
63	8 31·4	− 0·3	11 46·0	13 4·3	56·7
64	9 31·3	− 0·4	11 52·6	14 11·3	56·0
71	10 29·4	− 0·8	11 46·6	16 6·1	56·8
70	11 32·5	0·0	11 34·1	17 1·1	56·8

7° 30′ to 10° 30′ South Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
42	29·9	+ 1·2	11 18·7	18 4·9	57·8
58	1 29·0	+ 4·8	11 3·9	18 5·0	57·7
52	2 30·5	+ 5·2	10 49·1	17 8·2	57·3
51	3 27·3	+ 5·8	10 37·0	16 9·9	57·4
49	4 32·3	+ 5·6	10 29·5	15 1·8	57·4
46	5 32·7	+ 1·1	10 32·1	13 8·7	56·9
44	6 32·6	− 1·2	10 49·3	13 0·0	57·2
46	7 31·0	− 5·7	11 22·0	12 6·4	56·9
54	8 28·5	− 5·5	11 43·4	13 10·3	57·5
50	9 30·0	− 4·8	11 52·2	15 8·3	57·3
58	10 29·4	− 4·9	11 46·0	16 7·5	57·6
51	11 30·3	− 2·5	11 33·0	18 1·9	57·8

10° 30′ to 13° 30′ South Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
54	30·6	+ 1·4	11 16·3	18 4·7	57·9
57	1 30·3	+ 2·7	11 1·8	18 5·4	57·8
59	2 30·5	+ 4·0	10 46·8	17 8·1	57·7
48	3 30·3	+ 4·1	10 35·5	16 7·8	57·4
59	4 28·9	+ 3·1	10 27·3	15 1·9	57·1
57	5 30·8	+ 1·4	10 29·0	13 10·7	57·3
59	6 28·7	− 1·9	10 45·0	12 6·6	57·0
49	7 30·0	− 3·6	11 16·6	12 7·5	57·5
53	8 28·4	− 4·0	11 42·4	13 7·8	57·3
55	9 30·6	− 3·2	11 49·2	15 1·4	57·7
53	10 30·5	− 1·5	11 45·3	16 9·0	57·7
65	11 31·5	− 1·2	11 31·6	17 7·2	57·8

13° 30′ to 16° 30′ South Declination.

Number of Observations.	Moon's Transit. h m	Equation of Time. m	Interval. h m	Height of Tide. ft. in.	Hor. Par. ′
61	31·2	0·0	11 15·2	18 2·5	58·0
61	1 31·2	− 0·1	11 0·3	18 2·6	58·5
60	2 29·2	+ 0·6	10 45·0	17 6·6	57·6
62	3 30·6	+ 0·4	10 32·6	16 4·8	57·5
60	4 30·5	+ 0·7	10 23·5	15 1·7	57·3
64	5 29·0	+ 0·3	10 25·0	14 1·0	57·1
58	6 30·6	0·0	10 42·6	12 7·5	57·3
64	7 30·2	− 0·8	11 14·9	12 7·9	57·5
73	8 31·0	− 0·9	11 40·6	13 6·3	57·4
63	9 31·5	− 1·1	11 47·9	15 1·6	57·8
67	10 30·7	− 0·3	11 42·2	16 7·0	57·9
54	11 30·7	+ 0·2	11 29·5	17 4·6	58·0

TABLE X. (Continued.)

16° 30′ to 19° 30′ North Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
	h m	m	h m	ft. in.	′
92	32·6	− 0·4	11 18·4	17 1·0	56·3
95	1 31·6	− 3·0	11 1·0	17 2·1	56·5
97	2 31·3	− 4·0	10 45·0	16 8·7	56·5
93	3 30·8	− 3·8	10 31·0	15 10·9	56·1
100	4 31·6	− 2·6	10 21·7	14 4·3	56·1
94	5 30·7	− 1·5	10 21·6	12 9·5	56·0
95	6 29·5	+ 0·9	10 41·5	11 10·5	55·8
106	7 31·0	+ 2·7	11 16·8	12 0·0	56·2
89	8 31·1	+ 3·2	11 44·8	13 3·0	56·2
102	9 28·8	+ 4·2	11 52·8	14 7·0	56·2
95	10 30·6	+ 1·9	11 45·1	15 11·0	56·5
91	11 32·3	+ 1·2	11 33·3	16 9·9	56·4

19° 30′ to 22° 30′ North Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
	h m	m	h m	ft. in.	′
62	31·0	− 1·7	11 11·4	17 9·7	58·4
61	1 33·1	− 3·6	10 55·7	17 5·1	58·0
56	2 31·0	− 4·1	10 39·4	17 2·0	58·1
72	3 31·1	− 4·1	10 28·0	16 1·5	57·5
64	4 29·9	− 2·6	10 17·7	14 11·0	57·8
67	5 28·0	− 0·3	10 17·8	13 5·0	57·4
70	6 30·1	+ 0·8	10 33·6	12 2·7	57·6
63	7 32·0	+ 3·4	11 7·2	12 2·5	57·8
62	8 28·8	+ 4·3	11 36·7	13 3·6	57·7
65	9 29·8	+ 3·7	11 45·7	14 6·4	57·2
62	10 28·8	+ 3·6	11 39·2	16 2·1	58·1
62	11 30·6	+ 1·0	11 26·2	17 0·1	58·3

16° 30′ to 19° 30′ South Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
83	29·9	− 0·9	11 2·4	17 11·2	58·2
84	1 30·1	− 2·0	10 56·6	18 2·5	58·7
87	2 28·6	− 3·3	10 42·3	17 9·9	58·2
93	3 29·0	− 4·1	10 35·2	16 9·1	58·4
96	4 30·3	− 3·2	10 21·4	14 10·5	57·8
98	5 30·5	− 0·7	10 18·8	13 7·5	57·5
100	6 31·7	+ 1·0	10 35·5	12 9·9	57·5
92	7 31·1	+ 3·0	11 8·6	12 7·9	57·9
91	8 31·6	+ 3·1	11 36·4	13 8·0	57·9
80	9 31·2	+ 4·4	11 42·8	15 2·5	58·0
88	10 29·0	+ 2·8	11 39·1	16 5·7	58·3
83	11 31·9	+ 0·9	11 26·3	17 5·8	58·7

19° 30′ to 22° 30′ South Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
65	30·3	− 0·8	11 15·4	17 1·2	56·9
67	1 27·8	− 4·0	11 0·2	17 0·4	56·5
71	2 29·0	− 5·2	10 41·4	16 5·6	56·7
70	3 28·8	− 3·6	10 28·6	15 9·9	56·6
74	4 30·7	− 3·4	10 18·1	14 4·5	56·3
64	5 32·5	0·8	10 18·1	12 8·2	56·2
75	6 30·7	+ 1·3	10 35·3	11 5·2	56·1
65	7 28·5	+ 3·0	11 2·1	11 7·4	56·4
68	8 29·7	+ 4·5	11 44·7	12 7·5	56·1
67	9 28·7	+ 4·5	11 48·8	14 5·7	57·2
67	10 31·8	+ 3·4	11 45·0	15 8·5	56·6
63	11 31·3	+ 1·2	11 32·0	16 7·1	56·9

22° 30′ to 25° 30′ North Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
	h m	m	h m	ft. in.	′
48	30·8	− 1·3	11 7·6	17 5·3	58·7
57	1 33·3	− 4·7	10 51·8	17 8·9	58·8
45	2 32·0	− 6·6	10 35·4	17 4·4	58·5
54	3 29·5	− 6·0	10 23·5	16 5·5	58·2
51	4 30·0	− 5·5	10 13·5	15 0·5	58·2
56	5 29·2	− 0·4	10 14·0	13 5·7	57·7
54	6 31·9	+ 1·2	10 29·9	12 1·7	57·7
52	7 27·4	+ 4·2	11 2·5	11 11·4	58·0
61	8 34·6	+ 5·1	11 29·8	13 2·1	58·1
50	9 30·2	+ 7·2	11 38·5	14 11·1	58·8
52	10 30·5	+ 4·4	11 35·3	15 11·7	58·5
58	11 30·3	+ 1·4	11 25·6	16 9·0	58·0

Above 25° 30′ North Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
	h m	m	h m	ft. in	′
42	31·4	− 2·9	11 11·5	16 4·2	56·8
45	1 28·7	− 5·8	10 55·0	16 1·4	56·3
49	2 31·8	− 7·7	10 35·5	15 11·5	56·3
51	3 31·0	− 7·5	10 22·6	15 1·0	56·3
59	4 32·5	− 5·9	10 9·8	13 7·9	56·1
53	5 33·1	− 1·7	10 9·7	12 3·7	56·2
58	6 29·8	+ 1·9	10 25·4	10 9·0	56·1
54	7 31·4	+ 5·2	11 6·0	10 10·0	56·5
51	8 30·5	+ 8·5	11 44·3	12 1·6	55·9
44	9 30·2	+ 7·6	11 51·0	13 8·3	56·3
47	10 28·9	+ 6·0	11 43·2	15 0·0	56·2
41	11 30·4	+ 2·2	11 30·0	15 6·2	56·4

22° 30′ to 25° 30′ South Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
57	31·9	− 1·8	11 16·1	16 5·1	56·1
52	1 30·8	− 4·6	10 57·4	16 5·6	56·1
61	2 32·1	− 6·2	10 40·2	16 0·8	56·0
50	3 28·9	− 5·7	10 26·1	15 5·7	55·7
65	4 30·0	− 3·6	10 15·4	13 11·9	55·9
50	5 30·6	− 1·9	10 12·6	12 6·4	55·8
63	6 28·5	+ 0·7	10 30·1	11 2·6	57·2
60	7 32·3	+ 3·9	11 13·0	11 3·9	56·2
62	8 30·8	+ 6·3	11 44·1	12 6·9	56·0
57	9 32·4	+ 6·1	11 52·7	14 2·3	56·0
52	10 29·0	+ 5·3	11 46·8	15 1·3	56·2
51	11 32·4	+ 1·4	11 30·2	16 4·0	56·8

Above 25° 30′ South Declination.

Number of Observations.	Moon's Transit.	Equation of Time.	Interval.	Height of Tide.	Hor. Par.
42	28·5	− 2·0	11 6·9	17 2·5	58·3
41	1 30·0	− 5·4	10 50·0	17 5·6	58·4
43	2 28·6	− 7·5	10 34·3	17 0·1	58·4
49	3 30·7	− 7·9	10 20·9	16 1·0	58·0
50	4 31·8	− 6·7	10 9·5	14 8·2	57·7
52	5 28·8	− 1·9	10 8·1	13 2·8	57·7
59	6 29·2	+ 1·5	10 22·2	11 6·5	57·4
47	7 29·1	+ 6·0	10 54·3	11 6·8	58·0
52	8 30·0	+ 7·3	11 30·0	12 10·9	57·8
42	9 29·7	+ 8·6	11 40·0	14 3·6	58·2
44	10 32·7	+ 6·1	11 35·0	15 6·0	58·2
37	11 30·4	+ 2·9	11 20·0	16 5·6	58·7

In forming this Table, it has been assumed that Mr. HUTCHINSON's clock was regulated according to *apparent* solar time; if it was regulated according to *mean* solar time, the interval must be *diminished* by the equation of time given in the third column.

TABLE XI. (Interpolated from Table X.)

Showing the Interval between the Apparent Time of the Moon's Transit and the Time of High Water at the Liverpool Old Docks for every three degrees of her Declination north *and* south.

Moon's Transit.	0° Dec.	3° N. Dec.	6° N. Dec.	9° N. Dec.	12° N. Dec.	15° N. Dec.	18° N. Dec.	21° N. Dec.	24° N. Dec.	27° N. Dec.	Mean.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
30	11 22·1	11 20·4	11 19·9	11 20·3	11 19·5	11 19·2	11 19·0	11 11·7	11 7·8	11 11·9	11 17·2
1 30	11 5·0	11 5·7	11 5·7	11 4·9	11 4·0	11 2·5	11 1·4	10 56·5	11 52·7	10 54·6	11 1·3
2 30	10 51·4	10 51·2	10 50·6	10 49·4	10 49·2	10 47·0	10 45·3	10 39·6	10 35·9	10 36·0	10 45·6
3 30	10 39·1	10 38·1	10 38·5	10 37·0	10 35·3	10 32·8	10 31·2	10 28·2	10 23·4	10 22·8	10 32·6
4 30	10 31·6	10 28·1	10 31·2	10 30·3	10 28·1	10 25·0	10 21·8	10 17·7	10 13·5	10 10·0	10 23·8
5 30	10 34·5	10 35·3	10 35·3	10 32·2	10 27·4	10 27·1	10 21·6	10 17·5	10 14·1	10 9·7	10 25·5
6 30	10 54·7	10 54·0	10 52·2	10 50·7	10 48·0	10 44·5	10 41·6	10 33·6	10 29·4	10 25·4	10 43·4
7 30	11 23·4	11 24·0	11 24·2	11 23·2	11 20·2	11 19·7	11 16·3	11 6·2	11 3·7	11 5·0	11 16·6
8 30	11 47·1	11 47·4	11 49·5	11 45·6	11 48·5	11 45·6	11 44·0	11 37·1	11 30·7	11 43·0	11 43·6
9 30	11 53·8	11 50·0	11 53·7	11 54·8	11 52·0	11 52·5	11 52·8	11 45·7	11 38·5	11 51·0	11 50·5
10 30	11 48·1	11 44·0	11 47·0	11 47·0	11 48·6	11 46·5	11 45·2	11 39·1	11 35·4	11 43·1	11 44·4
11 30	11 34·4	11 36·5	11 34·4	11 35·7	11 36·0	11 34·7	11 33·9	11 26·4	11 25·6	11 30·1	11 32·8
		3° S. Dec.	6° S. Dec.	9° S. Dec	12° S. Dec.	15° S. Dec.	18° S Dec.	21° S. Dec.	24° S. Dec.	27° S. Dec.	Mean.
30		11 18·4	11 20·4	11 18·7	11 16·4	11 15·5	11 12·4	11 15·4	11 16·6	11 6·6	11 15·6
1 30		11 3·7	11 5·7	11 3·7	11 1·8	11 0·0	10 56·6	10 59·6	10 57·6	10 50·0	10 59·9
2 30		10 50·0	10 50·9	10 49·2	10 46·9	10 44·9	10 42·0	10 41·1	10 40·5	10 34·0	10 44·3
3 30		10 40·2	10 38·7	10 36·6	10 35·5	10 32·7	10 35·1	10 28·3	10 25·9	10 21·0	10 32·6
4 30		10 31·5	10 29·6	10 29·9	10 27·2	10 23·6	10 21·4	10 18·2	10 15·4	10 9·8	10 22·9
5 30		10 34·7	10 33·0	10 32·0	10 29·0	10 25·1	10 18·8	10 18·0	10 12·6	10 8·2	10 23·4
6 30		10 53·5	10 51·2	10 48·7	10 44·4	10 42·4	10 35·0	10 35·1	10 31·8	10 22·4	10 41·0
7 30		11 25·9	11 21·4	11 21·6	11 16·6	11 14·9	11 8·1	11 2·9	11 13·0	10 54·8	11 13·2
8 30		11 40·7	11 47·4	11 43·7	11 42·8	11 40·2	11 36·4	11 44·7	11 44·1	11 30·0	11 41·0
9 30		11 54·4	11 54·4	11 52·2	11 49·1	11 47·8	11 42·7	11 48·8	11 52·7	11 40·0	11 49·1
10 30		11 41·6	11 44·9	11 45·9	11 45·3	11 42·3	11 39·0	11 45·1	11 46·8	11 35·2	11 42·9
11 30		11 34·7	11 32·0	11 33·0	11 32·0	11 29·6	11 26·6	11 32·4	11 30·2	11 20·1	11 30·1

TABLE XII.

Showing the Interval between the Apparent Time of the Moon's Transit and the Time of High Water at the Liverpool Old Docks for every three degrees of her Declination north *or* south.

Moon's Transit.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.	Mean.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
30	11 22·1	11 19·4	11 20·1	11 19·5	11 17·9	11 17·3	11 15·7	11 13·5	11 12·2	11 9·2	11 16·7
1 30	11 5·0	11 4·7	11 5·7	11 4·3	11 2·9	11 1·2	10 59·0	10 58·0	10 55·2	10 52·3	11 0·8
2 30	10 51·4	10 50·6	10 50·8	10 49·3	10 48·0	10 46·0	10 43·6	10 40·4	10 38·2	10 35·0	10 45·3
3 30	10 39·1	10 39·1	10 38·6	10 38·9	10 35·4	10 32·8	10 33·2	10 28·2	10 24·6	10 21·9	10 33·2
4 30	10 31·6	10 29·8	10 30·4	10 30·1	10 27·3	10 24·3	10 21·6	10 17·9	10 14·4	10 9·9	10 23·7
5 30	10 34·5	10 35·0	10 34·1	10 32·1	10 28·2	10 26·1	10 20·2	10 17·7	10 13·3	10 9·0	10 25·0
6 30	10 54·7	10 53·8	10 51·7	10 49·7	10 46·2	10 43·4	10 38·3	10 34·3	10 30·6	10 23·9	10 42·6
7 30	11 23·4	11 25·0	11 22·8	11 22·4	11 18·4	11 17·9	11 12·2	11 4·6	11 8·3	10 59·9	11 15·5
8 30	11 47·1	11 44·0	11 48·4	11 44·7	11 45·6	11 42·9	11 40·1	11 40·9	11 36·3	11 36·5	11 42·7
9 30	11 53·8	11 52·2	11 54·0	11 53·5	11 50·6	11 50·1	11 47·7	11 47·2	11 45·6	11 45·5	11 50·0
10 30	11 48·1	11 42·8	11 46·0	11 46·4	11 46·9	11 44·4	11 42·1	11 42·1	11 41·1	11 39·2	11 43·9
11 30	11 34·4	11 35·6	11 33·2	11 34·3	11 34·0	11 32·1	11 30·2	11 29·4	11 27·9	11 25·1	11 31·6

Table XIII.

Showing the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to fifteen degrees Declination, for every three degrees of her Declination north *and* south.

Moon's Transit.	0	3° N. Dec.	6° N. Dec.	9° N. Dec.	12° N. Dec.	15° N. Dec.	18° N. Dec.	21° N. Dec.	24° N. Dec.	27° N. Dec.
h m	m	m	m	m	m	m	m	m	m	m
30	+ 2·9	+ 1·2	+ 0·7	+ 1·1	+ 0·3	0	− 0·2	− 7·5	−11·4	− 7·3
1 30	+ 2·5	+ 3·2	+ 3·2	+ 2·4	+ 1·5	0	− 1·1	− 6·0	− 9·8	− 7·9
2 30	+ 4·4	+ 4·2	+ 3·6	+ 2·4	+ 2·2	0	− 1·7	− 7·4	−11·1	−11·0
3 30	+ 6·3	+ 5·3	+ 5·7	+ 8·4	+ 2·5	0	− 1·6	− 4·6	− 9·4	−10·0
4 30	+ 6·6	+ 3·1	+ 6·2	+ 5·3	+ 3·1	0	− 3·2	− 7·3	−11·5	−15·0
5 30	+ 7·4	+ 8·2	+ 8·2	+ 5·1	+ 0·3	0	− 5·5	− 9·6	−13·0	−17·4
6 30	+10·2	+ 9·5	+ 7·7	+ 6·2	+ 3·5	0	− 2·9	−10·9	−15·1	−19·1
7 30	+ 3·7	+ 4·3	+ 4·5	+ 3·5	+ 0·5	0	− 3·4	−13·5	−16·0	−14·7
8 30	+ 1·5	+ 1·8	+ 3·9	0·0	+ 2·9	0	− 1·6	− 8·5	−17·0	− 2·6
9 30	+ 1·3	− 2·5	+ 1·2	+ 2·3	− 0·5	0	+ 0·3	− 6·8	−14·0	− 1·5
10 30	+ 1·6	− 2·5	+ 0·5	+ 0·5	+ 2·1	0	− 1·3	− 7·4	−11·1	− 2·4
11 30	− 0·3	+ 1·8	− 0·3	+ 1·0	+ 1·3	0	− 0·8	− 8·3	− 9·1	− 4·6
		3° S. Dec.	6° S. Dec.	9° S. Dec	12° S. Dec.	15° S. Dec	18° S. Dec.	21° S. Dec.	24° S. Dec	27° S. Dec.
30		+ 2·9	+ 4·9	+ 3·2	+ 0·9	0	− 3·1	− 0·1	+ 1·1	− 8·9
1 30		+ 3·7	+ 5·7	+ 3·7	+ 1·8	0	− 3·4	− 0·4	− 2·4	−10·0
2 30		+ 5·1	+ 5·1	+ 4·3	+ 2·0	0	− 2·9	− 3·8	− 4·4	−10·9
3 30		+ 7·5	+ 7·5	+ 3·9	+ 2·8	0	+ 2·4	− 4·4	− 6·8	−11·7
4 30		+ 7·9	+ 5·9	+ 6·3	+ 3·6	0	− 2·2	− 5·4	− 8·2	−13·8
5 30		+ 9·6	+ 7·9	+ 6·9	+ 3·9	0	− 6·3	− 7·1	−12·5	−16·9
6 30		+11·1	+ 8·8	+ 6·3	+ 2·0	0	− 7·4	− 7·3	−10·6	−20·0
7 30		+11·0	+ 6·5	+ 6·7	+ 1·7	0	− 6·8	−12·0	− 1·9	−20·1
8 30		+ 0·5	+ 7·2	+ 3·5	+ 2·6	0	− 4·1	+ 4·7	+ 3·9	−10·2
9 30		+ 6·6	+ 6·6	+ 4·4	+ 1·3	0	− 5·1	+ 1·0	+ 4·9	− 7·8
10 30		− 0·7	+ 2·6	+ 3·6	+ 3·0	0	− 3·3	+ 2·8	+ 4·5	− 7·1
11 30		+ 5·1	+ 2·4	+ 3·4	+ 2·4	0	− 3·0	+ 2·8	+ 0·6	− 9·5

Table XIV.

Showing the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to fifteen degrees Declination, for every three degrees of her Declination north *or* south.

Moon's Transit.	0	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.
h m	m	m	m	m	m	m	m	m	m	m
30	+ 4·8	+ 2·1	+ 2·8	+ 2·2	+ 0·6	0	− 1·6	− 3·8	− 5·1	− 8·1
1 30	+ 3·8	+ 3·5	+ 4·5	+ 3·1	+ 1·7	0	− 2·2	− 3·2	− 6·0	− 8·9
2 30	+ 5·4	+ 4·6	+ 4·8	+ 3·3	+ 2·0	0	− 2·4	− 5·6	− 7·8	−11·0
3 30	+ 6·3	+ 6·3	+ 5·8	+ 6·1	+ 2·6	0	+ 0·4	− 4·6	− 8·2	−10·9
4 30	+ 7·3	+ 5·5	+ 6·1	+ 5·8	+ 3·0	0	− 2·7	− 6·4	− 9·9	−14·4
5 30	+ 8·4	+ 8·9	+ 8·0	+ 6·0	+ 2·1	0	− 5·9	− 8·4	−12·8	−17·1
6 30	+11·3	+10·4	+ 8·3	+ 6·3	+ 2·8	0	− 5·1	− 9·1	−12·8	−19·5
7 30	+ 5·5	+ 7·1	+ 4·9	+ 4·5	+ 0·5	0	− 5·7	−13·3	− 9·6	−18·0
8 30	+ 4·2	+ 1·1	+ 5·5	+ 1·8	+ 2·7	0	− 2·8	− 2·0	− 6·6	− 6·4
9 30	+ 3·7	+ 2·1	+ 3·9	+ 3·4	+ 0·5	0	− 2·4	− 2·9	− 4·5	− 4·6
10 30	+ 3·7	− 1·6	+ 1·6	+ 2·0	+ 2·5	0	− 2·3	− 2·3	− 1·0	− 5·2
11 30	+ 2·3	+ 3·5	+ 1·1	+ 2·2	+ 1·9	0	− 1·9	− 2·7	− 4·2	− 7·0

Table XV. (Interpolated from Table X.)

Showing the Height of High Water at the Liverpool Docks for every three degrees of the Moon's Declination north *and* south.

Moon's Transit.	0° Dec.	3° N. Dec.	6° N. Dec.	9° N. Dec.	12° N. Dec.	15° N. Dec.	18° N. Dec.	21° N. Dec.	24° N. Dec.	27° N. Dec.	Mean.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
30	18·33	18·30	18·14	18·09	18·00	17·79	17·08	17·80	17·43	16·34	17·73
1 30	18·27	18·31	18·49	18·17	17·86	17·63	17·17	17·44	17·72	16·11	17·72
2 30	17·49	17·90	17·71	17·40	17·39	16·89	16·72	17·16	17·38	15·97	17·20
3 30	16·75	16·38	16·40	16·38	16·04	16·10	15·91	16·14	16·44	15·10	16·14
4 30	15·04	15·17	15·05	14·94	14·87	14·64	14·39	14·91	15·04	13·70	14·77
5 30	14·12	13·61	13·78	13·77	13·53	13·15	12·80	13·37	13·45	12·37	13·40
6 30	13·08	13·13	12·69	12·68	12·38	12·27	11·87	12·21	12·16	10·75	12·32
7 30	12·91	13·24	13·19	12·71	12·56	12·13	12·00	12·19	11·97	10·82	12·97
8 30	14·13	13·86	13·48	14·11	13·63	13·33	13·01	13·32	13·07	12·12	13·41
9 30	15·14	15·52	15·21	15·11	14·92	14·90	14·67	14·53	14·92	13·69	14·86
10 30	16·86	16·71	16·64	16·49	16·22	16·51	15·95	16·19	15·97	15·02	16·26
11 30	16·83	17·69	17·23	17·56	17·25	17·07	16·81	17·00	16·75	15·52	16·97
		3° S. Dec.	6° S. Dec.	9° S. Dec.	12° S. Dec.	15° S. Dec	18° S Dec	21° S. Dec.	24° S Dec.	27° S. Dec	
30		18·54	18·00	18·40	18·38	18·19	17·93	17·10	16·41	17·21	17·80
1 30		18·28	18·50	18·41	18·44	18·21	18·20	17·04	16·45	17·46	17·89
2 30		17·60	17·73	17·68	17·67	17·54	17·80	16·41	16·08	16·99	17·03
3 30		16·55	16·68	16·81	16·64	16·40	16·74	15·80	15·45	16·09	16·35
4 30		15·43	15·43	15·20	15·13	15·14	14·87	14·39	13·99	14·64	14·91
5 30		13·75	14·06	13·76	13·89	14·06	13·64	12·74	12·53	13·20	13·51
6 30		12·94	12·86	13·03	12·55	12·64	12·83	11·43	11·22	11·54	12·34
7 30		13·00	12·95	12·54	12·62	12·65	12·65	11·63	11·31	11·57	12·33
8 30		13·13	13·85	13·88	13·67	13·51	13·64	12·62	12·56	12·90	13·31
9 30		15·31	15·48	15·69	15·11	15·09	15·18	14·51	14·14	14·30	14·96
10 30		16·69	16·95	16·63	16·75	16·56	16·49	15·67	15·12	15·45	16·26
11 30		17·73	18·00	18·15	17·58	17·37	17·45	16·57	16·30	16·46	17·29

Table XVI.

Showing the Height of High Water at the Liverpool Docks for every three degrees of the Moon's Declination north *or* south.

Moon's Transit	0° Dec.	3° Dec	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.	Mean.
h m	feet	feet	feet.	feet	feet.	feet.	feet	feet.	feet	feet.	feet.
30	18·33	18·42	18·07	18·24	18·19	17·99	17·50	17·45	16·92	16·77	17·79
1 30	18·27	18·30	18·50	18·29	18·15	17·92	17·69	17·24	17·08	16·79	17·82
2 30	17·49	17·75	17·72	17·54	17·53	17·22	17·26	16·78	16·73	16·49	17·25
3 30	16·75	16·47	16·54	16·60	16·34	16·25	16·33	15·97	15·95	15·60	16·28
4 30	15·04	15·30	15·24	14·98	15·00	14·89	14·53	14·65	14·52	14·17	14·83
5 30	14·12	13·68	13·92	13·76	13·71	13·60	13·22	13·06	12·99	12·78	13·48
6 30	13·08	13·04	12·77	12·86	12·46	12·45	12·35	11·80	11·69	11·15	12·36
7 30	12·91	13·12	13·07	12·62	12·59	12·39	12·32	11·91	11·64	11·19	12·38
8 30	14·13	13·50	13·66	14·00	13·65	13·42	13·33	12·97	12·80	12·51	13·40
9 30	15·14	15·41	15·35	15·40	15·01	15·00	14·93	14·52	14·53	14·00	14·93
10 30	16·86	16·70	16·80	16·56	16·49	16·53	16·22	15·93	15·54	15·24	16·29
11 30	16·83	17·71	17·61	17·86	17·41	17·22	17·13	16·79	16·52	15·99	17·71

Table XVII.

Showing the Difference between the Height of High Water and the Height corresponding to fifteen degrees of the Moon's Declination, for every three degrees of her Declination north *and* south.

Moon's Transit.	0° Dec.	3° N. Dec.	6° N. Dec.	9° N. Dec.	12° N. Dec.	15° N. Dec.	18° N. Dec.	21° N. Dec.	24° N. Dec.	27° N. Dec.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
30	+·54	+ ·51	+·35	+·30	+·21	0	—·71	+·09	—·36	—1·45
1 30	+·64	+ ·68	+·86	+·54	+·23	0	—·46	—·19	+·09	—1·52
2 30	+·60	+1·01	+·82	+·51	+·50	0	—·17	+·27	+·49	— ·92
3 30	+·65	+ ·28	+·30	+·28	—·06	0	—·19	+·04	+·34	—1·00
4 30	+·40	+ ·53	+·41	+·30	+·23	0	—·25	+·27	+·40	— ·94
5 30	+·97	+ ·46	+·63	+·62	+·38	0	—·35	+·22	+·22	— ·78
6 30	+·81	+ ·86	+·42	+·41	+·11	0	—·40	—·06	—·11	—2·52
7 30	+·78	+ ·11	+·06	+·58	+·47	0	—·13	+·06	—·16	—1·31
8 30	+·80	+ ·53	—·15	+·78	+·30	0	—·32	—·01	—·26	—1·21
9 30	+·24	+ ·62	+·31	+·21	+·02	0	—·23	—·37	+·02	—1·21
10 30	+·35	+ ·20	+·13	—·02	—·29	0	—·56	—·32	—·54	—1·49
11 30	—·24	+ ·62	—·02	+·49	+·18	0	—·26	—·07	—·32	—1·29
		3° S. Dec.	6° S. Dec.	9° S. Dec.	12° S. Dec.	15° S. Dec.	18° S. Dec.	21° S. Dec.	24° S. Dec.	27° S. Dec.
30		+·35	—·19	+·21	+·19	0	—·26	—1·09	—1·78	— ·98
1 30		+·07	—·16	+·20	+·23	0	—·01	—1·17	—1·76	— ·75
2 30		+·06	+·19	+·14	+·13	0	+·26	—1·13	—1·46	— ·55
3 30		+·15	+·28	+·41	+·24	0	+·34	— ·60	— ·90	— ·31
4 30		+·29	+·29	+·06	—·01	0	—·27	— ·75	—1·15	— ·50
5 30		—·31	00	—·30	—·17	0	—·42	—2·32	—1·53	— ·86
6 30		+·30	+·22	+·39	—·10	0	+·19	—1·21	—1·42	—1·10
7 30		+·35	+·30	—·11	—·03	0	00	—1·02	—1·34	—1·08
8 30		—·38	+·34	+·37	+·16	0	+·13	— ·89	— ·95	— ·61
9 30		+·22	+·30	+·60	+·02	0	+·09	— ·60	— ·95	— ·79
10 30		+·13	+·39	+·07	+·19	0	—·07	— ·89	—1·44	—1·11
11 30		+·36	+·63	+·78	+·21	0	+·08	— ·80	—1·07	— ·99

Table XVIII.

Showing the Difference between the Height of High Water and the Height corresponding to fifteen degrees of the Moon's Declination, for every three degrees of her Declination north *or* south.

Moon's Transit.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
30	+·44	+·43	+·08	+·25	+·20	0	—·49	—·54	—1·07	—1·22
1 30	+·35	+·38	+·58	+·37	+·23	0	—·23	—·68	— ·84	—1·13
2 30	+·27	+·53	+·50	+·32	+·31	0	+·04	—·44	— ·49	— ·73
3 30	+·50	+·22	+·29	+·35	+·09	0	+·08	—·28	— ·30	— ·65
4 30	+·15	+·41	+·35	+·09	+·11	0	—·36	—·24	— ·37	— ·72
5 30	+·52	+·08	+·32	+·16	+·11	0	—·38	—·54	— ·61	— ·82
6 30	+·63	+·59	+·32	+·41	+·01	0	—·10	—·65	— ·76	—1·30
7 30	+·52	+·73	+·68	+·23	+·20	0	—·07	—·48	— ·75	—1·20
8 30	+·71	+·08	+·24	+·58	+·23	0	—·09	—·45	— ·62	— ·91
9 30	+·14	+·41	+·35	+·40	+·01	0	—·07	—·48	— ·47	—1·00
10 30	+·33	+·17	+·23	+·03	—·04	0	—·31	—·60	— ·99	—1·29
11 30	—·39	+·49	+·39	+·64	+·19	0	—·09	—·43	— ·70	—1·23

Table IV. has been formed on the supposition that Mr. HUTCHINSON'S *clock was regulated according to Apparent Solar Time. The following*

TABLE XIX.

Results from Table I., if Mr. HUTCHINSON'S clock was regulated according to Mean Solar Time, showing the Difference in the Interval between the Mean Solar Time of the Moon's Transit and the Time of High Water, and the Mean Interval, for every month in the year.

Moon's Transit.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
h m	m	m	m	m	m	m	m	m	m	m	m	m
30	−13·3	−13·7	− 6·5	+2·8	+ 3·9	− 3·2	− 8·5	− 3·4	+8·5	+16·7	+16·2	0·0
1 30	−11·2	− 7·3	− 5·6	+0·7	+ 1·6	− 4·8	− 7·7	+ 0·3	+7·3	+15·6	+12·9	− 1·6
2 30	− 7·7	−10·1	− 4·5	−1·1	− 2·2	− 4·7	− 4·3	+ 3·3	+9·2	+12·4	+ 9·5	+ 0·3
3 30	− 4·5	− 9·2	− 7·6	−1·4	− 2·5	− 2·0	+ 0·8	− 0·1	+8·3	+ 9·3	+ 7·6	+ 1·2
4 30	− 1·3	−10·2	−11·2	−8·5	− 2·5	+ 2·4	+ 5·6	+ 3·7	+3·8	+ 6·1	+ 7·2	+ 5·4
5 30	+ 1·2	−11·5	−15·5	−9·8	+ 0·1	+ 8·8	+ 6·5	+ 2·0	−0·8	+ 3·5	+ 9·6	+12·0
6 30	− 1·4	−17·8	−19·1	−8·7	+ 5·0	+12·2	+ 5·9	− 6·3	−3·5	+ 2·8	+14·8	+16·4
7 30	− 8·0	−21·0	−17·4	+0·2	+10·3	+10·4	− 7·0	−10·1	−4·1	+11·5	+22·0	+13·7
8 30	−14·0	−19·5	− 9·8	+1·8	+ 8·8	+ 1·9	− 7·9	− 8·7	+2·2	+17·9	+20·5	+ 7·4
9 30	−15·5	−14·5	− 9·0	+3·6	+ 6·0	− 1·5	− 7·1	− 7·6	+4·9	+17·5	+18·2	+ 1·7
10 30	−15·7	−15·3	− 8·3	+4·2	+ 6·7	− 2·3	− 9·6	− 5·8	+7·2	+18·8	+17·7	+ 2·4
11 30	− 7·6	−15·2	− 7·2	+4·4	+ 3·5	− 3·0	−10·0	− 5·6	+8·1	+17·6	+16·0	− 0·1

Table XIII. has been formed upon the supposition that Mr. HUTCHINSON'S *clock was regulated according to Apparent Solar Time. The following*

TABLE XX.

Results from Table X., if Mr. HUTCHINSON'S clock was regulated according to Mean Solar Time, showing the Difference in the Interval between the Mean Solar Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to fifteen degrees Declination, for every three degrees of her Declination north *and* south.

Moon's Transit.	0	3° N. Dec.	6° N. Dec.	9° N. Dec.	12° N. Dec.	15° N. Dec.	18° N. Dec.	21° N. Dec.	24° N. Dec.	27° N. Dec.
h m	m	m	m	m	m	m	m	m	m	m
30	+ 0·5	− 0·9	− 0·8	− 0·1	− 0·8	0	+ 0·3	− 5·7	−10·0	− 4·3
1 30	− 3·5	− 3·1	− 3·3	− 1·9	− 0·8	0	+ 1·9	− 2·4	− 5·1	− 2·1
2 30	− 3·3	− 2 5	− 1·7	− 2·1	− 0·1	0	+ 3·8	− 1·8	− 3·0	− 1·8
3 30	− 2·4	− 3·2	− 2·0	− 1·6	− 1·3	0	+ 2·3	− 0·4	− 3·3	− 2·4
4 30	+ 0·6	− 2·9	+ 0·8	+ 1·2	+ 0·6	0	− 0·5	− 4·6	− 5·9	− 9·0
5 30	+ 5·6	+ 6·6	+ 6·9	+ 4·9	0	0	− 3·0	− 5·3	−11·6	−14·7
6 30	+12·5	+ 7·4	+10·5	+ 7·7	+ 4·7	0	− 3·8	−11·7	−16·3	−21·0
7 30	+ 8·5	+ 9·9	+ 8·7	+ 6·8	+ 1·3	0	− 7·1	−10·8	−21·2	−20·9
8 30	+10·3	+ 9·4	+10·8	+ 6·3	+ 6·8	0	− 5·1	−13·1	−20·3	−11·4
9 30	+10·2	+ 6·0	+ 8·5	+ 8·1	+ 2·0	0	− 4·3	−10·9	−21·6	− 9·5
10 30	+ 7·3	+ 3·1	+ 4·9	+ 4·2	+ 3·9	0	− 4·0	−11·8	−16·2	−10·2
11 30	+ 1·9	+ 4·3	+ 1·4	+ 2·1	+ 2·7	0	− 2·0	− 9·3	−10·5	− 6·8
		3° S. Dec.	6° S. Dec.	9° S. Dec.	12° S. Dec.	15° S. Dec.	13° S. Dec.	21° S. Dec.	24° S. Dec.	27° S. Dec.
30		− 0·1	+ 2·4	+ 2·0	− 0·5	0	− 2·2	+ 0·7	+ 2·9	− 6·9
1 30		− 4·1	− 0·4	− 1·2	− 1·0	0	− 1·5	+ 3·5	+ 2·1	− 4·7
2 30		− 2·4	− 1·4	− 0·3	− 1·4	0	+ 1·0	+ 2·0	+ 2·4	− 3·8
3 30		− 0·4	− 1·5	− 1·5	− 0·9	0	+ 6·9	− 0·4	− 0·7	−10·3
4 30		+ 0·8	+ 1·7	+ 1·4	+ 1·2	0	+ 1·7	− 1·3	− 3·9	− 6·4
5 30		+ 7·6	+ 5·1	+ 7·1	+ 2·8	0	− 5·3	− 6·0	−10·3	−14·7
6 30		+10·0	+10·9	+ 7·5	+ 3·9	0	− 8·4	− 8·6	−11·3	−13·5
7 30		+16·7	+10·8	+11·6	+ 4·5	0	−10·6	−15·8	− 6·6	−26·9
8 30		+ 7·8	+13·1	+ 8· 1	+ 5·7	0	− 7·8	− ·9	− 3·3	−10·4
9 30		+14·1	+13·0	+ 8·1	+ 3·4	0	−10·6	− 4·6	− 2·3	− 6·9
10 30		+ 5·0	+ 7·9	+ 8·2	+ 4·2	0	− 6·4	− 0·9	− 1·1	−13·5
11 30		+ 9·9	+ 4·5	+ 6·1	+ 3·8	0	+ 3·7	+ 1·8	− 0·6	−12·2

Tables to be used in predicting the Time of High Water at Liverpool.

Table XXI.

Showing the Semimenstrual Inequality + a constant, or the Interval between the Moon's Transit and the Time of High Water, her Parallax being 57′, and her Declination 15°. (This Table has been formed by interpolation from the column corresponding to Parallax 57′ in Table VII.)

Moon's Transit.	Interval.	Moon's Transit.	Interval.	Moon's Transit.	Interval	Moon's Transit.	Interval.	Moon's Transit.	Interval.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 0	11 25	2 30	10 45	5 0	10 23	7 30	11 16	10 0	11 50
0 10	11 23	2 40	10 43	5 10	10 23	7 40	11 20	10 10	11 49
0 20	11 21	2 50	10 41	5 20	10 23	7 50	11 25	10 20	11 48
0 30	11 18	3 0	10 39	5 30	10 23	8 0	11 30	10 30	11 47
0 40	11 16	3 10	10 37	5 40	10 24	8 10	11 34	10 40	11 44
0 50	11 13	3 20	10 35	5 50	10 26	8 20	11 39	10 50	11 41
1 0	11 10	3 30	10 33	6 0	10 28	8 30	11 44	11 0	11 39
1 10	11 8	3 40	10 32	6 10	10 32	8 40	11 46	11 10	11 37
1 20	11 5	3 50	10 31	6 20	10 36	8 50	11 48	11 20	11 35
1 30	11 2	4 0	10 30	6 30	10 40	9 0	11 50	11 30	11 32
1 40	11 0	4 10	10 28	6 40	10 46	9 10	11 51	11 40	11 30
1 50	10 57	4 20	10 26	6 50	10 52	9 20	11 51	11 50	11 28
2 0	10 54	4 30	10 25	7 0	10 58	9 30	11 52		
2 10	10 51	4 40	10 24	7 10	11 4	9 40	11 52		
2 20	10 48	4 50	10 24	7 20	11 10	9 50	11 51		

Table XXII.

Showing the Semimenstrual Inequality + a constant, or the Height of High Water at Liverpool, the Moon's Parallax being 57′, and her Declination 15°, from the Sill of the Old Dock Gates.

Moon's Transit.	Height of High Water	Moon's Transit	Height of High Water	Moon's Transit.	Height of High Water.	Moon's Transit	Height of High Water	Moon's Transit	Height of High Water.
h m	feet.	h m	feet	h m	feet	h m	feet	h m	feet.
0 0	17·25	2 30	17·04	5 0	14·12	7 30	12·33	10 0	15·40
0 10	17·35	2 40	16·90	5 10	13·90	7 40	12·40	10 10	15·60
0 20	17·45	2 50	16·75	5 20	13·65	7 50	12·55	10 20	15·80
0 30	17·53	3 0	16·60	5 30	13·39	8 0	12·70	10 30	16·00
0 40	17·55	3 10	16·45	5 40	13·21	8 10	12·90	10 40	16·20
0 50	17·57	3 20	16·30	5 50	13·05	8 20	13·10	10 50	16·35
1 0	17·59	3 30	16·17	6 0	12·90	8 30	13·36	11 0	16·50
1 10	17·61	3 40	15·95	6 10	12·77	8 40	13·60	11 10	16·68
1 20	17·62	3 50	15·73	6 20	12·65	8 50	13·80	11 20	16·85
1 30	17·63	4 0	15·51	6 30	12·53	9 0	14·10	11 30	17·00
1 40	17·59	4 10	15·30	6 40	12·44	9 10	14·30	11 40	17·10
1 50	17·50	4 20	15·10	6 50	12·40	9 20	14·55	11 50	17·20
2 0	17·40	4 30	14·89	7 0	12·37	9 30	14·79		
2 10	17·30	4 40	14·60	7 10	12·34	9 40	15·00		
2 20	17·17	4 50	14·37	7 20	12·32	9 50	15·20		

The two following Tables have been made by arbitrary alterations in Tables VIII. and XI., in order to get rid of the irregularities, and may, I think, be considered as showing the effect of changes in the Moon's parallax upon the tides at Liverpool.

Table XXIII.

Showing the Correction for the Moon's Parallax in the Time of High Water at Liverpool.

Moon's Transit.	H. P. 54′.	H. P. 55′.	H. P. 56′.	H. P. 57′.	H. P. 58′.	H. P. 59′.	H. P. 60′.
m	m	m	m	m	m	m	m
0	+ 8	+ 5	+ 2	0	− 2	− 5	− 8
1	+ 7	+ 4	+ 2	0	− 2	− 4	− 7
2	+ 5	+ 3	+ 1	0	− 1	− 3	− 5
3	+ 3	+ 2	+ 1	0	− 1	− 2	− 3
4	+ 2	+ 1	+ 0	0	0	− 1	− 2
5	+ 2	+ 1	+ 0	0	0	− 1	− 2
6	+ 4	+ 2	+ 1	0	− 1	− 2	− 4
7	+ 10	+ 6	+ 3	0	− 3	− 6	− 10
8	+ 15	+ 10	+ 5	0	− 5	− 10	− 15
9	+ 15	+ 10	+ 5	0	− 5	− 10	− 15
10	+ 12	+ 8	+ 4	0	− 4	− 8	− 12
11	+ 10	+ 6	+ 3	0	− 3	− 6	− 10

Table XXIV.

Showing the Correction for the Moon's Parallax in the Height of High Water at Liverpool.

Moon's Transit.	H. P. 54′.	H. P. 55′.	H. P. 56′.	H. P. 57′.	H. P. 58′.	H. P. 59′.	H. P. 60′.
h	feet.	feet.	feet.	feet.	feet	feet.	feet.
0	−1·15	− ·76	−·38	0	+·38	+ ·76	+1·15
1	−1·25	− ·82	−·41	0	+·41	+ ·82	+1·25
2	−1·40	− ·92	−·46	0	+·46	+ ·92	+1·40
3	−1·50	−1·00	−·50	0	+·50	+1·00	+1·50
4	−1·50	−1·00	−·50	0	+·50	+1·00	+1·50
5	−1·50	−1·00	−·50	0	+·50	+1·00	+1·50
6	−1·50	−1·00	−·50	0	+·50	+1·00	+1·50
7	−1·45	− ·98	−·48	0	+·48	+ ·96	+1·45
8	−1·35	− ·90	−·45	0	+·45	+ ·90	+1·35
9	−1·30	− ·86	−·43	0	+·43	+ ·86	+1·30
10	−1·25	− ·82	−·41	0	+·41	+ ·82	+1·25
11	−1·20	− ·80	−·40	0	+·40	+ ·80	+1·20

The following Table has been formed from Table XIV.

Table XXV.

Showing the Correction for the Moon's Declination in the Time of High Water at Liverpool, *if Mr.* Hutchinson's *Clock was regulated according to Apparent Solar Time.*

Moon's Transit.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.
h	m	m	m	m	m	m	m	m	m	m
0	+ 4	+ 3	+ 2	+ 1	+ 1	0	− 1	− 3	− 5	− 7
1	+ 5	+ 4	+ 2	+ 2	+ 1	0	− 2	− 4	− 6	− 8
2	+ 6	+ 4	+ 3	+ 2	+ 1	0	− 2	− 4	− 6	− 9
3	+ 7	+ 5	+ 3	+ 2	+ 1	0	− 2	− 4	− 7	−10
4	+ 9	+ 7	+ 5	+ 3	+ 1	0	− 3	− 6	− 9	−13
5	+11	+ 8	+ 6	+ 4	+ 2	0	− 4	− 8	−12	−16
6	+13	+10	+ 7	+ 4	+ 2	0	− 4	− 8	−13	−18
7	+ 9	+ 7	+ 5	+ 3	+ 1	0	− 4	− 8	−13	−18
8	+ 6	+ 4	+ 3	+ 2	+ 1	0	− 4	− 8	−12	−17
9	+ 4	+ 3	+ 2	+ 1	+ 1	0	− 1	− 3	− 5	− 7
10	+ 3	+ 2	+ 1	+ 1	+ 1	0	− 1	− 2	− 3	− 4
11	+ 3	+ 2	+ 1	+ 1	+ 1	0	− 1	− 2	− 3	− 4

The following Table has been formed from Table XVIII. Although it would seem from Table XVII. that a difference does exist in the correction for north and south declination, yet the numbers in the latter Table are so irregular as almost to defy any attempt to reduce them to uniformity.

Table XXVI.

Showing the Correction for the Moon's Declination in the Height of High Water at Liverpool.

Moon's Transit.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.
h	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0	+·40	+·32	+·24	+·16	+·08	0	−·25	−·50	−·75	−1·00
1	+·55	+·44	+·33	+·22	+·11	0	−·25	−·50	−·75	−1·00
2	+·64	+·52	+·39	+·26	+·13	0	−·19	−·37	−·56	− ·75
3	+·60	+·48	+·36	+·24	+·12	0	−·12	−·25	−·37	− ·50
4	+·40	+·32	+·24	+·16	+·08	0	−·10	−·20	−·30	− ·40
5	+·38	+·29	+·21	+·14	+·07	0	−·16	−·32	−·48	− ·65
6	+·50	+·40	+·30	+·20	+·10	0	−·21	−·42	−·63	− ·85
7	+·70	+·56	+·42	+·28	+·14	0	−·22	−·44	−·66	− ·90
8	+·60	+·48	+·36	+·24	+·12	0	−·20	−·40	−·60	− ·80
9	+·50	+·40	+·30	+·20	+·10	0	−·14	−·28	−·42	− ·55
10	+·40	+·32	+·24	+·16	+·08	0	−·14	−·28	−·42	− ·55
11	+·30	+·24	+·18	+·12	+·06	0	−·17	−·34	−·51	− ·70

Index to the Tables formed from Mr. HUTCHINSON'S *Observations.*

In forming these Tables it has been assumed that the Observations were recorded in Apparent Solar Time. The Time of the Moon's Transit at Greenwich, which constitutes the principal argument of the Tables, is always given in Apparent Solar Time.

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Table II. (Interpolated from Table I.), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at Liverpool Old Docks, for each month in the year.

Table III. (Interpolated from Table I.), showing the Height of High Water at Liverpool Old Docks, corresponding to the Apparent Solar Time of the Moon's Transit, in each month of the year.

Table IV., showing the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Mean Interval, for every month in the year.

Table V., showing the Difference in the Height of High Water and the Mean Height for every month in the year.

Table VI., showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Old Docks, corresponding to the Apparent Solar Time of the Moon's Transit, for every minute of her Horizontal Parallax.

Table VII. (Interpolated from Table VI.)

Table VIII., showing the Difference in the Interval between the Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to fifty-seven minutes of the Moon's Horizontal Parallax.

Table IX., showing the Difference between the Height of High Water and the Height corresponding to fifty-seven minutes of the Moon's Horizontal Parallax.

Table X., showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Old Docks, corresponding to the Apparent Solar Time of the Moon's Transit, for every three degrees of her Declination north and south. The Equation of Time to be added to Apparent Time. (In forming this Table, it has been assumed that Mr. HUTCHINSON'S clock was regulated according to *apparent* solar time; if it was regulated according to *mean* solar time, the interval must be *diminished* by the equation of time given in the third column.)

Table XI. (Interpolated from Table X.), showing the Interval between the Apparent Time of the Moon's Transit and the Time of High Water at the Liverpool Old Docks for every three degrees of her Declination north *and* south.

Table XII., showing the Interval between the Apparent Time of the Moon's Transit and the Time of High Water at the Liverpool Old Docks for every three degrees of her Declination north *or* south.

Table XIII., showing the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to 15° Declination, for every three degrees of her Declination north *and* south.

Table XIV., showing the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to 15° Declination, for every three degrees of her Declination north *or* south.

Table XV. (Interpolated from Table X.), showing the Height of High Water at the Liverpool Docks for every three degrees of the Moon's Declination north *and* south.

Table XVI., showing the Height of High Water at the Liverpool Docks for every three degrees of the Moon's Declination north *or* south.

Table XVII., showing the Difference between the Height of High Water and the Height corresponding to fifteen degrees of the Moon's Declination, for every three degrees of her Declination north *and* south.

Table XVIII., showing the Difference between the Height of High Water and the Height corresponding to fifteen degrees of the Moon's Declination, for every three degrees of her Declination north *or* south.

Table XIX. results from Table I., if Mr. HUTCHINSON's Clock was regulated according to Mean Solar Time, showing the Difference in the Interval between the Mean Solar Time of the Moon's Transit and the Time of High Water, and the Mean Interval, for every month in the year.

Table XX. results from Table X., if Mr. HUTCHINSON's Clock was regulated according to Mean Solar Time, showing the Difference in the Interval between the Mean Solar Time of the Moon's Transit and the Time of High Water, and the Interval corresponding to 15° Declination, for every three degrees of her Declination north *and* south.

Table XXI., showing the Semimenstrual Inequality + a constant, or the Interval between the Moon's Transit and the Time of High Water, her Parallax being 57′, and her Declination 15°. (This Table has been formed by interpolation from the column corresponding to Parallax 57′ in Table VII.)

Table XXII., showing the Semimenstrual Inequality + a constant, or the Height of High Water at Liverpool, the Moon's Parallax being 57′, and her Declination 15°, from the Sill of the Old Dock Gates.

Table XXIII., showing the correction for the Moon's Parallax in the Time of High Water at Liverpool.

Table XXIV., showing the Correction for the Moon's Parallax in the Height of High Water at Liverpool.

Table XXV., showing the Correction for the Moon's Declination in the Time of High Water at Liverpool, *if Mr.* HUTCHINSON'S *Clock was regulated according to Apparent Solar Time.*

Table XXVI., showing the Correction for the Moon's Declination in the Height of High Water at Liverpool.

The following Table has been formed from Table XIV.

TABLE XXV.

Showing the Correction for the Moon's Declination in the Time of High Water at Liverpool, *if Mr.* HUTCHINSON's *Clock was regulated according to Apparent Solar Time.*

Moon's Transit.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec.	21° Dec.	24° Dec.	27° Dec.
h	m	m	m	m	m	m	m	m	m	m
0	+ 4	+ 3	+ 2	+ 1	+ 1	0	− 1	− 3	− 5	− 7
1	+ 5	+ 4	+ 2	+ 2	+ 1	0	− 2	− 4	− 6	− 8
2	+ 6	+ 4	+ 3	+ 2	+ 1	0	− 2	− 4	− 6	− 9
3	+ 7	+ 5	+ 3	+ 2	+ 1	0	− 2	− 4	− 7	−10
4	+ 9	+ 7	+ 5	+ 3	+ 1	0	− 3	− 6	− 9	−13
5	+11	+ 8	+ 6	+ 4	+ 2	0	− 4	− 8	−12	−16
6	+13	+10	+ 7	+ 4	+ 2	0	− 4	− 8	−13	−18
7	+ 9	+ 7	+ 5	+ 3	+ 1	0	− 4	− 8	−13	−18
8	+ 6	+ 4	+ 3	+ 2	+ 1	0	− 4	− 8	−12	−17
9	+ 4	+ 3	+ 2	+ 1	+ 1	0	− 1	− 3	− 5	− 7
10	+ 3	+ 2	+ 1	+ 1	+ 1	0	− 1	− 2	− 3	− 4
11	+ 3	+ 2	+ 1	+ 1	+ 1	0	− 1	− 2	− 3	− 4

The following Table has been formed from Table XVIII. Although it would seem from Table XVII. that a difference does exist in the correction for north and south declination, yet the numbers in the latter Table are so irregular as almost to defy any attempt to reduce them to uniformity.

TABLE XXVI.

Showing the Correction for the Moon's Declination in the Height of High Water at Liverpool.

Moon's Transit.	0° Dec.	3° Dec.	6° Dec.	9° Dec.	12° Dec.	15° Dec.	18° Dec	21° Dec.	24° Dec.	29° Dec.
h	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0	+·40	+·32	+·24	+·16	+·08	0	−·25	−·50	−·75	−1·00
1	+·55	+·44	+·33	+·22	+·11	0	−·25	−·50	−·75	−1·00
2	+·64	+·52	+·39	+·26	+·13	0	−·19	−·37	−·56	− ·75
3	+·60	+·48	+·36	+·24	+·12	0	−·12	−·25	−·37	− ·50
4	+·40	+·32	+·24	+·16	+·08	0	−·10	−·20	−·30	− ·40
5	+·38	+·29	+·21	+·14	+·07	0	−·16	−·32	−·48	− ·65
6	+·50	+·40	+·30	+·20	+·10	0	−·21	−·42	−·63	− ·85
7	+·70	+·56	+·42	+·28	+·14	0	−·22	−·44	−·66	− ·90
8	+·60	+·48	+·36	+·24	+·12	0	−·20	−·40	−·60	− ·80
9	+·50	+·40	+·30	+·20	+·10	0	−·14	−·28	−·42	− ·55
10	+·40	+·32	+·24	+·16	+·08	0	−·14	−·28	−·42	− ·55
11	+·30	+·24	+·18	+·12	+·06	0	−·17	−·34	−·51	− ·70

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Table VI., showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Old Docks, corresponding to the Apparent Solar Time of the Moon's Transit, for every minute of her Horizontal Parallax.

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Table XXVI., showing the Correction for the Moon's Declination in the Height of High Water at Liverpool.

XVI. *Remarks on the difficulty of distinguishing certain Genera of Testaceous* Mollusca *by their Shells alone, and on the Anomalies in regard to Habitation observed in certain Species.* *By* John Edward Gray, *Esq. F.R.S. &c.*

Received June 11,—Read June 18, 1835.

It has been a very common error, both among conchologists and geologists, to regard all shells in which no remarkable difference of form and character can be distinguished as inhabited by one and the same genus of animals; and not less usual to assume that all the species of the same genus inhabit similar localities. Many geologists have still further enlarged the boundaries of error, by taking for granted that all the fossil species of shells which are referrible by the characters of the shell to recent genera, must have been formed by animals which, in their recent state, possessed the same habits as the most commonly observed species of the genus to which they appear to belong. These theories were, indeed, quite consistent with our former ignorance of the habits of the animals of this class; but since the works of Poli, Müller, Montagu, Lamarck, and Cuvier have induced zoologists again to turn their attention, as was the practice among the older writers, to the animals of shells, and their habits, and no longer to confine themselves, as was too often the case with the followers of the Linnean system of conchology, to the study of the shells as mere pieces of ornament, classed without reference to their inhabitants, the acknowledged importance of the subject is daily bringing to our knowledge some animal unknown before, and adding to our stock of information facts which prove the fallacy of the opinions so hastily taken up. Thus, although even at the present day the animals of less than one twentieth part of the well-known species of shells have been observed, —and of those which are known the greater part have been very imperfectly described,—numerous exceptions to the theories in question have been brought to light, which deserve to be collected into one point of view, and made the subject of serious consideration.

The exceptions which it is the object of the present paper to notice may be arranged under the two following heads:

1. Shells having every appearance of belonging to the same natural genus, but inhabited by animals of a very different character.

2. Species of testaceous *Mollusca* living in very different situations from the majority of the known species of the genus to which they belong, or having the faculty of maintaining their existence in several different situations.

These two classes of exceptions I shall proceed to notice in detail.

1. *Of Shells apparently similar, but belonging, on a comparison of their Animals, to very different Genera.*

In a note on my former paper on the structure of shells *, I pointed out the perplexity in which the extreme similarity of the shells belonging to the genera *Patella* and *Lottia* must involve the geologist and the conchologist, intending at some future time to pursue the subject further, and to show that similar difficulties existed in regard to several other genera. The two genera above referred to are probably, however, the most remarkable example of this complete resemblance, on account of the extreme dissimilarity of their animals, which are referrible to two very different orders of *Mollusca*, while the shells are so perfectly alike, that after a long-continued study of numerous species of each genus, I cannot find any character by which they can be distinguished with any degree of certainty. Both genera present a striking discrepancy from all other univalve shells, in having the apex of the shell turned towards the head of the animal, the genera to which they are immediately related in both the orders to which they belong offering no variation in this respect from the usual structure of the class. The agreement in the internal structure of their shells is equally complete; yet the animal of *Patella* has the branchiæ in the form of a series of small plates disposed in a circle round the inner edge of the mantle, while that of *Lottia* has a triangular pectinated gill seated in a proper cavity formed over the back of the neck within the mantle, agreeing in this respect with the inhabitants of the *Trochi*, *Monodontæ*, and *Turbines*, from which it differs so remarkably in the simple conical form of its shell. This difference in the respiratory organs of animals inhabiting shells so strikingly similar is the more anomalous, inasmuch as those organs commonly exercise great influence on the general form of shells; a circumstance readily accounted for when we reflect that a principal object of the shell is to afford protection to those delicate and highly important parts.

To the practical conchologist it will be sufficient to mention *Pupa* and *Vertigo*, *Vitrina* and *Nanina*, *Rissoa* and *Truncatella*, as affording numerous and perplexing instances of the difficulty of distinguishing between genera of shells, inhabited by very different animals.

A similar difficulty exists with regard to *Siphonaria* and *Ancylus*, genera belonging to two different families, one inhabiting the sea-shores, while the other lives in rivers and brooks. The only distinction between the shells of these two genera consists in the *Ancyli* being generally of a thinner substance than the *Siphonariæ*; but this is by no means an adequate character, some species of *Siphonaria* (*S. Tristensis*, for example,) being quite as thin in texture as any *Ancylus*. Both have the muscular impression interrupted by the canal through which the air passes to the respiratory organs; yet the animal of *Ancylus* has long tentacles, and eyes placed as in the *Lymnææ*, to which it is closely allied, while *Siphonaria* has no distinct tentacles, and

* Philosophical Transactions, 1834, p. 800.

in these respects agrees with the equally marine genus *Amphibola,* confounded by LAMARCK with the *Ampullariæ.*

About fifteen years since, I first observed, in the marshes near the banks of the Thames between Greenwich and Woolwich, in company with species of *Valvata, Bithynia,* and *Pisidium,* a small univalve shell, agreeing with the smaller species of the littoral genus *Littorina* in every character both of shell and operculum; yet this very peculiar and apparently local species has an animal which at once distinguishes it from the animal of that genus, and from all other Ctenobranchous *Mollusca.* Its tentacles are very short and thick, and have the eyes placed at their tips; while the *Littorinæ,* and all the other animals of the order to which they belong, have their eyes placed on small tubercles on the outer side of the base of the tentacles, which are generally more or less elongated. The shell in question and its animal were described and figured by Dr. LEACH, in his hitherto unpublished work on British *Mollusca,* under the name of *Assiminia Grayana*; and as this name has been referred to by Mr. JEFFRIES and other conchologists, it may be regarded as established, and that of *Syncera hepatica,* proposed by myself in the Medical Repository, vol. x. p. 239, will take the rank of a synonym. A second species of this genus has lately been made known by Mr. BENSON, by whom it was found in ponds in India. Its shell is banded like that of *Littorina 4-fasciata* and several others of the smaller *Littorinæ,* and had been figured in the Supplement to WOOD's Catalogue, t. 6. f. 28, under the name of *Turbo Francesiæ.*

Taking this in conjunction with the preceding, we have here two instances of univalve shells apparently belonging to the same genus, the one found in fresh and the other in salt water, but proving, when their animals are examined, to belong to genera essentially distinct. My next illustration will show that a similar fact has been observed among the bivalves.

The *Mytilus polymorphus* of CHEMNITZ is truly a freshwater species, having been first observed in the Wolga by the illustrious PALLAS. It has recently been introduced, doubtless with the Russian timber, (for this species, in common with the *Ampullariæ, Paludinæ,* and *Neritinæ* of fresh water, and the *Littorinæ, Monodontæ,* and *Cerithia* of salt, has the faculty of living for a very long time out of water,) into the Lake of Haarlem and the Commercial Docks at Rotherhithe; in both of which it appears to increase with great rapidity. I am aware that Mr. LYELL has given another explanation of the mode of introduction of this remarkable species; but from experiments which I have myself made on the animal's power of living out of water, I cannot hesitate in giving the preference to the suggestion advanced above, rather than supposing it to have made its passage from one river to the other, across the sea, attached to the bottom of a vessel. The shell in question differs from the shells of other *Mytili* in no character of more than specific importance; but the animal is essentially distinct. In the genus *Mytilus* the lobes of the mantle are free throughout nearly their whole circumference, as in *Unio, Cardita, Pecten, Ostrea,* &c.; while in

the animal of *Mytilus polymorphus* they are united through nearly their whole extent, leaving only three small apertures, one for the passage of the foot and beard, and the other two for the reception and rejection of the water, from the contents of which the animal derives its sustenance. This shell must consequently form a new genus, to which the name of *Dreissena* has been appropriated by VAN BENEDEN *. As a proof of the importance attached to this character, it may be observed that CUVIER considered the adherence or non-adherence of the lobes of the mantle so essential a distinction as to found on it his division of the bivalves into families. In his system, therefore, the genus *Dreissena* would be placed with the family of *Chamacées*, while the genus *Mytilus* forms the type of the preceding family of *Mytilacées*. The genus *Iridina*, however, and one or two others, show that this character cannot be implicitly relied on for the natural classification of animals of this class, although it forms a very good generic mark of distinction.

The genus *Iridina* † above referred to affords a second instance of this anomaly; for though the animals of the *Iridinæ* and *Anodontæ* differ in the adhesion and non-adhesion of the lobes of the mantles, yet the shells are so alike that they cannot be distinguished by any external character; so much so, that one of the species now referred to the genus by M. DESHAYES, who first pointed out this peculiarity in the animal, was considered as an *Anodon* by LAMARCK.

The animals of *Cytherea*, *Venus*, and *Venerupis* have, like those of most of the allied genera, a lanceolate foot projecting at the anterior part of the shell; while the genus *Artemis* of POLI, which has generally been confounded with *Cytherea*, from which it is not easily to be distinguished except by its usually more rounded form, is provided with a crescent-shaped foot, exserted at the middle of the lower edges of the valves.

Again, there is but little difference in external characters and habit between *Cyclas* and *Pisidium*; but the animals of the latter have elongated siphons which are not found in the former.

In reference to Univalves it may also be observed, that it is frequently impossible to distinguish some of the genera of that class without an examination of their opercula. This is the case, for instance, as regards the smaller and more solid *Paludinæ*, inhabitants of fresh water, and some species of *Littorina* living on the coast; several of the shells described as *Paludinæ* by DRAPARNAULD and others appearing rather to belong to the latter genus. A similar difficulty exists with respect to other *Littorinæ* as distinguished from *Phasianella*, and with the *Neritinæ* as distinguished from the *Neritæ*. In the latter case the characters derived from the operculum are so essential

* Institut, 1835, p. 130; and Annales des Sciences Naturelles, N. S., tom. iii. p. 193.

† LAMARCK formed this genus on a specimen which had its hinge margin accidentally tubercular and slightly crenated; but this character is not found in most of the specimens of the species which he describes. The English conchologists, misled by this character, have referred to the genus a very different African shell, with a long series of transverse teeth on the hinge margin, which has lately been separated by Mr. CONRAD under the name of *Pleiodon*.

to the discrimination of the two genera, that M. RANG, looking only to the characters of the shell, has proposed to reunite them into one. In proof of the little attention that has hitherto been paid to this very important part, I may mention that three species referred by LAMARCK to the genus *Solarium* are each furnished with a different kind of operculum; and it is deserving of notice that the *Monodonta canaliculata*, according to the observations of M. QUOY, has an operculum very different from the rest of the shells of that genus.

In some shells, again, the differences in character are so slight as almost to throw an air of ridicule on the attempt to separate them generically from the structure of the shells alone; and yet when the animal is examined the necessity of their separation becomes so obvious as to be immediately acknowledged. This is especially the case with my genus *Bullia* compared with *Terebra*: the shells of these two genera are so similar, that LAMARCK and all other conchologists have retained them in one group, no other distinction being observable except that in the former there is a more or less distinct callous band winding round the volutions just above the suture, and produced by a slight extension of the inner lip beyond the part of the shell occupied by the whorl. This extension of the lip is probably deposited by the foot of the animal, which in the genus *Bullia* is very large and expanded, while that of *Terebra* is small and compressed. This, however, is not the only difference between the two animals, that of the former genus having rather large and eyeless tentacles, while the *Terebræ* have very small and short tentacles, bearing the eyes near their tips.

A second example of a similar kind is derived from the genus *Rostellaria*, in which LAMARCK includes the *Strombus Pes Pelecani* of LINNÆUS. The animal of this shell has been figured by MÜLLER, and very much resembles that of *Buccinum*, having long slender tentacles with the eyes sessile on the outer side of their base; while, as Dr. RÜPPELL informs me, the *Rostellaria curvirostris* has an animal allied to *Strombus*, with the eyes on very large peduncles, which give off from the middle of one of their sides the small tentacles. Notwithstanding this difference in the form of their animals, I am not, however, aware of any essential character by which the shell of *Aporrhais* (as the *Strombus Pes Pelecani* has been generically named) can be distinguished from the other *Rostellariæ*.

With all this uncertainty with regard to the generic characters of the recent species of shells, of which the animals can be subjected to examination, how much must the difficulty of deciding their genera with certainty be enhanced with reference to the fossil species, and especially to those which have no strictly analogous form existing in the recent state. Considerations like these tend greatly to disturb the confidence formerly reposed in the opinion that every difference in the form and structure of the animal was accompanied by marks permanently traced upon the shell, by which it might be at once distinguished, and which it was therefore the great object of the conchologist to point out. But another source of error, particularly interesting to the geologist, is included under my second head, to the elucidation of which I shall now proceed.

2. *Of Species belonging to the same natural Genus, inhabiting essentially different situations.*

The general belief that all the species of the same genus inhabit the same kind of situation, undoubtedly holds good with reference to most of the genera of shells; but many exceptions have already been observed, and we may anticipate that many more will be discovered as the natural habits of the different species become better known. In bringing together a number of these exceptions, I have been under the necessity of placing considerable reliance on the observations of others, who have noted in foreign countries facts similar to those which I have myself witnessed at home; but these observations have been chiefly collected from the works of Professor NILSSON of Sweden, of Mr. SAY of the United States of North America, and of MM. LESSON, QUOY, and RANG of Paris, writers who, from their extensive knowledge of conchology, are fully capable of accurately recording their observations, and whose statements may therefore be received as deserving of the most implicit confidence. It is moreover to be observed, that all their observations on this subject were made simply with the view of extending the knowledge of the history of the species to which they refer, and without reference to the establishment of any preconceived theory.

These observations may be classed under the four following subdivisions: 1st, where species of the same genus are found in more than one kind of situation, as on land, in fresh and in salt water; 2nd, where one or more species of a genus, most of whose species inhabit fresh water, are found in salt or brackish water; 3rd, where, on the contrary, one or more species of a genus, whose species generally inhabit the sea, are found in fresh water; and 4th, where the same species is found both in salt and fresh water.

Of the first of these classes the genus *Auricula*, as defined by LAMARCK, may be quoted as a striking example. Of its species, *A. Scarabus* and *A. minima* are found in damp places on the surface of the earth; *A. Judæ* lives in sandy places overflowed by the sea; *A. Myosotis*, *A. coniformis*, *A. nitens*, &c. (separated by DE MONTFORT under the name of *Conovulus*,) are found only in the sea in company with Chitons, *Littorinæ*, and other truly marine shells; and the South American species which I distinguished some time since under the name of *Chilina*, including *A. Dombeyi* of LAMARCK, and *A. fluviatilis* of LESSON, inhabit freshwater streams, having most of the habits of the *Lymnææ*. This disparity of habitation has been in some degree overcome by dividing the genus into several, as noticed above; but the characters employed for their distinction are very slight, and species apparently intermediate between them are constantly occurring.

The genus *Lymnæa* has usually been considered as confined to fresh water; but M. NILSSON describes a species under the name of *L. Balthica*, which is found "in aquâ parùm salsâ Maris Balthici ad littora Gothlandiæ et Scaniæ, &c. In maris juxta Esperöd fucis et lapidibus adhærens frequenter obvenit simul cum *Paludinâ Balthicâ* et *Neritinâ fluviatili*;" and a second under the name of *Lymnæa succinea*, which is

found on the shores of the sea near Trelleborg. All the species of *Paludina* and *Bithynia* which have fallen under my own observation are essentially fluviatile; but M. NILSSON refers in the paragraph above quoted to a species of the former genus inhabiting the sea. This may, however, like some of the smaller *Paludinæ* of DRAPARNAULD, be truly a *Littorina*, having a horny and spiral, and not an annular, operculum.

According to the observations of my sister, Mrs. INCE, of Mr. BENSON, of MM. QUOY and GAIMARD, and of M. LESSON, the Indian species of *Neritina*, like the European, are found only in fresh water; yet M. RANG, in his Manuel des Mollusques, p. 193, states that the *Neritina viridis* is a marine species found on rocks covered by the sea at Martinique, and that a larger variety of this species is found in similar situations at Madagascar; General HARDWICKE marks on his drawing of the *Neritina crepidularis*, that it was found in "saltwater lakes, April 1816;" and SAY has described the *Neritina Meleagris* of LAMARCK (*Theodoxus reclinatus*, SAY,) as living both in fresh and salt water. This is most probably the species to which Mr. GUILDING refers*, when he observes that he has kept *Neritina* for some time alive in a close vessel of salt water, which they appear to purify. The animals of some of the tropical species often quit the stream and crawl up the trunks of neighbouring trees, on which, like the species of *Littorina, Planaxis,* and *Bulla*, which creep up the rocks on the sea-coast, they attach themselves, and remain exposed to the influence of the sun. It may be added, that M. RANG has found *Neritina Auricula* in brackish marshes near the sea in the Island of Bourbon, in company with *Aviculæ* and *Aplysiæ*; and I have little doubt that *Neritina Pupa* inhabits the sea, it being uniformly brought to this country in company with marine shells.

Many species of *Melania*, as, for example, *M. amarula, M. fasciolata,* and *M. lineata*, are found in the freshwater streams of India and its islands. Mr. SAY mentions species found in similar situations in North America; he also describes one (*M. simplex*) as found in a stream running through the saltwater valley near the salt-works, but does not state whether the water of the stream is salt or fresh. On the other hand, M. QUOY asserts that they are sometimes taken in brackish water; M. CAILLIAUD states that *Melania Oweni* is found in brackish water; and M. RANG has found other species in the Island of Bourbon under the same circumstances with the *Neritina* just adverted to. The genus *Melanopsis* has the same habits; its species are often found in large inland lakes. I have myself received *M. buccinoidea* from the sea of Galilee; and Dr. CLARK, in his Travels, vol. ii. p. 243, figures *M. Dufourii* under the name of *Buccinum Galileum*. The water of this lake, however, unlike that of the neighbouring Dead Sea, is, according to the statement of FULLER, perfectly fresh and sweet. M. LESSON, on the other hand, states that he found the *Pyrena terebrans*, regarded by M. DE FÉRUSSAC as a *Melanopsis*, in great abundance in brackish marshes in New Guinea, and at the Island of Bourou.

I am informed by Mr. SOWERBY that some species of the fluviatile genus *Cyrena* are found in the sea on the coast of South America; but he thinks it probable that

* See Zoological Journal, vol. v. p. 33.

the part of the sea in which they are met with may be fresh, like certain parts of the ocean described by Dr. ABEL in his voyage to China. It would be highly interesting to procure a verification of this observation. Similar phenomena may not be uncommon, for I have myself observed in Torbay a small space in the neighbourhood of Brixham, the water of which was of a different colour and much fresher than that of other parts of the bay. With reference to another species of the same genus, *Cyrena Vanikorensis*, M. QUOY observes: "Ne l'ayant pas trouvée dans les lieux marécageux, mais sur les bords de la mer, il est probable qu'elle vit à l'embouchûre des rivières qui sont saumâtres à marée haute*."

The third class of cases, in which species of *Mollusca* that are generally found in the sea are taken in fresh water, is much more rare than the preceding. It is obvious that in such instances the animal must be possessed of the capability of adapting itself to the different characters of the two fluids. This capability exists in much more highly organized animals, such as fishes, many species of which constantly migrate from the sea and ascend the rivers to deposit their spawn; but in these cases it is the result of a regular and determinate habit, while in the *Mollusca* it appears to be entirely dependent on accidental circumstances.

In some marshes in the Island of Bourbon, in which the water is almost fresh, M. RANG has observed specimens of *Aplysia dolabrifera* in company with *Neritinæ* and *Melaniæ*.

The greater number of species of the genus *Cerithium* are truly marine, chiefly living in sandy bays, like our own *Cerithium reticulatum*. M. LESSON, however, found *C. sulcatum*, and ADANSON the African species figured by him, in the pools of brackish water, sometimes overflowed by the sea, which are situated between the weeds and the belts of mangrove trees on the shore; and Mr. SAY observes that the small species, called by him *Pyrena scalariformis*, but which is a true *Cerithium*, is found in great abundance in the fresh water of Florida Keys. He adds: "it is most certainly a freshwater shell, yet it is destitute of an epidermis."

The genus *Bulla* is also truly marine; but the Rev. Mr. HENNAH some time since presented to the British Museum specimens of one of its species, resembling the *Bulla Hydatis*, found by him in brackish pools on the coast of Chili; and Mr. SAY has described a *Bulla fluviatilis* found by Mr. AARON STONE deeply imbedded in the mud of the river Delaware†.

The *Littorinæ*, again, are all found either on the sea-shore or in the very brackish water of the mouths of rivers, except two, which although described as *Paludinæ* by PFEIFFER and DE FÉRUSSAC, and formed into a distinct genus by ZIEGLER under the name of *Lithoglyphus*, agree with *Littorina* in every character of shell and operculum, and, as far as I can ascertain from the descriptions, of the animal also. These are the *Paludina fusca* of PFEIFFER, and the *P. naticoides* of DE FÉRUSSAC: they are truly fluviatile.

* Voyage de l'Astrolabe, tom. iii. p. 516.

† See for this latter instance the Journal of the Academy of Natural Sciences of Philadelphia, vol. ii. p. 179.

These anomalies are not restricted to the univalves: bivalves have also their share. Thus, the genus *Solen* is generally and properly considered as marine; but Mr. Benson has lately discovered a species inhabiting the mud on the banks of the Ganges; and conceiving, from the nature of its habitation, that it ought to be separated from the common species, he has formed a genus for its reception under the name of *Novaculina*. On comparing, however, some specimens of the shell presented to the British Museum by Mr. Royle, I can scarcely distinguish it as a species from the *Solen Dombeyi* of Lamarck, which is found on the coast of Peru; and I have two other species, very nearly related, one from the rivers of China, and the other from pools of brackish water on the coast of America. In like manner M. Nilsson has found his *Tellina Balthica*, which appears to be little more than a variety of the *Tellina solidula* of our coast, in the brackish water of the shores of the Baltic. *Avicula margaritifera*, the mother-of-pearl shell, commonly found in the ocean, has been taken by M. Rang in marshes in the Isle of Bourbon in the neighbourhood of the sea in which the water is nearly fresh. Specimens of *Mya arenaria* also are often found so high up the rivers that the water in which they live is brackish only during high tides. They are found, moreover, with freshwater shells on the coasts of the Baltic, while all the other species of the genus are found only where the water is quite salt.

By far the greater part of the species of *Corbulæ* are truly marine; but there is a large species of the genus, called by Dr. Maton* *Mya labiata*, brought with freshwater shells from the mouth of the Rio de la Plata; and this agrees in many respects with the fossil *Corbula Gallica*, which occurs in what are called the upper freshwater strata of the Isle of Wight.

The transitions to which the oysters intended for the London market are exposed may be mentioned as an additional illustration. Many of these are collected in the sea on the coasts of Guernsey and of France, and are brought to situations in the mouth of the river where the water is merely brackish during the ebb of the tide, and where they are consequently subjected to the alternate action of salt and brackish water twice in each day. It is even affirmed that oysters can exist in water absolutely fresh; for in the Museum of the Bristol Institution there is a large group said to have been dredged up in a river on the coast of Africa where the stream was so sweet as to have been used to water the ship. To these shells are attached specimens of *Cerithium armatum*; and the person by whom they were presented to the collection stated that *Cardium ringens* was found abundantly in the same situation.

The genus *Cucullæa*, again, is universally considered as truly marine; but Mr. Benson has found in the Ganges a small shell belonging to it, regarded by him as an *Arca*, but on account of its freshwater origin formed into a new genus under the name of *Scaphula*.

On this subject I may observe, that I was some time ago informed that *Arca senilis* was found in the rivers of Africa in company with *Galatea radiata*: M. Cailliaud,

* Linnean Transactions, vol. x. p. 326, t. 24, f. 3.

however, assures me that this is by no means the case, the shells in question being found near the mouths of the rivers, but never in the rivers themselves.

One of the most decisive facts regarding the finding of the same species of shell in both salt and fresh water is noticed by SAY*. Speaking of *Theodoxus reclinatus*, he observes, "I found this species in great plenty, inhabiting St. John's river in East Florida, from its mouth to Fort Picolata, a distance of one hundred miles, where the water is potable. It seemed to exist equally well where the water was as salt as that of the ocean, and where the intermixture of that condiment could not be detected by the taste." The shell in question is determined, by specimens which I received from my late friend himself, (to whom science is so deeply indebted, and especially for his researches into the zoology of North America,) to be the *Neritina Meleagris*, obtained in such abundance from the West Indian Islands. NILSSON too, as before mentioned, has noticed the *Neritina fluviatilis*, which in this country is not observed to inhabit ditches in the neighbourhood even of brackish water, living on the coasts of the Baltic, in brackish situations, in company with *Lymnæa Balthica* and *L. succinea*; and M. RANG found *Neritina auriculata* in similar situations.

According to the observations of OLIVIER, the *Ampullaria ovata* inhabits Lake Mareotis, where it is taken in company with marine shells found also in the Mediterranean; and I have lately received (dead) specimens from the locality indicated. The same species was found by M. CAILLIAUD in freshwater lakes in the Oasis of Siwah, where it is called Bozue and eaten as food. It thus appears to be found both in fresh and brackish water. Two of the species referred to this genus by LAMARCK, his *Ampullaria Avellana* and *A. fragilis*, are truly marine; but they differ from the others in animal and operculum, as well as in the sinuated form of the outer lip of their shell.

The common cockle of the shops, *Cardium edule*, is constantly to be seen in the ditches of brackish water in the neighbourhood of Tilbury Fort, which gradually become more or less fresh in proportion to the quantity of rain that falls between the periods of opening the sluices. It is to be observed that the specimens found in this situation are rather thinner and more produced posteriorly than those usually found in the sea. The species in question is also, according to NILSSON, found in the brackish water on the shores of the Baltic, but I am not aware whether or not it is there subject to a similar variation in form. NILSSON observes, however, that the marine species found in those localities are generally smaller than those found in other situations.

From this list of exceptions to the general rules which have commonly been regarded as decisive of the localities inhabited by recent shells, and of the nature of the deposits in which the fossil species are found, it is manifest that those rules cannot safely be made use of for practical purposes without considerable reservation.

* Journal of the Academy of Natural Sciences of Philadelphia, vol. ii. p. 258.

XVII. *On the supposed existence of Metamorphoses in the* Crustacea. *By* J. O. Westwood, *F.L.S. & Sec. Ent. Soc. Communicated by* J. G. Children, *Esq., Sec. R.S.*

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PERHAPS none of the phenomena of natural history have attracted a greater share of the attention of mankind in all ages than those exhibited by insects in their passage to the perfect state, and to which it is not surprising that the name of metamorphoses should have been applied. If this were the case in the darker days of zoological knowledge, when the true nature of these changes was not understood, it is not strange that the subject should have lost none of its interest when, owing to the admirable researches of Redi and Swammerdam, De Geer and Reaumur, all of the marvellous has been removed, and a series of gradual developments exposed, far exceeding in peculiarity those exhibited in any of the other tribes of animals.

It will not perhaps be considered out of place if we here shortly glance at those general principles which regulate these metamorphoses amongst the *Annulosa.* "Si nous voulons concevoir," observes Latreille, "d'une manière claire et positive le sens qu'il faut attacher au mot de *métamorphose,* il est necessaire que nous nous formions une idée exacte de celui de *mue*; car leurs significations paraissent avoir beaucoup d'affinité, et il est essentiel de les déterminer aussi rigoureusement qu'il est possible*."

It would, however, lead us to far too great a length were we at the outset to enter into the question of the *gradual formation* of the various organs of annulose animals from the rete mucosum, as insisted upon by Dr. Heroldt in opposition to the generally received opinion of Swammerdam, that these various organs are, from the first exclusion of the insect from the egg, in a state of existence, but enveloped in various coverings which are successively cast off; although the determination of this question must be considered as having a material influence upon the subject under consideration, more especially as it seems difficult to account for the reproduction of the limbs of the *Crustacea* when torn off if we do not adopt the theory of Dr. Heroldt.

Every animated being in its passage to the perfect development of its species undergoes a certain but varied series of changes. In man and most of the vertebrated animals there is a gradual action of the vital forces in different organs till they are fitted for reproduction, accompanied, as progress is made to the adult state, by the acquisition of various appendages, as teeth, horns, pubes, feathers, &c. In addition

* Cours d'Entomologie, p. 271.

to this gradual action, the greater portion of the *Vertebrata* are subject to that particular species of ecdysis which Mr. MacLeay has termed *incomplete*, consisting simply in the integuments, hairs, skin, feathers, &c., scaling off piece by piece, or one by one, as distinguished from that *complete* change in the identity of the envelope of other less perfect animals which prevails in various degrees amongst the *Annulosa* and some few of the *Vertebrata*, and by means of which the entire envelope of the animal is shed at once. And here we may be allowed to notice the rationale of this complete shedding of the envelope which so peculiarly distinguishes the *Annulosa*. In them we find the internal vertebræ of the higher animals converted into a hard horny or crustaceous external covering, to the inner surface of which the muscles are attached. This covering would of course, from its very nature, offer an insurmountable obstacle against the growth of the animal, were it persistent. It is therefore necessary that, in order to ensure the due increase of size in the animal, its old covering should be cast off and a new and enlarged one obtained. And this is what takes place throughout the *Annulosa*, the shedding of the shell of the Lobster and the moulting of the Caterpillar being but modified examples of the same principle.

These modifications may be reduced to three principal heads, of which the Spider, the Grasshopper, and the Butterfly may be cited as well-known examples.

In the first of these, the animal is produced from the egg in a form which it is destined to retain throughout its existence, its only change consisting in a series of moultings of the outer envelope, by which an increase of size, but not an addition of new organs, is acquired.

In the second, the animal at its exclusion exhibits the form which it retains through life, but it is subject to a series of moultings, during several of the last of which certain new organs are gradually developed.

In the third, the form of the animal at its exclusion from the egg is totally different from that in which it appears in its imago state, this change of form taking place during two or three of its final moultings, and consisting not only in the variation of the form of the body, but also in a complete change in the nutritive and digestive systems, and in the acquisition of various new organs. This constitutes what has generally been termed metamorphosis.

Now, since the *Ptilota* of Aristotle are preeminently the types of the invertebrated animals, and as such more distantly removed from the various groups of the *Vertebrata* than the remainder of the *Invertebrata*, (owing this preeminence not only to the superiority of their instincts, but also to the development of organs of flight during the latter moultings,) it will necessarily follow that those *Annulosa* which are less typical, or, in other words, more nearly allied to the lowest of the higher groups of animals, will not exhibit in so remarkable a degree those metamorphoses which, as we have seen, the *Ptilota* so peculiarly undergo.

Hence, since the organization of the *Crustacea* is more clearly analogous to that of the *Vertebrata* than that of the *Ptilota*, we arrive at one of the chief grounds for the

generally received opinion amongst naturalists, that the transformations of the *Crustacea* consist merely in the periodical shedding of the outer envelope, without any metamorphosis being undergone or additional organs acquired.

The object of the following pages is therefore to endeavour to ascertain whether this opinion be correct or not; and in order to do this satisfactorily we shall be obliged to test such observations which may negative its correctness, by the application of those general principles which, as we have seen, regulate the transformations or other changes of the *Annulosa*.

The non-existence of transformations in the *Crustacea* in general has been asserted by every crustaceologist, with the exception of a recent author, JOHN V. THOMPSON, Esq., F.L.S., (the accuracy of whose beautiful figures deserves the highest praise,) and by whom, in the first and succeeding numbers of the Zoological Researches, the discovery that the greater number of the *Crustacea* do actually undergo metamorphoses of a very peculiar kind, and of a totally different description from those of insects, has been announced. "So little has this been suspected by naturalists," observes this author*, "that the contrary has been assigned as one of the distinctive characters of the class, and been used as an argument for their separation from insects."

Mr. THOMPSON'S views are founded upon some circumstances exhibited by some of the most singular animals hitherto ascertained to belong to the class, (which constitute the genus *Zoea* of Bosc,) as recorded by SLABBER or Mr. THOMPSON himself, as well as upon some other circumstances respecting other portions of the class. These consist,

In the first place, in a supposed change which the Zoes are reputed to undergo; respecting which Mr. THOMPSON (after alluding to the observations of SLABBER, which he thinks erroneous,) thus expresses himself: "After keeping a full-grown Zoe for more than a month, it died in the act of changing its skin and of passing into a new form, but one by no means similar to that expected [from the previous observations of SLABBER], as appears evidently by its disengaged members, which are changed in number as well as in form, and now correspond with those of the *Decapoda* (Crabs, &c.), viz. five pair, the anterior of them furnished with a large claw or pincer: the metamorphosis not having been completed, prevented any knowledge being acquired of its general form; enough, however, has been gained to show that the distinctive characters of *Zoea* and of SLABBER'S changed *Zoea* were entirely lost; that the members, from being natatory and cleft (as shall shortly be shown), become simple and adapted to crawling only. On the 1st of May another large *Zoea* was taken, and dying towards the end of the month without having the strength to disengage itself from the exuvium, presented precisely the same results with the former†." In the account of the figures of this full-grown Zoe, "behind the corselet the rudiments of the limbs of the perfect animal, or Crab," are described as "beginning to show themselves‡;" but on comparing this figure with that of the newly hatched "Zoe,

* Zool. Illustr., p. 7. † Ibid., pp. 8, 9. ‡ Ibid., p. 33.

or larva of the common or edible Crab," (Pl. VIII. fig. 1.) "the disparity in size is shown between a Zoe newly hatched and one which has attained its full development, and the changes which the various parts undergo during the growth of the animal," (No. 2. Addendum,) as in the total absence of subabdominal fins, and in the natatory division of the two pairs of feet having only four plumose setæ in the younger animal: and in a former passage he observes, that "the larger specimens may be supposed to differ from such as occur of smaller size in the greater degree of development of all its parts; thus, the eyes are more distinctly pedunculate, the natatory division of the feet have an increased number of plumose setæ, the rudiments of the subabdominal fins are quite obvious, and the mandibles show the rudiment of a palp: in other respects they are essentially the same." (p. 10.)

In the second place, our author states that he had succeeded in hatching the ova of the common Crab (*Cancer Pagurus*), which presented exactly the appearance of *Zoea Taurus*, with the addition of lateral spines to the corselet. And in the addenda to his second number he has again stated this circumstance, adding somewhat more precisely, that he had protected a female Crab with spawn apparently ready to hatch until the young burst from their envelopes and swam about in myriads under the exact form of *Zoea* represented in the Plate.

In the third place, Mr. Thompson has stated that the common Lobster undergoes metamorphosis, "but less in degree" than any of the other genera in which he states that he had observed this to take place, and "consisting in a change from a cheliferous Schizopode to a Decapode, in its first stage being what I call a *modified* Zoe, with a frontal spine, a spatulate tail, and wanting subabdominal fins, in short, such an animal as would never be considered what it really is, were it not obtained by hatching the spawn of the Lobster*."

The only figure which accompanies this remark is given in tab. xv. fig. 13. of the same work, of "the cheliferous member of the larva of the Lobster, in which *a* is the claw; *b*, the outer division of the limb, or *future flagrum*; and *c*, the rudimentary branchia." In this figure, three organs are represented as arising from a large basal joint: first, the chelate organ, composed of two joints and a large didactyle chela; second, a three-jointed organ, of which the terminal joint is long, slender, and strongly setose; and third, a small rudimental branchia.

In the fourth place, "this curious piece of economy," according to Mr. Thompson, "explains what has ever appeared paradoxical to naturalists, viz. the annual peregrinations of the land Crabs to the sea-side, which, although acknowledged to be true by several competent observers, could never before be satisfactorily accounted for." (p. 9.) And again, in the Addenda to his second number: "Hitherto the rationale of this long and dangerous journey did not appear; naturalists have thought it strange that an animal entirely terrestrial should not spawn in its native haunts, and rear its young at home, instead of putting them to the trouble of a tedious and unknown

* Zoological Journal, No. xix. p. 383.

route back again in their very tender age. Scarcely a stronger confirmation than this very circumstance, of the universality of metamorphosis, could be adduced: for if there were any exception, it would be in the terrestrial species; but no, they are, when first hatched, incapable of living out of water with swimming members; hence the parent is impelled by instinct to seek that element for its progeny which Nature has designed for the whole of the tribe to which they belong. Having lived amongst West Indian islands, where these facts were constantly before him, neither he, nor any other person, could invent any plausible reason for this curious piece of economy."

In the fifth and last place are to be noticed Mr. THOMPSON's general statements. In the Addenda to his second number he states that he has had a confirmation of his views in one of the West Indian land Crabs, and in some other of our most widely separated native genera, authorizing his previous assertion that the greater number of the *Crustacea* do actually undergo transformations, of which, in addition to the facts adduced in his first memoir, further instances will be given in future memoirs. On the wrapper of his fourth number he has given a list of some of these promised memoirs, in which we find the *Paguri*, the Shrimp and Prawn, the genera *Porcellana*, *Gegarcinus*, *Hydrodomus*, and other genera of land Crabs and *Pinnotheres*, all stated to undergo various remarkable metamorphoses; and in the nineteenth number of the Zoological Journal he states that the newly hatched young of the following Brachyurous genera, *Cancer*, *Carcinus*, *Portunus*, *Eryphia*, *Gegarcinus*, *Thelphusa*?, *Pinnotheres*, and *Inachus*, have been ascertained to be Zoes by himself; and that the following Macrourous genera are likewise subject to metamorphosis, viz. *Pagurus*, *Porcellana*, *Galathea*, *Crangon*, *Palemon*, *Homarus*, *Astacus*.!

Such are the various circumstances upon which Mr. THOMPSON has built his theory of metamorphosis. I have given them at rather an inconvenient, but not an unnecessary length, and as far as possible in his own words, in order that I might be free from any charge of misrepresentation in the observations which I may think it necessary to make upon each of them, with a view to prove that the theory is without foundation.

For this purpose I propose, in the first place, to enter into a review of Mr. THOMPSON's observations, whence alone I conceive that no sufficient ground is raised for the establishment of the theory in question. In the second place, I propose to collect the recent views of the most celebrated crustaceologists, all of whom have advanced opinions to the like effect. And in the third place, I shall bring forward some circumstances observed by myself having a precisely similar tendency.

In the first place, therefore, I have to endeavour to prove from Mr. THOMPSON's own statements and figures, that there is not sufficient foundation for the theory of metamorphosis, and for this purpose I shall take in review *seriatim* the several circumstances which he has mentioned and above alluded to.

And first with respect to the metamorphosis into Crabs which the Zoes are stated to undergo, against which six arguments may be adduced.

1. It is to be observed that the account given of the mode in which this metamorphosis is supposed to be effected, is as vague and indefinite as it is possible to be. It is stated that the Zoe died in the act of casting its skin, but its metamorphosis not being completed, prevented any knowledge being acquired of its general form; and yet it is added that five pairs of legs had become disengaged, and that the characters of *Zoea* were entirely lost. Plate II. fig. 2., however, proves nothing like this. The limbs of the future Crab are asserted to be beginning to show themselves, and yet the Zoe retains its original form, without *losing* a single character which it previously possessed,—without our being able to trace the least appearance of the animal having commenced the shedding of the skin,—or without our being able to gain the least idea how "the members, from being natatory and cleft (*as shall shortly be shown*), become simple and adapted to crawling only." But Mr. THOMPSON has omitted to fulfill this promise, which, if it mean anything, must be understood as an assertion that the two pairs of natatory and cleft legs are transformed into five pairs of simple crawling legs.

2. The appearance of these limbs (represented as perfectly disengaged in Mr. THOMPSON'S Plate II. fig. 11.) previous to the shedding of the cephalothoracic shield and anterior parts of the body, is totally at variance with the principles of ecdysis observable throughout the *Annulosa*, in which the locomotive organs, at least the legs, are the last which are disengaged, and the thoracic shield of the inclosed animal the first portion exposed to view. It would, in fact, be impossible for the Zoe to disengage the thoracic limbs without the thorax itself being previously withdrawn from its covering.

3. But we will look more precisely at the nature of this supposed disengagement of the five pairs of legs. This, in the absence of any precise explanation given by Mr. THOMPSON, may be presumed to be effected in three different ways.

Firstly,—as indeed Mr. THOMPSON appears to suppose by his statement that the large natatory limbs "become" simple ones,—this may be effected by the two pairs of large natatory limbs entirely throwing away their outer covering, whereby the five pairs of small simple legs, which had been previously inclosed within them, are disengaged. This I take to be the true nature of the disengagement of the organs of motion in the *Annulosa*; but if we regard this to take place in *Zoea*, we shall necessarily have two conditions totally at variance with the principles of ecdysis, viz. that an existing organ in a state of incomplete development incloses only a single organ,—thus, the wings of the Grasshopper are not inclosed within the legs of its larva; and that an organ disengaged by the shedding of its envelope is always larger than such envelope. This, in fact, is the very end of the metamorphoses of the annulose animals, the hardness of their outer covering preventing their growth, except by the shedding of such covering.

Secondly, We may imagine that the five pairs of minute rudimental legs of the future Crab are not transformed from the two pairs of natatory limbs, but are totally

unconnected with them, being, as Mr. Thompson himself says, "disengaged from beneath the clypeus", (and his Plate II. fig. 2. represents the same idea,) and having no previous existence in the young Zoe. Now if this be the case, setting aside its disagreement with the recognised conditions of development, we arrive at once at this startling and important conclusion, namely, that the large natatory organs of the Zoe are instrumenta cibaria, foot-jaws, in fact, and that the animal in its Zoe state has no true legs. Without, however, asserting (which might reasonably be done) that every annulose animal which in its immature state is furnished with locomotive organs, is also furnished with instrumenta cibaria, which latter legitimately represent the instrumenta cibaria of the imago, whilst the former as truly represent the *true* legs of the imago, we may assert, that where an immature annulose animal is furnished with locomotive organs, these, or at least some of them, represent the true thoracic legs of the imago, and are not, in such immature state, merely rudimental trophi of the perfect animal. Now on applying this principle to the case in question, we find the Zoe furnished both with trophi and natatory organs; and if we regard the trophi, although few in number, as representatives of the trophi, and the two pairs of natatory organs as representatives of the locomotive organs of the future Crab, we can only regard the five pairs of disengaged limbs either as representing the subabdominal appendages of the Crab, or as simple thoracic appendages (distinct from legs), or as supplemental limbs. But each of these suppositions is so contrary to nature with reference to the organization of Zoe or the Crab as distinct animals, that in order to show their futility it will be sufficient to notice the determinate leg-like form of these disengaged limbs, the first pair of which is cheliferous; the fact that the *Zoea* has distinct subabdominal appendages; that the *Crustacea* are not, like the *Myriapoda*, furnished with auxiliary limbs; and that true thoracic locomotive organs (which in Zoe, according to the principles above stated, must still remain undeveloped,) are constantly developed at the same time as, or even before, supplemental ones.

Thirdly, We may imagine the disengagement of these "future limbs" to take place in a mixed manner, by considering that the two pairs of natatory limbs of the Zoe produce the first, or chelate, and second pairs of legs, and that the three posterior pairs are simply disengaged from beneath the clypeus. Against this idea many of the preceding observations may be conjointly adduced; to which it may be added, that the similar size of these disengaged limbs is sufficient to prove that they must have undergone an equal degree of development. Moreover, in such case the chelate members, which are larger than the following limbs, must be produced from the first pair of natatory limbs of the Zoe, which are much smaller than the second pair.

I have in these observations left unnoticed the small member anterior to the claws, observed by Mr. Thompson, and considered by him as the rudiment of the outer foot-jaw, which offers still greater difficulties as to its nature if we adopt Mr. Thompson's views, but which, as I shall subsequently show, is a necessary organ of the Zoe.

4. In the next place it is to be observed, that Mr. THOMPSON's figures and statements relative to the gradual development of the Zoes are totally at variance with one of the received principles of ecdysis, which may be thus stated. When an animal undergoes a variety of moultings, attended by alteration in form or development of organs, there is a gradual tendency towards the organization of the perfect animal. Now Mr. THOMPSON expressly states that his large Zoes differed from the smaller ones in the greater degree of development of all their organs. This therefore is precisely what would occur in case the large Zoes were perfect animals; and it is precisely what would not take place if the subsequent state of the Zoe were a Crab.

5. It is worthy of notice that there are several peculiarities in *Zoea* so evidently partaking of the Macrourous type, that it is surprising that Mr. THOMPSON should not have noticed that these characters present themselves in so complete a state of development, when compared with the *Macroura*, as to negative the opinion that these animals would ever become Brachyurous. The elongated tail, the rostrated cephalothorax, but more especially the structure of the mandibles and two pairs of maxillæ, may especially be noticed.

6. If, as we shall subsequently perceive, there be no pretence for doubting the correctness of RATHKE's researches upon the Cray-fish, which is clearly proved to undergo no metamorphosis, I think we are fully warranted from analogy in considering that the other Decapods do not undergo metamorphosis. Mr. THOMPSON, indeed, seems inclined to consider that in such case the Cray-fish "can only be regarded as one *solitary exception* to the generality of metamorphosis*;" although he had previously given his opinion of the weight of analogy in the second number of his Researches, by stating that "metamorphosis having been proved in a *single instance* amongst animals so uniform in structure as the *Homobranchia*, *we may safely infer from analogy*, as far as regards the particular tribe alluded to, that it is general."

These six considerations induce me to adopt the opinion that no sufficient ground has been shown by Mr. THOMPSON for supposing that a metamorphosis of Zoes into Crabs takes place.

Secondly, therefore, we will proceed to notice Mr. THOMPSON's statements relative to the hatching of the young Zoes from a female of the common Crab, and which he states took place under his own eye. It is much to be regretted that Mr. THOMPSON, having such ample opportunity, did not dissect the ova in various states, so as to ascertain in the most satisfactory manner the gradual development of the embryo, as RATHKE has done in the Cray-fish. The statement, although short, has, however, such sufficient precision, that we are compelled to believe either that (notwithstanding whatever may be advanced to the contrary) the young of the common Crab are Zoes, or that the latter are parasitic animals, which in some unexplained manner are introduced in the embryo state beneath the abdomen of the Crab; and if we consider the large Zoes observed by Mr. THOMPSON to be perfect animals, there is some ground

* Zoological Journal, No. xix. p. 383.

for the latter opinion in their comparatively less perfect organization, a circumstance to which a completely analogous case exists amongst the *Hyperiidæ* in the order *Amphipoda*, &c. Zoe, indeed, is not the only animal respecting which this kind of parasitic obscurity exists; the genus *Meloe* amongst the coleopterous insects is perfectly analogous, the young of which, according to some authors, are *Acari*, whilst others state them to resemble the perfect insect. I am the more anxious to offer this explanation of Mr. THOMPSON's argument, in as much as the facts subsequently stated respecting the ova and young of the *Brachyura* are totally at variance with Mr. THOMPSON's assertions.

Thirdly, As respects Mr. THOMPSON's statements relative to the young of the common Lobster, we have again to regret the slightness of the information given to us upon this branch of the subject. The young is called a modified Zoe, a cheliferous Schizopode, with a frontal spine, a spatulate tail, and wanting subabdominal fins, undergoing a metamorphosis *less in degree* than the other mentioned genera. We are left in uncertainty whether there are eight pairs of locomotive organs, as in the true Schizopods, or whether these organs are all divided into two parts; the only evidence of such Schizopod nature being the chelate limb figured; and yet this is precisely where information was required. Examine the other characters given of this "modified Zoe" without reference to its undescribed legs, and we are able to trace (notwithstanding Mr. THOMPSON's assertion to the contrary) precisely such an animal as might be expected for an immature Lobster. But if we examine the nature of the cheliferous member figured, we shall find the strongest reason for considering that this "larva" is not a Schizopode. Fig. *a*. represents the cheliferous member of the perfect Lobster, as well as of its larva; but this organ is not provided in the perfect state with any lateral appendage. And Mr. THOMPSON himself does not attempt to prove the connexion of the lateral appendage which he figures with the cheliferous limb of the perfect Lobster, since he describes this lateral appendage as the *future flagrum* of the Lobster, that is, the lateral division of the exterior pair of foot-jaws; consequently, unless Mr. THOMPSON is prepared to prove that the lateral appendage of one organ in the immature state becomes the lateral appendage of a totally distinct organ in the perfect state of the same animal, it must follow that this gentleman has erred in his dissections of the immature Lobster, and mistaken the lateral appendage of the outer foot-jaw for a Schizopodous appendage of the Cheliferous limb.

Fourthly, As respects the explanation or "excuse" which this principle of metamorphosis enables us to give for the annual migrations of the land Crabs of the West Indies to the ocean to deposit their spawn, the young produced from which being natatory animals—Zoes, in fact,—are incapable of living in the same element as their parents in their early stages, I can very well agree with Mr. THOMPSON that if any exception existed amongst the *Crustacea*, in which the young should not undergo any change from aquatic to terrestrial habits, accompanied of course by a

corresponding modification of structure, it would be amongst the land Crabs; but I cannot agree with this gentleman that scarcely a stronger confirmation than this very circumstance could be adduced of the universality of metamorphosis, in as much as it appears to me that Mr. THOMPSON has arrived somewhat too suddenly at the conclusion that the young must consequently be Zoes, or even that, although incapable of living out of the water, they are necessarily furnished with natatory members. Examine the sea Crab, and no material difference in the structure of its locomotive organs is to be observed from that of the land Crab whence a different kind of motion can be inferred; hence there can be no actual necessity for the existence of natatory apparatus in the young land Crab, which must be just as able to support itself in the water without any such as an ordinary sea Crab. If, moreover, we examine the structure of the branchial apparatus of the land Crabs, we find still further evidence in support of this argument. MM. AUDOUIN and M. H. EDWARDS, in the Annales des Sciences Naturelles for September 1828, have given an account of this organization: the exterior of the branchial cavity is furnished with a reservoir for containing a supply of water; and in the land Crab there is moreover a second vessel destined for the like purpose, whence it is evident that in this respect the land Crabs do not materially differ from the sea Crabs. I will not dwell upon this subject further than to refer to the conclusive facts subsequently stated in proof that the young of the land Crabs is neither a Zoe nor furnished with natatory apparatus.

Fifthly, I will only observe with reference to Mr. THOMPSON's general assertions, that no great weight ought to be attached to them until the necessary details shall have been given to the public, more especially if, as I have shown to be the case, we find cause in those already published to distrust the views of the author.

Having thus gone through the various statements made by Mr. THOMPSON in support of his theory, and ascertained from them the apparent want of confirmation of such theory, I proceed to notice the opinions of crustaceologists whose writings have established for them some degree of weight as authorities upon the question.

These observations will be confined, firstly, to such as bear directly upon Mr. THOMPSON's statements, and secondly, to such as relate to facts noticed respecting the transformations in the early stages of various animals in the class; since it is the more necessary in endeavouring to ascertain the correctness of the views of an author, to reject all general assertions made by others to the contrary which have not been made in reference to such opinion, or which do not rest upon direct observation. And it is to be regretted that Mr. THOMPSON's memoirs have been far from generally known; this will account for the slight degree of attention which has been bestowed upon the interesting subject upon which they treat, and for the paucity of notices respecting them.

We find LATREILLE*, however, stating that the opinion of Mr. THOMPSON "a grand besoin d'être étayée par des expériences positives, si toutefois elle n'est pas erronée."

* Cours d'Entomologie, p. 385.

M. EDWARDS, the most celebrated of modern living crustaceologists, observes, that as the *Malacostraca Podophthalma* are divisible from the presence or absence of branchiæ to the thorax, inclosed in a peculiar cavity, "on n'aura plus d'incertitude sur la place que doit occuper un genre très curieux, *Zoea*; en effet, un examen attentif de ces petits animaux m'a convainçu que non seulement leurs yeux sont portés sur des péduncles, mais aussi de chaque côté de leur thorax il existe sous le carapace une cavité respiratoire renfermant des branchies semblables par leur structure et leur position à celles d'autres Macroures. Il est donc evident pour moi que le Zoe est réellement un Crustace de l'ordre des Décapodes. Mr. THOMPSON assure que cet animal n'est autre chose que le jeune de Crabe commun. Cette opinion me ne paraît pas soutenable, mais neanmoins il serait possible que les Zoes observés jusqu'ici ne soient pas des animaux adultes, et alors il se pourrait bien que par les progrès de l'âge ils deviennent assez semblables aux Megalops; question que nous nous proposons de traiter plus au long dans une autre occasion*."

The talented editor of the Zoological Journal has also, in his review of Mr. THOMPSON's work, expressed his doubts as to the universality of the fact of metamorphosis taking place in the *Crustacea*; and in the eighteenth number of that work he has stated the confirmation which his doubts had received by the publication of Dr. RATHKE's work, adding, that if there existed no optical delusion or other cause of error in the isolated observations which Mr. THOMPSON has given us, the difference of organization between a Macrourous and a Brachyurous Decapod is much greater than either analogy or anatomy would have led him to suspect.

And lastly, Mr. KIRBY has communicated to me his conviction that the researches of Mr. THOMPSON are to be regarded with distrust, the grounds for which opinion will appear in his forthcoming Bridgewater Treatise.

I now proceed to notice, as concisely as possible, the direct observations made by various authors upon different Crustaceous animals in the young state.

And in the foremost place are to be mentioned the elaborate researches of RATHKE upon the development of the ova of the common Cray-fish, a work which for minute and delicate investigation is rivalled only by LYONNET's celebrated memoir upon the larva of *Cossus*. Some idea may be entertained of the extent of these inquiries, from the fact that five large folio plates are completely filled with details of the structure, internal and external, of the ova in various states of development, and of the newly hatched animal. And so beautifully clear are the representations of these objects, and so completely is the development of the embryo to be traced through all its stages, that unless we believe the whole to be the work of a fanciful imagination, it is impossible to arrive at any other conclusion than that the Cray-fish does not undergo any change which can in the least degree merit the name of metamorphosis. A full abstract of this valuable memoir is inserted in the eighteenth number of the Zoological Journal, and in the Annales des Sciences Naturelles for August 1831,

* Ann. Sc. Nat., April 1830.

the latter of which is accompanied by four plates. I will therefore content myself with referring the student to these accessible sources, without attempting to give even an outline of the elaborate investigations in question.

LATREILLE, speaking of the young of the Cray-fish, says: "Les jeunes écrevisses, très molles au moment de leur naissance, et toute-à-fait semblables à leurs mères, se réfugient sous leur queue, et y restent pendant plusieurs jours et jusqu'à ce que les parties de leurs corps soient raffermies *."

Mr. THOMPSON himself, in the genus *Mysis*, has clearly shown that these animals, which he has proved to be most intimately allied to the Decapod *Macroura*, undergo a series of changes, which he states "cannot be considered as metamorphoses, but simply a gradual development of parts †."

The above appear to be all the direct observations hitherto made upon the *Podophthalma* (with the exception of those subsequently detailed from my own researches); but amongst the sessile-eyed *Malacostraca* we have more numerous observations.

Of these, as in the former, the researches of RATHKE again stand foremost; since, in a series of memoirs, very recently published, upon the development of the ova and embryos of various animals, we find the common *Asellus aquaticus* to have been investigated by him, with the result that no material alteration takes place in the form of the animal ‡.

In the Annales des Sciences Naturelles for December 1833, is published a valuable report, by M. ISIDORE GEOFFROY ST. HILAIRE, upon a memoir of M. H. MILNE EDWARDS, entitled "Observations sur les changemens de forme que les Crustacés éprouvent dans le jeune âge." Passing over the more generalized views deduced by M. ST. HILAIRE from the facts noticed by M. EDWARDS, I shall merely state the latter. The genus *Cymothoa*, and some other Isopodous genera, afforded to M. EDWARDS an easy opportunity of examining the development of the eggs and the structure of the young, in consequence of their being inclosed within the large subthoracic pouch. Hence he was enabled to ascertain that some organs which are fully developed in the adult animal, are either rudimental or absolutely wanting in the early state: thus, in the latter the animal has only six thoracic segments, and six pairs of legs, although when adult it has seven segments and seven pairs of legs. On the contrary, other organs, which are fully developed in the young, become rudimental in the adult state: thus, in the former we find a large head, furnished with two large oval black eyes, and the abdominal segments nearly as large as the thoracic ones; whilst in the adult state the head is extremely small, the eyes are not externally visible, and the abdominal segments are very short and linear. M. EDWARDS has also made similar observations upon many other genera, especially upon *Anilocra*, in which a pair of legs is also developed after birth (the same likewise takes place in

* Règne Animal, tom. iv. p. 90. 2nd edit.

† Zoological Researches, p. 16.

‡ Abhandlungen zur Bildungs ant Entwickelung-geschichte des Menschen und der Thiere. 4to. 2 parts. 1832, 1833.

Oniscus, as De Geer long ago remarked); upon *Cyamus*, a genus of *Læmodipoda*, which in the young state is of a slender and cylindric form, but which afterwards becomes much enlarged and depressed; also upon *Phronyma*, a genus of *Amphipoda*, remarkable for its large head, conical thorax, and singular construction of the fifth pair of thoracic legs, but which in the young state exhibits a head of ordinary size, a thorax larger in the centre than at the extremities, and the fifth pair of legs not unlike the others, and not didactyle.

From the modification in form which the *existent* organs undergo in the passage to the adult state, M. Edwards deduces this curious theory,—that the changes of form which the *Malacostraca* undergo constantly tend to remove the animal to a greater distance from the type which is common to the greatest number of individuals in the group, so as to individualize it more and more completely. Thus the form of the immature *Cymothoa* or *Phronyma*, for instance, is referrible to the general typical form of the *Isopoda* or *Amphipoda*; but, by the gradual change of form, these animals are exhibited in forms the furthest removed from the types of their respective orders.

It is evident, however, from these remarks, that the *Edriophthalma* undergo no change worthy of the name of metamorphosis; and this is most fully supported by the observations of Latreille upon the *Isopoda* in general, viz. that the progeny "naissent avec la forme et les parties propres à leur espèce, et ne font que changes de peau en grandissant*"; of Mr. Montague upon *Caprella Phasma*, who states that he observed ten young ones crawl from the abdominal pouch of the female, "all perfectly formed"†; of Mr. Coldstream in an admirable paper upon *Limnoria terebrans*, inserted in the Edinburgh New Philosophical Journal of Professor Jameson for April 1834; and, lastly, of Professor Zencker in his memoir "De Gammari Pulicis Historia Naturali," 4to, 1832.

Hitherto I have confined these observations to the *Malacostraca*, because it is in that division of the class that the non-existence of metamorphoses has been denied. The *Entomostraca* are admitted on all hands to undergo very material modifications of form, as may be seen from the researches of Jurine, Strauss, Prevost, &c.; whilst Mr. Thompson's recent memoir upon *Artemia* (not *Artemis*,) is an additional evidence of the same fact, although the nature of the various alterations is very far from being detailed in that satisfactory manner which the author seems so capable of doing.

I have therefore now to detail, as the third portion of this essay, such circumstances as have fallen under my own observation relative to this interesting inquiry, the tendency of which is precisely similar to that exhibited by the two preceding portions of my treatise.

We have seen that Mr. Thompson's chief argument is founded upon the supposed transformations of the Zoe into a Crab. His Zoe, figured as the just-hatched larva of the common Crab, is not so large as a large pin's head. Slabber's "changed Zoe" is represented as three lines long; and Mr. Thompson's Zoe, which died on the sup-

* Règne Animal, tom. iv. p. 131.

† Linnean Transactions, vol. vii. p. 66.

posed point of transformation into a Crab, is nearly four lines long between the tips of the spines. Now if Mr. THOMPSON's views be correct, and these latter Zoes are to be regarded as the larvæ of Crabs, they must be considered as having acquired the maximum of their Zoe form; but so far is this from being the case, that I have obtained from the collection of the late Rev. LANSDOWN GUILDING, specimens of a species of *Zoea ten* lines long between the points of the spines; a size far too large to allow us to suppose that they would subsequently put off their Zoe form, and appear as Crabs; bearing at the same time in mind the minute size of the latter animals in the very young state, although possessing their ordinary form.

Of this West Indian species I have given, in the accompanying sketch, figures in detail of the various organs, which I shall not describe at length. The palpigerous mandibles, the two pairs of antennæ, one pair of which is bipartite, the multilobed inner maxillæ, are all characters found in the *Macroura* and *Schizopoda*, but not in the *Brachyura*. The natatory apparatus of the tail, observed in my species and unnoticed by Mr. THOMPSON, is also similarly characteristic, but the locomotive organs are those to which the highest importance attaches with respect to the real nature of the animal. At first sight, in addition to and immediately succeeding the two pairs of maxillæ, there appear only two pairs of large locomotive bipartite organs. These therefore, on the supposition that the Zoe is the young of a Decapod animal, must either be legs, or outer foot-jaws greatly developed; and from their bipartite structure, the latter may be partly assumed; but upon carefully dissecting the animal, a series of organs were found, which not only fully proved this to be the case, but also led at once to the discovery of the real nature of these animals, and gave a clue for the correction of Mr. THOMPSON's ideas upon the supposed disengagement of the thoracic limbs. Immediately succeeding the outer pair of the natatory organs, and, in fact, lying between them when at rest, was discovered a pair of slender minute organs, composed apparently of two joints, one long and one short, and furnished at the base with a still more minute lateral appendage. Beyond these, in succession, were found the five pairs of organs precisely similar to Mr. THOMPSON's "limbs of the future Crab disengaged from beneath the clypeus." Moreover, a number (undetermined) of minute fleshy elongated masses were found near and attached to the base of these limbs. Are we therefore, with Mr. THOMPSON, to suppose that in this Zoe (and all the specimens were alike) metamorphosis had commenced, whilst not the slightest trace of such a process could be observed beyond this acquisition of rudimental limbs, which can otherwise be much more satisfactorily accounted for? The researches of recent authors, and those particularly of M. H. MILNE EDWARDS, have clearly proved that in some species of Decapods (*Acetes*, *Sergestes*,) one or more pairs of legs become rudimental, and that their place is supplied by highly developed foot-jaws.

Now upon applying this theory, to the correctness of which Mr. THOMPSON bears witness, to *Zoea*, we find that the two large pairs of natatory organs represent the

first and second pairs of foot-jaws of the typical Decapods immensely developed; the minute pair of organs following these to be the third pair of foot-jaws; the five pairs of "limbs of the future Crab" to be the real thoracic legs of the Zoe, and that the minute fleshy masses are evidently branchiæ. Thus we perceive that the possession of these limbs, instead of being an evidence of the imperfect state of the Zoe, is a proof of its anomalous perfection; and thus we arrive at the unexpected conclusion that *Zoea* is a genus of Decapod *Crustacea*, for the reception of which amongst the *Macroura* a distinct section must be established.

With reference to Mr. Thompson's statements respecting the hatching of the Zoes from the eggs of the common Crab, and the arguments adduced from the habits of the West Indian land Crabs, I am able to offer the following as, I trust, very conclusive observations to the contrary.

In the collection above alluded to were contained, in spirits, the abdomens of several female Crabs, having the interior surface covered with hundreds of eggs or newly hatched young. One of the bottles in which one of these was deposited was labelled by Mr. Guilding, "Eggs and young of a land Crab not undergoing a metamorphosis." From this specimen I obtained eggs, and young Crabs evidently just hatched, and others at a rather later stage of their growth.

The eggs are of a dark reddish colour, showing through the outer integument the rudimental limbs of a future animal of a paler colour. On removing the thin transparent pellicle which surrounded one of these eggs, the eyes of the future animal were most conspicuous, the tail was seen extended as a narrow plate, nearly reaching to the eyes, and along its sides lay the large anterior cheliferous and the four following simple pairs of limbs. The existing organs, although perfectly discernible, occupied only a small portion of one side of the egg, its greater part being filled with hardened matter composed of minute molecular grains. The animal was in a sufficiently forward state of development not to allow the least doubt to be entertained as to the nature of these limbs, nor did any organs appear answering to the two large split pairs of natatory organs of *Zoea*. The branchiæ, in a fleshy and unorganized state, were also found at the base of the legs. The eggs are 1½ line in diameter.

In the accompanying sketches I have represented one of the Crabs evidently just hatched, being about 1¾ line long, and having the upper part of the cephalothorax considerably swollen. From my figures, the very rudimental state of the two pairs of antennæ, and of the feelers or flagrums of the outer foot-jaws, will be perceived; but the general form of the animal is thus early exhibited, and the developed state of its branchiæ and the want of subabdominal appendages are especially noticeable. In the following sketch the animal is seen in a somewhat more advanced stage of its growth, being rather more than 2 lines long, and in which the upper surface of the cephalothorax has acquired its ordinary shape, and the antennæ have attained a greater degree of perfection.

These circumstances are, I trust, amply sufficient to prove that the land Crab does not undergo any metamorphosis.

It is to be observed that Mr. GUILDING has not stated the precise species of land Crab of which the above-mentioned individuals were the offspring; but his well-known acquirements in crustaceology put the question of its being at all events a species of land Crab to rest. Should this, however, be nevertheless called in question, the argument which I would deduce from it will be but little diminished even were it a sea Crab.

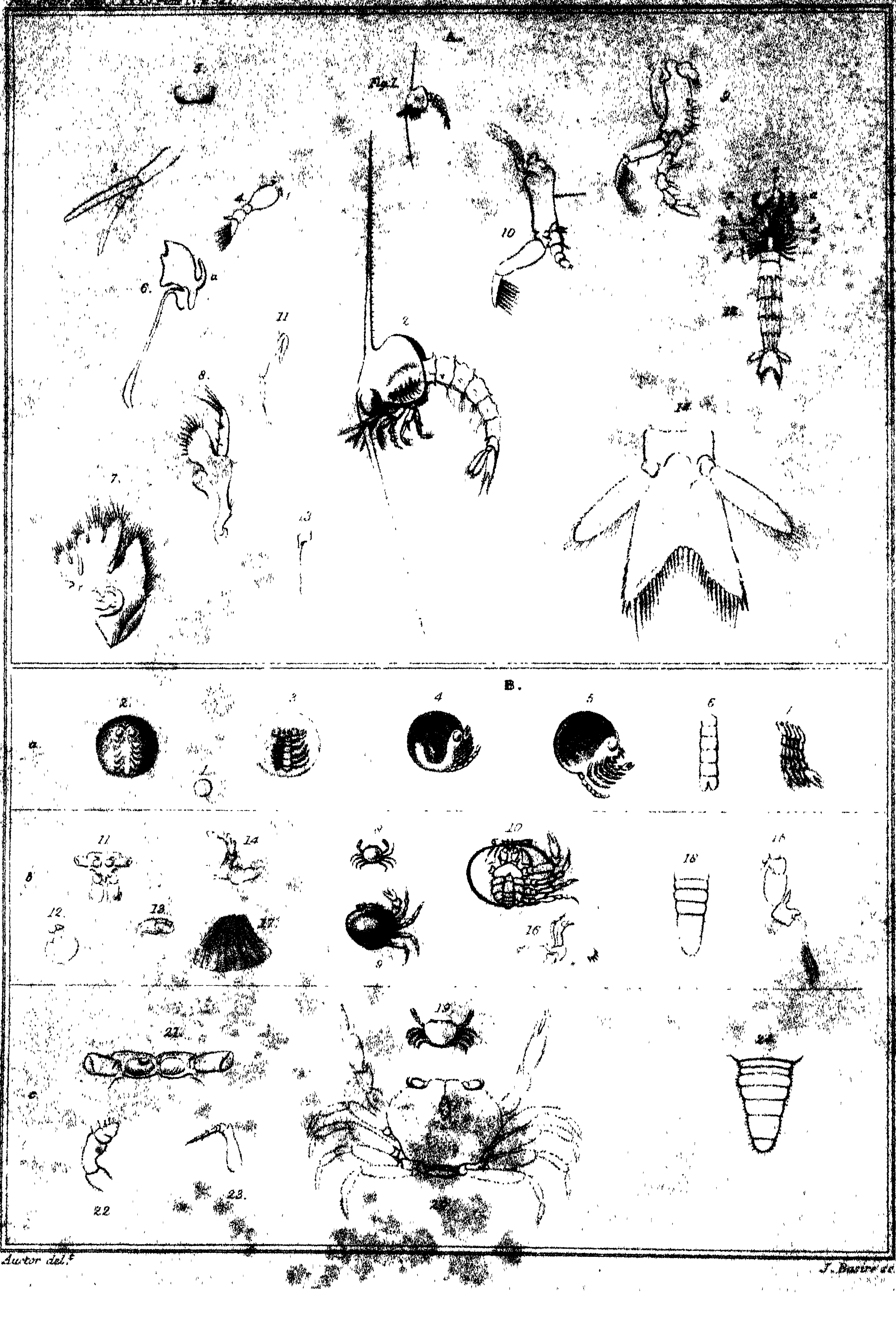
We have seen that Mr. THOMPSON's supposed full-grown Zoe, which died on the point of undergoing its supposed metamorphosis, was 3 lines long between the points of the spines, and the length of which, from the head to the tail, must have been at least $1\frac{1}{2}$ line. But the young of the common Crab is found of a much smaller size than this, exhibiting at the same time all the form of the full-grown Crab. I have myself captured the young of *Cancer Mœnas* not more than $\frac{1}{2}$ a line long, yet perfectly formed, and capable of running about with much quickness.

Although disagreeing with Mr. THOMPSON in respect to his theory, I have already stated that his figures are very faithful delineations of nature. I have therefore the more pleasure in stating that his representations of the young of *Mysis* are (as I have ascertained by extracting them from the subthoracic pouch of the female) correct.

Hence, by taking the preceding observations into consideration, we find that one or more types of each of the great groups of the typical Malacostracous *Crustacea* have been ascertained to undergo no change of form sufficiently marked to warrant the employment of the term metamorphosis. Thus,

The *Brachyura* are represented by the Land Crab.		
The *Macroura*	———	Cray-fish.
The *Schizopoda*	———	*Mysis*.
The *Amphipoda*	———	*Gammarus* and *Phronyma*.
The *Læmodipoda*	———	*Caprella* and *Cyamus*.
The *Isopoda*	———	*Asellus*, *Cymothoa*, and *Limnoria*.

Note.—Since the preceding pages were written, Mr. THOMPSON has published a memoir upon the genus *Pinnotheres*, belonging to the *Brachyura*, in which the ova are stated to have been seen to hatch in great numbers under the form of a new kind of Zoe, without the circumstances attending their development being recorded. And, on the other hand, some of the late Mr. GUILDING's MSS. have been published in the Magazine of Natural History, in which it is distinctly stated that the land Crabs do not undergo transformations.



Autor del. J. Bastre sc.

Explanation of the PLATE.

PLATE IV. A.

Fig. 1. *Zoea Gigas*, WESTW. natural size.
Fig. 2. Ditto, magnified.
Fig. 3. The outer antenna.
Fig. 4. The inner antenna.
Fig. 5. The labrum.
Fig. 6. One of the mandibles, a representative of the the palpus.
Fig. 7. One of the interior maxillæ.
Fig. 8. One of the second pair of maxillæ.
Fig. 9. One of the first pair of foot-jaws, developed into a natatory organ.
Fig. 10. One of the second pair of foot-jaws, developed into a natatory organ.
Fig. 11. One of the third pair of foot-jaws, minute and rudimental.
Fig. 12. View of the underside of the body, as extracted from the cephalothoracic shield.
a. The first pair of foot-jaws.
b. The second pair.
c. The third pair.
d. The anterior pair of chelate members.
e. The rudimental branchiæ.
f. The four posterior pairs of simple members.
Fig. 13. One of the subabdominal appendages.
Fig. 14. The tail, developed.

PLATE IV. B.

" Eggs and young of a land Crab not undergoing a metamorphosis."—GUILDING, MSS.

a. The egg.
Fig. 1. Natural size.
Fig. 2. Magnified, seen in front.
Fig. 3. The same, seen in front, having the outer pellicle stripped off: the legs on one side extended laterally, the branchiæ visible on the other side.
Fig. 4. The same, seen in front, having the outer pellicle stripped off, seen sideways.
Fig. 5. The same, seen in front, having the outer pellicle stripped off, seen sideways, with the limbs and tail extended.
Fig. 6. The tail.
Fig. 7. The legs, with the branchiæ at the base, not organized.

b. The young in its earliest state.

Fig. 8. Natural size.

Fig. 9. Magnified.

Fig. 10. Ditto, seen from beneath.

Fig. 11. Anterior portion of the body, from beneath, showing the outer foot-jaws, two pairs of antennæ, and eyes at the extremity of the peduncles.

Fig. 12 & 13. One of the rudimental internal antennæ attached to a large fleshy tubercle.

Fig. 14. One of the rudimental external antennæ.

Fig. 15. One of the outer foot-jaws.

Fig. 16. One of the intermediate foot-jaws.

Fig. 17. The branchiæ.

Fig. 18. The abdomen, unfurnished with internal appendages.

c. The young at a rather more advanced period.

Fig. 19. Natural size.

Fig. 20. Magnified.

Fig. 21. The anterior part of the body seen from beneath.

Fig. 22. One of the internal antennæ separated from its large basal lobe.

Fig. 23. One of the external antennæ.

Fig. 24. The abdomen.

XVIII. *On the Ice formed, under peculiar circumstances, at the bottom of running Water.* By the Rev. JAMES FARQUHARSON, *of Alford, F.R.S.*

Received March 17,—Read April 2, 1835.

ICE formed at the bottom of rivers and streams, frequently in great quantities, is a phenomenon quite common in this climate. I made for several years past a number of incidental and desultory observations upon it, and became convinced that the principal explanation of its occurrence is the radiation of heat from the solid opake materials of the bottom; but as I conceived this to be also the generally admitted one, I took no note of the observations, with the view of vindicating the theory of the radiation. It appears, however, from a paper of M. ARAGO upon the subject, translated and published in the Edinburgh New Philosophical Journal, vol. xv. p. 123, from the Annuaire for the year 1833, that he entirely rejects the theory of the radiation of heat through a thick layer of water. In the same paper, although he does not, in conclusion, pretend to give a complete explanation of the phenomenon, he brings forward, as explanations in part, three circumstances, which, although accurately stated by him, appear to be not exclusively appropriate to ice formed at the bottom, and cannot therefore aid us in solving the main question which we have to discuss here, which I apprehend to be, *Why is ice formed sometimes on the surface of running water, and sometimes at the bottom?*

On reading M. ARAGO's paper, I became desirous of offering some remarks in answer to it, as without some one doing this, on proper data, a misapprehension concerning the cause of a natural phenomenon, so much at variance with our most frequent experience of the formation of ice only on the surface of all waters, as to have often greatly excited the attention and even called forth the astonishment of scientific men, would continue to be propagated under the authority of a distinguished name. Having, however, no record of my former observations to enable me to refer accurately to the time, place, and other circumstances of them, I delayed till a renewed occurrence of ice on the bottoms of our streams should enable me to repeat them.

Such an occurrence, on a great scale, took place in the beginning of this month of January (1835); and I now have the honour of presenting to the notice of the Royal Society a brief account of the observations I have been enabled to make, and of the conclusions to which they appear to direct us.

Previously to entering on this detail and discussion, it seems proper to describe the appearance and quality of the ice formed at the bottoms of streams. A misapprehension regarding these may have been one cause of the incredulity of its existence,

entertained by some persons who have never witnessed it, and which M. ARAGO, in the paper referred to, has deemed it necessary to remove, by bringing forward the testimony of many distinguished men to its reality. The ice formed at the bottom does not resemble the solid glass-like plates which are formed on the surface. It has nearly the aspect of the aggregated masses of snow as they are seen floating in rivers during a heavy snow shower; but, on taking it out of the water, it is found to be of a much firmer consistence than these, although never approaching to the firmness and solidity of surface ice. It is a cavernous mass of various-sized, but all small, pieces or crystals of ice, adhering together in an apparently irregular manner by their sides, or angles, or points, promiscuously. Both the firmness of the adhesion and the dimensions of the interstices (the latter filled with water, and their volume easily estimated by the quantity of it which is discharged when the ice is lifted out of the stream,) are, however, greatly modified by the intensity and continuance of the previous cold. When the ice begins first to form on the bottoms of the streams, it presents a rudely symmetrical appearance, which, for illustration, may be compared to little hearts of cauliflowers, fixed on the bottom, having a similar uniform circular outline and protuberance in the centre, with coral-like projections. These pieces have a shining silvery aspect; they are dispersed, at first irregularly, in small numbers, but increase both in size and numbers, till the whole bottom is covered, and, if the frost continues severe, grow in height, but in a very irregular manner, so as to obliterate the earlier somewhat symmetrical shapes, till the streams are raised high above their former levels, and frequently made to overflow their banks. And here I take the opportunity to notice the incorrectness of an observation of DESMAREST, quoted by M. ARAGO, and which, M. ARAGO observes, no one has corroborated, "that it was from the lower parts, which touched the bottom, that the flakes of ice successively increased." On the contrary, the forms of the surface of the earlier masses are continually obscured, in succession, by new ice added to the top.

This congealed mass being thus very different in appearance and consistence from the sheets or plates generally known by the name of ice, it were no doubt well that, like the Germans, who, M. ARAGO informs us, name it *grundeis*, we too designated it by another name, to prevent confusion or misapprehension when we refer to it. The inhabitants of this part of the country will furnish us with a better one than even that of the Germans. In a district where it occurs almost every winter, and often repeatedly during that season, and where many of the rivers are crossed by means of fords, its existence influences too much their economical arrangements not to excite their particular attention, especially as many horses refuse to enter any stream even slightly impeded by it, being greatly alarmed by the pieces which break and float up from the bottom by the action of their feet. A body with which all are so well acquainted is known by an appropriate name. They call it *ground-gru*; *gru* being the term by which they designate snow saturated with, or swimming in water. I shall venture to use their term for the ice formed at the bottom.

It will be better here also to state, generally, the conditions of temperature and phases of the weather under which the ground-gru is formed. I have seen it occur only when the temperature of the whole mass of water was reduced to, or nearly to, 32° FAHR., and when the temperature of the air was several degrees below that point. I have observed it an invariable condition, that it was preceded by a continuance, for some time, of a clear, or very nearly clear, state of the sky. This is at variance with another observation of DESMAREST, quoted by M. ARAGO, that "when, in consequence of a cloudy sky, the atmospherical temperature experiences little variation throughout the day and night, the ice at the bottom of the water uniformly increases every twenty-four hours; on the contrary, when the sun shows itself, the ice does not increase during the day." It is the fact, that while it is forming under the continuance of a cloudless sky, its increase is impeded during the day. It may be possible, amidst the infinite variety of measures of cold that may exist at the time, that the increase of the gru may go on for a little time after the sun has been obscured by a thin cloud; but I have always seen, that when a densely clouded state of the sky supervened, and continued for the space of even only twenty-four hours, the gru became detached from the bottom, and floated down the stream. Should the temperature of the air continue low, with the clouded sky, or get lower, the ground-gru is not renewed, but the river is speedily frozen over at the surface. It is, in fact, a matter of frequent occurrence, in frosty winters, that our rivers, filled, and so impeded, by ground-gru, as to be raised above their banks, are found returned into their natural channels, and there frozen over at the surface, but flowing over a clear bottom, in a space of time so short as to appear very wonderful to those who have not investigated the cause. The process is named, by the country people, the *flitting* of the ice. In opposition to the observation of DESMAREST, and in confirmation of those which I have made, on this point, I may refer to the Rev. Mr. EISDALE, who, not satisfied with the explanations of M. ARAGO, has published one of his own, in the Edinburgh New Philosophical Journal, vol. xvii. p. 167. His explanation appears equally unsatisfactory, as will be shown afterwards; but the part of his statement we have to do with here is his notice of this observation of DESMAREST. The formation of the ground-gru, under a cloudy sky, is so much at variance with the information which Mr. EISDALE had received, that he resolves DESMAREST's "cloudy sky" into "an atmosphere loaded with hoar frost, and rendered hazy by its condensation *." The state of the air, in respect of being windy or calm, deserves also to be noticed. The ground-gru occurs most frequently during calm, with a deposition of hoar frost upon the ground at the time; and this was the condition of matters during the observations now to be detailed. But it also occurs during a frosty wind, when there is no hoar frost, which is formed only in a calm state of the atmosphere. The formation of the gru during wind, and consequently without any deposition of hoar frost on the ground, is especially to be noticed in reference to Mr. EISDALE's explanation,

* p. 172.

as will be afterwards seen. It occurred to M. HUGI, as quoted by M. ARAGO, in the Aar, on the 16th February 1827, with a west wind, after the river had been completely open on the 15th; and one of Mr. EISDALE's correspondents ascribed its occurrence in one particular instance, which he related to him, to the prevalence of a very sharp north-east wind, which had blown during the night of its formation.

The following observations were made in the rivers Don and Leochal. The former, having an easterly course, is about 120 feet broad, and a foot or two deep at the shallows and fords. The latter, one of the small tributaries of the former, having a northerly course, is about 20 feet broad, and a foot deep at the shallows. Both rivers possess a like character of very clear water, and alternating rapids and pools. The rapids in the Don are reaches, where the water falls two or three, or more, feet, from a higher to a lower level, within a distance of fifty or a hundred, or sometimes two or three hundred, yards. They are generally impeded with many large stones, some of them projecting above the water. The depth varies greatly, but seldom exceeds two or three feet. The pools between the rapids are on an average much longer reaches, in which there is little fall, and a greatly diminished velocity of the stream, which often, in them, flows so equably as to give rise to no ripple on the surface. They too have in them large stones, but fewer in number. The depth in them too varies greatly, from two or three to four or five feet. The rapids and pools in the Leochal are of a similar kind, but both much less deep in this smaller stream. The bed of this river has however, on the whole, a steeper descent, and owing to this there is more broken water and spray in the rapids. The character of alternating rapids and pools, in both streams, is owing to the varying hardness of the granitic and micaceous-schistose rocks in which their beds are formed. Where the rocks are hard, there is a rapid; where more friable, a pool. In the parts of the rivers observed, the original rocks themselves do not anywhere form the immediate bed of the stream. That, to the depth of two or three, or more, feet, is composed of the debris of these rocks, broken up and sometimes much waterworn, and reduced to the size of a very large gravel, by the action of the stream, but not so small as to deserve to be named sand. No part of the bottom is muddy.

On the night between the 31st of December 1834 and the 1st of January 1835, after the mean temperature of the air had continued for three days at 47° FAHR., and when there had been little frost in the season before, there commenced a hard frost, with a calm and perfectly cloudless sky, which continued with little abatement till the 5th of January, at 10 A.M. On the night between the 3rd and 4th, the temperature of the air was 23° FAHR.; and on the 4th, the bottoms of the rapids in the Leochal were seen coated in some places with silvery cauliflower-shaped clusters of ground-gru. I neglected at this time to examine the temperature of the water.

Between the 4th and 5th, the temperature of the air was down to 19° FAHR.; and on the 5th I examined the Don and the Leochal along half a mile of each, beginning the examination at half-past 8 o'clock A.M. The examination began at the bridge of

Alford, built of granite over the Don, in the middle of one of the rapids. At this rapid, the whole bottom, with the exceptions to be immediately stated, was covered with silvery gru, appearing from two or three to five or six inches deep. My attention was particularly directed to the exceptions, as throwing a clear light on the question of the radiation of heat from the bottom. Round each of the piers, and in front of the abutments of the bridge, there was a space quite clear of all frozen matter, excepting at a side of one pier under an arch, where a piece of very still water, caused by an obstruction at the bottom, was covered by clear sheet ice. On the south side of the river, two embanking walls, one up and the other down the stream, each twelve yards long, are built in a line with the water-courses of the abutment. Close to the bridge these walls are eight feet high from the bottom of the stream, but as they recede from the bridge the masonry slopes gradually to a lower level, till the extremities are little above the level of the water. The bottom in front of these walls was clear of ground-gru, as well as that in front of the abutments; but the breadth of the clear space in front of the walls narrowed gradually towards their extremities, in proportion as the masonry became lower, till at the extremity of the downward wall especially, which ends at a sloping gravelly bank, the gru came to the edge of the water. The space of the bottom clear of gru was about five or six feet broad at the high parts of the walls next the bridge; and the water runs on the place at the medium depth and velocity of the rapid. There was another clear space in the bottom of this rapid. About twenty-five yards above the bridge there is, in the middle of the stream, a piece of still water, caused by an elevated bed of gravel, just below it, over which the stream is very shallow. The still water, for an extent of two or three square poles, was covered with sheet ice, and that again covered by a very thin, but white, opake deposition of hoar frost. From under this ice the water, flowing rapidly over the gravel bed below, had no ground-gru for a space of eight or ten yards downwards.

Above this rapid, a pool of moderate stillness, about three or four feet deep, extends a hundred and fifty yards in length. Over the bottom of this there were scattered, in an irregular manner, many cauliflower-shaped clusters of silvery gru, most of them very small, and none that were observed covering more of the bottom than a square foot or two at one place. In the deepest and stillest part of the pool there were several tufts of water starwort, with sooty-coloured decaying leaves, forming the darkest-coloured objects seen at the bottom. These were all densely tangled with fringes of silvery gru. At the head of the pool, where the velocity acquired by the water in the rapid immediately above it was not yet greatly diminished, an appearance of a different kind presented itself. There are here several large stones in the bed of the stream, but none of them projecting above the water. On the faces of these opposed to the stream there were seen quantities of gru of a different aspect from that further down. It was not arranged in the same cauliflower shapes, but in angular masses, like wreaths of snow blown by the wind. It wanted, too, the silvery glance of the other, and had more the appearance of a pale ash-coloured mud. On

reaching it with the end of a pole, its consistency was found to be less firm; in fact, it was only a heap of detached uncemented spiculæ pressed against the stones, and retained there mechanically by the action of the water, in a certain modified state of its velocity. The source of these heaps of uncemented spiculæ will soon be noticed. This pool, as indeed was the case with all the pools in the river, had at its edges and in its little bays narrow pieces of surface-ice, extending a foot or two from the banks.

The rapid immediately above this, not unlike that at the bridge, was covered at the bottom with silvery gru, with one exception. The river was low at the time from long-continued deficiency of rain, and the water had deserted the south side of the channel, leaving many little pools among the stones, communicating more or less freely by irregular little currents with the main stream. The pools were covered over with sheet-ice, and that with a thin opake deposit of hoar frost like snow. In the little currents returning from under this ice there was no frozen matter.

At the head of this rapid there is a pool much deeper and stiller than that above the bridge-rapid already described. The depth is five feet, and the stillness such that, at many points of it, there is no ripple or wave on the surface. None of the silvery cauliflower-like ice was seen on the bottom here; but near the head of it, in a modified state of the current pouring in from the rapid above it, there were, on the faces of several large stones opposed to the stream, collections of uncemented icy spiculæ.

The source of these collections was very readily observed in a great rapid immediately above this. In that rapid the water has a much quicker descent than in the others referred to. It is about a hundred yards long, and cumbered with many large stones, over which, at many points, through its whole length, the water breaks with a great deal of spray. Here an immense quantity of gru occupied the bottom, impeding much the course of the stream. At the time of observation many pieces of this gru were seen edging up, and in some instances breaking quite away from the bottom, apparently by the increasing pressure of the water, as it became dammed back by the increase of the gru itself. This at least was the appearance, although there may have been another cause for the disengagement of it from the bottom, and that is, the impeding, by the imperfectly translucent gru, of that radiation of heat from the bottom which, I trust in conclusion to demonstrate, is the immediate chief agent in the whole phenomenon.

It is now to be observed, that a number of pieces of loose gru, the origin of which was so clearly ascertained at this last rapid, were floating down in all parts of the river. In passing through the rapids, they were broken into fragments, and, where the fall was violent, shivered into minute pieces. The larger pieces that remained after passing through the rapids floated at the surface, immediately as they got into the uniformly flowing currents at the heads of the pools; but the minuter ones, mixed with the water to all depths by the plunging whirls in the rapids, not being so speedily disentangled from their cohesion with the water, by the action of gravity, floated

for a greater distance immersed in the water, and were intercepted by, and mechanically retained against, the faces of the stones by the action of the stream at the heads of the pools. Further down, and in stiller water, where no such intercepted heaps were seen, their buoyancy had, no doubt, by degrees, overcome the cohesion and raised them to the surface; and in fact, in the still water, many minute icy fragments were floating in the surface.

Mr. KNIGHT, the celebrated botanist, quoted by M. ARAGO, has obviously, in part, but not completely, distinguished between the "frozen matter which reflected a silvery kind of whiteness," which covered the stones in the rocky bed of the river, and "floating spiculæ under water," which he found to "accumulate much more abundantly upon such parts of the stones as stood opposed to the current, where that was not very rapid, below the little falls or very rapid parts of the river."

In the smaller stream of the Leochal, the quantity of ground-gru was comparatively much more abundant, occupying the bottoms both of the pools and rapids in close masses, and in the latter, at many parts, forming such an impediment as to urge the water over its usual banks. But there were two remarkable exceptions. One of the pools flows close to the foot of a steep bank about fifteen feet high, and in the side next the bank there was little ground-gru. In a rapid, which at a turn of the river has an easterly course, there was a very dense fringe of *Phalaris arundinacea* standing, with its dense foliage of withered leaves, in the south edge of the water. Its height was four feet, and it extended fourteen feet in length along the stream. At the foot of it the bottom of the rapid was clear of ground-gru to the breadth of three feet.

The temperature of the air and water, at the time of these observations, was particularly ascertained. That of the air at sunrise, about an hour before the observations commenced, had been 23° FAHR.; but it was rising rapidly during their progress, and was at 36° FAHR. before their conclusion. The temperature of the water in the Don varied from 32° to 33° FAHR.; but the variation could not be distinctly traced as depending on the depth or velocity, as there was a temporary variation in the same place, both in the pools and rapids. At one of the small streams, returning from under the sheet-ice on the little pools at the edge of one of the rapids, the temperature was nearly steady at 33° FAHR. In the Leochal the temperature was nearly steady everywhere at 32° FAHR.

By 10 o'clock A.M. on the same day, a cloud obscured the whole sky, and at 2 o'clock P.M. the temperature of the air was 40° FAHR. At this time much gru rose from the bottom and floated down the streams of both rivers. The relaxation of the frost, however, was of very brief continuance. Before sunset the temperature of the air was again down to 31° FAHR., with a perfectly calm air and clear sky; and the clear sky continued till the evening of the 7th of January, the thermometer during the two intermediate nights being at 23°, and during the intermediate day at 26°.

The same parts of the Don and Leochal were again examined at 10 o'clock A.M. on

the 7th. In the Don the ground-gru now covered all the bottoms of the pools as well as of the rapids. It was of less depth in the deep still pool below the great rapid; but everywhere else it formed a great impediment to the stream, raising it so much above its former level that it covered deeply the pieces of sheet-ice formed at the edge on the 5th. New pieces of similar ice were now forming at the same places on the more elevated surface. The Leochal was still more impeded by the gru than the Don.

But, what is worthy of particular notice, the clear spaces of the bottom, at the piers, abutments, and embanking-walls of the bridge on the Don, and at the Phalaris grass in the Leochal, still continued so, but were now considerably narrowed in their lateral dimensions, the ground-gru having encroached upon them on the sides next the streams. The temperature of the air was 24° Fahr.; of the water, everywhere nearly steady at 32°.

Several circumstances occurred on some subsequent days which deserve to be noticed, as throwing light, by the contrast which they exhibit, on the phenomenon now under consideration. On the 8th of January there occurred a thaw, when the thermometer suddenly rose to 47° Fahr. The rivers were speedily cleared of ice and ground-gru, which last rose from the bottom and floated away with the stream. The atmosphere at the time was considerably clouded, with a brisk S.W. wind. On the 9th of January the temperature of the air fell to 36° Fahr.; and on the morning of 10th of January, with a temperature of the air at 29° Fahr., there was a fall of snow, of about an inch deep, which ceased by 8 o'clock A.M. The snow that fell into the rivers was observed to be entangled, and stuck fast, in irregular crushed masses, in many parts of the rapids; and there were collections formed of loose spiculæ of a muddy aspect, at the sides of the stones opposed to the streams, in the heads of the pools, where the velocity of the currents was intermediate between that of the rapids and that of the stiller parts of the pools; but there was no appearance on any part of the bottom resembling the symmetrical cauliflower-shaped ground-gru. On the evening of the 10th the temperature of the air fell to 23°, and continued at from 23° to 21° till the morning of the 12th, with a densely clouded state of the sky. During this time extensive sheets of surface-ice were formed on the pools of the Don, and many of the pools of the Leochal were quite frozen over, but the ground-gru was nowhere renewed; on the contrary, the masses of snow entangled in the rapids on the 10th disappeared to a great extent, obviously floating away in the stream. In this state of the river and weather, the collections of uncemented spiculæ, on the faces of the stones opposed to the streams in the heads of the pools, appeared in their places the same as before, neither increasing nor diminishing in size.

M. Arago, in his paper, refers to three circumstances, as partly, at least, explanatory of the formation of ground-gru in running water.

1st. The inversion, by the motion of the current, of the hydrostatic order, by which the water at the surface, cooled by the colder air, and which at all points of the tem-

perature of water under 39° Fahr. would, in still water, continue to float on the surface, is mixed with the warmer water below, and thus the whole body of water to the bottom is cooled alike by a mechanical action of the stream:

2nd. The aptitude to the formation of crystals of ice on the stones and asperities of the bottom, in the water wholly cooled to 32°, similar to the readiness with which crystals form on pointed and rough bodies in a saturated saline solution:

3rd. The existence of a less impediment to the formation of crystals in the slower motion of the water at the bottom, than in the more rapid one near, or at the surface.

There is no denying the justness of these three positions, and yet the slightest reflection teaches us that neither singly nor combined do they aid us in answering the main question before us, "Why is ice formed sometimes at the surface of running water, and sometimes at the bottom?" All the circumstances, or conditions, referred to by M. Arago, are present when ice, as most frequently takes place, is in the course of being formed only on the surface, as well as when the formation is going on at the bottom. Were we to admit them as an answer to our question, then running water ought always to freeze first at the bottom. But a most extensive experience teaches us that this is not the case. The illustrations of M. Arago, indeed, just and true in themselves, are not to be overlooked when we would investigate and explain the formation of ice either at the bottom or at the surface. They will serve to enlighten us greatly in both these events, but they have no exclusive relevancy to either, and we must therefore look out for another solution of the problem.

M. Arago, in his conclusion, does not present these three circumstances as a complete explanation; but he says, the reader may ask why he has not done so, and he answers to this, "that we have no observations which prove that this kind of ice is seen, until the temperature of the whole of the water is at zero" (centigr.); and that it is not certain that the little icy particles, seen by Mr. Knight, floating on a milldam, at the time ground-ice was forming in the stream, and which may have acquired in contact with the air a temperature below zero (centigr.), do not play an important part in the phenomenon which he has overlooked.

In regard to the former of these points, I cannot say what M. Arago would have deduced from it, had it been established in one way or the other. The observations made on the Don on the 5th of January show that the temperature of the whole water was not quite down to 32° Fahr. when the ground-gru was forming in large quantity. In regard to the latter, the little icy particles seen by Mr. Knight, the same condition belongs to them that belongs to the circumstances professedly adduced by M. Arago as explanations; that is, they occur as well when the ice is forming on the surface only as when it is forming on the bottom. They account well, however, for the collections of frozen matter seen by him at the sides of the stones opposed to the stream, in parts where its velocity had a certain modification.

And here I may advert to the explanation offered by the Rev. Mr. Eisdale, in his paper already referred to. From the information he received, he was led to believe

the ground-gru does not occur but when there is a hoar frost on the ground; and he explains the ground-gru to be particles, or crystals as he afterwards names them, of hoar frost precipitated into the water, retaining there the shapes in which they descended, brought into contact with the rocks by the agitation of the water, and forming nuclei for the accumulation of ground-gru. Could it be proved that such crystals are precipitated into the water, they would serve no more for explanation than the icy particles of Mr. Knight. We have learnt, indeed, from travellers in high northern regions, that, in certain states of cold and moisture of the air, such crystals, as Mr. Eisdale assumes, are there seen and felt floating in it; but nothing of that kind was observed in January last; and when Mr. Eisdale, from the existence of spiculæ of hoar frost on the ground, would infer the like may be formed in the air to fall into the water, he neglects to take into the account, that the spiculæ of hoar frost have not fallen from above, but that their symmetrical arrangement, round on all sides of the bodies on which they are found, and their slow increase, prove they have been deposited on their places by a gradual deposition of invisible watery vapour, owing to the substances to which they are attached being cooled below the temperature of the surrounding air, by the radiation made known to us by the experiments of Dr. Wells. Besides this we have to remark, that the ground-gru sometimes takes place, agreeably to the information of one of Mr. Eisdale's own correspondents, in a windy state of the atmosphere, at which time no hoar frost is seen.

The interesting experiments of Dr. Wells just referred to enable us to give, after all, a very satisfactory explanation of the ground-gru; and Mr. M'Keever, quoted by M. Arago, had gone far to illustrate it by means of them, although he had overlooked some conditions necessary to be taken into the account for a complete explanation. M. Arago, however, entirely rejects the explanation of Mr. M'Keever, and it is fair to set down the terms in which he does so.

After having shown that the ground-gru cannot be explained by the action of the moon*, according to the sailors, nor by the friction of running water producing more heat at the surface than at the bottom, nor by referring its source to the smaller tributaries of the streams, nor to different layers of ice formed at the several surfaces, when the water in the river, from whatever cause, is in a state of varying fullness, all of which have been assigned as causes of the ground-gru, M. Arago proceeds:

"We come now to Mr. M'Keever, who, confining himself closely to the most subtle principles of the theory of heat, has not on this account been more fortunate than his predecessors. According to this author, 'the rocks, stones and gravel, which generally cover the bottom of rivers, have powers of radiation superior to those of mud, perhaps on account of their peculiar nature, but chiefly because they

* This explanation of the sailors is a confirmation of what I have stated, that the gru never appears but under a clear sky. The constant observation of the sailors has associated, in their minds, the shining of the moon with the ground-gru; but the moon never shines, to excite great attention, but in a clear sky.

have rough surfaces. Thus rocks in large or small masses will become much cooler in consequence of radiation: when the atmospherical temperature is very low, they of course freeze the water which touches them.' It is unnecessary to examine here whether heat radiates through a thick layer of water, as Mr. M'KEEVER supposes, as the most simple observation is sufficient to overthrow it. Where is the person who has not observed that the strong radiation, which the Irish philosopher admits, would be more plainly manifested, or as completely, in still water than in running water? but no one has seen a piece of still water frozen at the bottom*."

But there is nothing more easy of experimental proof than that heat radiates through water. I do not mean, however, to vindicate the reasoning of Mr. M'KEEVER respecting the more powerful radiation of it from stones than from mud. His reasoning respecting that matter is, on his own part, conjectural, to explain the readier formation of gru on a stony or gravelly bottom; but the gru also forms on a muddy bottom, a fact which M. ARAGO notices, when he brings the attachment of mud to the under side of the floating flakes as a proof that they have been formed at the bottom. Mr. M'KEEVER was driven to his conjecture from having overlooked the more complete and sudden inversion of the hydrostatic order that takes place over stones than over mud; which last is deposited only in places where the water has a stiller and more equable motion. In such places the ground-gru is later in forming, and therefore is more rarely seen; and it is doubtful whether Mr. M'KEEVER had a proper opportunity for noticing it in them.

But to return to the main point which we have here to maintain in opposition to the reasoning of M. ARAGO, the radiation of heat through a body of water. When we construct an achromatic object-glass for a telescope, it does not the less remain a burning-lens when we have included in it a transparent fluid, and no experiment has proved that were the fluid water the case would be altered. We are aware of the danger that has been incurred of setting fire to an apartment by an ornamental glass globe filled with water, and placed in the sun at a window. But as I cannot particularly refer to circumstances of time and place of the cases now mentioned, I made an experiment on the subject with such apparatus as I could find readily at hand, having no access to better in a remote country place†. In a room, of which the temperature was 50° FAHR., a semiglobular tumbler filled with water, containing about a pint and a half, was placed inside a window, in the rays of the low but clear winter sun. The bulb of a thermometer, which had been previously placed in a similar situation till it rose and remained steady at 61°, was shifted into the brightest part of the fan-shaped focus of rays, into which the light was refracted through the tumbler. In this position it was raised in four minutes to 72°. It was again shifted into the unconcentrated rays passing through the window, when it fell, but more

* Edinburgh New Philosophical Journal, vol. xv. pp. 132, 133.

† It may seem absurd to have had recourse to experiment in a case so plain; but the procedure seemed, at the same time, indispensable, to meet reasonings promulgated with the authority of such a distinguished name.

slowly than it had risen; and the experiment was repeatedly renewed with similar results, leaving no doubt that the heat, like the light, radiated through, and was refracted by the water. If the fact is so in regard to the radiation of heat through a mass of water four or five inches thick, where ought we to set the limits of thickness of the mass through which it cannot pass? Obviously only where the thickness is so great, that the aggregation of the fluid, and of its minute impurities, prevents the transmission of light, as in the deeps of the sea, but not within the ordinary depths of our clear streams.

Of the effect of radiation in cooling down the surface of the ground, and substances placed upon it, during a clear sky, we cannot give a more lucid account than that of M. ARAGO, in his paper "On the supposed Influence of the Moon on Vegetation." "No one had supposed," says he, "before Dr. WELLS, that terrestrial substances, excepting in the case of a very rapid evaporation, may acquire during the night a different temperature from that of the surrounding air. This important fact is now well ascertained. On placing little masses of cotton down, &c. in the open air, it is frequently observed that they acquire a temperature 6°, 7°, or even 8° centigr. below that of the surrounding atmosphere..... These differences of temperature between solid bodies and the atmosphere only rise to 6°, 7°, or 8° of the centesimal scale, when the sky is perfectly clear. If the sky is clouded they become insensible." This lucid statement, however, requires one modification; for the greater cooling of the solid substances, under a clear sky, takes place not only during the night, but also during the day, in places not directly exposed to the sun's rays.

This radiation, as it passes freely through the transparent atmosphere, may, as we learn from the above experiment, pass also through the transparent water, to cool down the solid substances at the bottom below the temperature of the surrounding fluid. That fluid is permeable to radiating heat as well as the atmosphere. The application of the thermometer, in the hands of Dr. WELLS, instructed us regarding the cooling of the surface of the ground; but the water of a river, placed under the very same condition of a clear sky, fluid above and freezing below, is a great natural thermometer, teaching us that a corresponding cooling is going on on the surface of the solid opake substances of the bottom. In fact, if we may so speak, the phenomenon of the ground-gru is the result of an experiment in the water, entirely similar to that of Dr. WELLS on the land, performed by nature on a large scale, and presented to us for our interpretation and instruction. And when we look back to the observations made in the month of January, we find the results of the modifications of this great natural experiment corresponding with those of similar modifications of the experiment on the dry land.

The cooling of the surface of the ground by radiation, discovered by Dr. WELLS, takes place only under a clear sky. It is therefore greatly modified on parts of the ground screened from a part of the sky by opake objects, as walls, trees, hedges. In illustration of the extent to which a screening or shading body, near at hand,

modifies the radiation, I shall detail some observations I made on the 7th of January last, incidentally in the first instance, but then extended, in reference to the observations on the ground-gru, which I was making at the time. Having occasion that day to dig into recently hoed ground, in the middle of a garden, remote from shade, the soil was observed to be frozen to the depth of four inches, by the clear frost, which had continued from the 1st of January, with the trifling intermission above mentioned. On digging into similar ground at the north base of a wall six feet high, the soil was found, close at the foot of the wall, frozen to the depth of only half an inch; at a foot distance from it, about an inch; at two feet, little more; and it was only at the distance of ten or twelve feet that it was frozen hard to the depth of three inches. A similar modification of the effect of radiation was observed in the shade of trees. Under the Scotch fir the soil, slightly covered with decaying herbage, was not at all frozen; although in similar ground similarly covered, but remote from shade, it was hard frozen to the depth of two or three inches.

Now the ground-gru in the rivers was modified in a way strictly similar by the effect of shade. The bridge of Alford, over the Don, is happily situated for illustrating this, being on one of the rapids, where the ground-gru is earliest and most abundantly formed. While the other rapids, and the unshaded parts of this one, were quite occupied by gru on both the 5th and 7th of January, spaces in the shade of the masonry at this bridge were quite clear of it. It cannot be admitted as an explanation of this fact, that heat may have been there laterally transmitted to the water by contact with the piers and walls; for if this took place, why then did the clear spaces on the bottom narrow gradually towards the low extremities of the embanking walls? Besides, the transmission of heat laterally had not hindered the formation of surface-ice, in contact with a pier, on a piece of still water under one of the arches. The modification of the radiation by shade was also exhibited in the absence of all gru on the bottom, along the foot of the dense tuft of Phalaris grass in the Leochal, where there could be no more transmission of heat laterally, than at the general line of the grassy banks of this stream.

The water, too, returning warmer from under the surface-ice, on the little pools at the edge of one of the rapids, is another instance of the modification of the radiation by shade. The thin white opake covering of hoar frost on the ice prevented radiation, at least in a great measure, and the heat of the bed of the river, in the course of continual transmission upwards, from strata not yet cooled to much depth by the frost, finding no outlet by the radiation, was expended in heating the water by contact.

There was another phenomenon observed on the 5th of January, (although no longer seen on the 7th, being then concealed by the immense formation of gru,) which can be readily explained by the admission of the radiation of heat through the water, and therefore goes to support the justness of the theory. The tufts of water starwort, in the deepest and stillest parts of one of the pools, were the darkest-

coloured objects seen at the bottom, and they were fringed in every part with spiculæ of gru, at a time while it yet occupied little of the bottom of this pool. The experiments of BOYLE, FRANKLIN, RUMFORD, LESLIE (although he denies the conclusion himself), DAVY, and STARK appear too uniform in their results to leave any doubt remaining, that dark-coloured bodies both absorb and radiate heat more freely than those which are light-coloured. It is in consistency, then, with an ascertained law of the radiation of heat, that the very dark-coloured tufts of the water starwort should have been the first bodies in the pool cooled to a very low temperature, and of course first covered with gru.

In arguing the whole question, let us not forget to assign a proper value to the illustrations of M. ARAGO. The first of them suggests a ready and satisfactory answer to one of the objections which he brings against the theory of radiation, which is, that the effect of it should be as readily manifested in still as in running water, and yet no one has seen a piece of still water frozen at the bottom*.

In still water, that hydrostatic order, which M. ARAGO has so well illustrated as belonging to water when reduced to a temperature under 39° FAHR., has free play to establish itself, and is not inverted by the mechanical action of a stream. When the temperature of a body of water is under 39°, then the coldest portions of it are the lightest, and naturally rise and float on the surface. When in a still pond the water nearest the bottom has been cooled below the general temperature by contact with the solid materials cooled by radiation, it is displaced by the heavier warmer water above. Hence ice forms first on the surface by the meeting there of both the cold of radiation and that acquired by contact with the incumbent cold atmosphere.

M. ARAGO's illustrations also furnish us with a satisfactory explanation of the curious facts, that the ground-gru makes its first appearance in the more rapid and agitated parts of the stream, and begins to show itself on the bottoms of the stiller parts, and to accumulate there in quantity, only after a longer continuance of the clear frosty weather. In the rapids the hydrostatic order is overturned, and the colder, which is also the lighter, water not only mixed with the warmer below, but, at the whirls of the greatest rapids, brought suddenly, without much mixing, into direct contact with the bottom, cooled still lower than itself by radiation. If the water is at the temperature of 32° FAHR. it can give out no heat to the colder bottom

* There is an exception to the universality of this position, which, although rare, I have sometimes witnessed; and as the phenomenon is in accordance with the theory of the radiation of heat from the bottom, it deserves notice. In little ponds of a foot or two deep, dug to obtain the materials for building or agricultural purposes, of which there are many examples in this neighbourhood, after they have been covered, owing to hard and long-continued frost, by a thick sheet of ice, that is sometimes nearly melted off, and the remaining fragments driven to the lee side by a strong westerly gale of high temperature. Such a gale, in this climate, frequently, towards its conclusion, shifts to N.W., when the temperature of the air falls again below the freezing-point of water, with a generally clear sky. In such peculiar circumstances the little ponds are suddenly filled with gru, commencing at, and shooting up from the bottom. The whole water is here at 32° FAHR. when the gru begins forming, and the hydrostatic order is deranged by the wind.

without part of it being converted into ice, the spiculæ and crystals of which find a solid body for their attachment at the very point where the heat is given out*.

But while in this manner we can explain some of the incidents, may it not be held, as above demonstrated, that the chief cause of the ground-gru is the radiation of heat from the bottoms of the rivers? Every branch of the phenomenon is of easy explanation when we admit the radiation; and among the rest a circumstance to which I have yet made no reference, and that is, the disappearance at the bottom of the water of the immense quantity of heat, 140° of Fahr., which constitutes the caloric of fluidity disengaged, when water at 32° Fahr. is converted into ice at the same temperature.

The answer to our original question then is, That ice is formed sometimes on the surface of running water, and sometimes at the bottom, because frost sometimes takes place with a clouded sky, which is incompatible with radiation of heat from the bottom of the stream, and sometimes with a clear sky, when that radiation takes place through the water, in the same manner as the experiments of Dr. Wells prove it goes on, under a like sky, through the atmosphere. The bottom is by this cooled down below the freezing point of water, before the water itself; ice is formed on it, and its detachment by transmitted heat from below prevented, as long as the radiation continues.

* We may observe also, that there is a local source of greater cold of the water in the rapids, in its being brought into more active and extensive contact with the air by the sharp ripple and spray.

XIX. *Observations on the Theory of Respiration.* By WILLIAM STEVENS, *M.D. D.C.L. Fellow of the Royal College of Physicians in Copenhagen, Fellow of the Royal College of Surgeons in London, &c. &c.* *Communicated by* W. T. BRANDE, *Esq. V.P.R.S.*

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THE cause of the dark colour of the blood in the venous circulation has long been a subject of discussion; but even at the present moment the question has not been satisfactorily decided.

It is universally admitted that the expired air contains carbonic acid, but it is still doubtful in what part of the system this acid is formed. LAVOISIER maintained that carbon was the cause of the dark colour of the venous blood, and that the acid was formed in the pulmonary organs, by the combination of that carbon with the oxygen of the air. At one period this theory was generally adopted, though the evidence in its favour is almost entirely hypothetical; for hitherto there has not been even one well-conducted experiment which proves the existence of any form of free carbon in the venous blood.

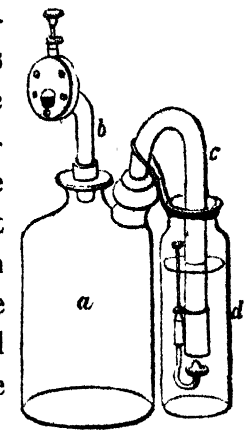
Another class of physiologists maintain that the carbonic acid is formed, not in the lungs, but in the general round of the great circulation; in proof of which some experimentalists have asserted that they had obtained carbonic acid from venous blood; but others, of equal respectability, who have repeated the same experiments, deny the existence of this acid in the venous current. The air-pump has hitherto been almost exclusively used for the purpose of deciding this question; but the positive proofs which have been brought forward by the one class of physiologists have been so completely contradicted by the negative proofs of the other, that a great majority remains still in favour of the old theory. In fact, TIEDEMANN and GMELIN, the latest writers on this subject, are decidedly of opinion that venous blood does not contain carbonic acid. As this is a very important question, the following experiments were made in reference to it.

1. A glass vessel containing a small quantity of warm and fluid venous blood was put under the receiver of an air-pump. In proportion as the air was exhausted, a number of globules appeared to escape from the blood, which were at first small, but in proportion as the air was removed they became larger in size.

2. A small quantity of venous blood, contained in a glass vessel, was covered with a layer of barytic water. This was put under the receiver of an air-pump. When the

pump was used, globules were observed to escape from the blood, which passed through the barytic water, but the transparency of the latter was not affected.

The annexed sketch is a representation of an apparatus which was invented by Mr. SQUIRE, of Duke Street, Grosvenor Square, and used in the following experiments. To a double-necked pint bottle, *a*, two glass tubes were fitted, one, *b*, ascending, the other, *c*, descending. The ascending tube is terminated by an air-tight box, having an aperture, over which a slide is moved by means of a strong wire. The descending tube is terminated by a brass orifice, which is closed at pleasure by a brass cap, having a cone in the centre surrounded with leather, and moved with a sliding wire. *d* represents a four-ounce phial with a loop of wire round the neck, by which it is connected with the descending tube. The phial was filled to the double line with distilled water.



3. The pint bottle and both tubes being filled with pure hydrogen gas, the orifice of the upper tube was placed on the skin, near the bend of the arm; a vein was then opened, and the orifice slid carefully along until it included the incision which had been made by the lancet. The valve was opened, and as the blood passed into the bottle, hydrogen was expelled through the descending tube, the orifice of which was immersed in distilled water. As soon as five or six ounces of blood had entered the bottle, both the orifices were carefully closed. The orifice of the descending tube was then immersed in barytic water, and the valve being opened, the whole was placed under the receiver of an air-pump. In proportion as the air was removed, the hydrogen, as well as any gas that might escape from the blood, passed through the barytic water, without, however, producing the slightest change in its transparency.

From the first of these experiments it might be inferred that a gas is capable of being removed from venous blood by the air-pump: but this supposition may possibly be erroneous; for similar globules appear to arise when we use water, even after it has been boiled and then cooled in a close vessel to 98°. The second experiment shows that this appearance is not due to the escape of carbonic acid; and from the third experiment it is very obvious that carbonic acid cannot be so obtained from venous blood which has not been exposed to air.

4. About four ounces of serum were put into a HOPE's eudiometer, the upper division of which contained four tenths of a cubic inch of carbonic acid: they were agitated together, and after a few minutes the serum had absorbed the whole of the acid. This impregnated serum, without being exposed to the air, was transferred into the double-necked pint bottle, which had previously been filled with hydrogen, and which was immediately put under the receiver of the air-pump. When the pump was used, the hydrogen, as well as the gas which appeared to escape from the serum, passed through the barytic water; but its transparency was not affected.

From this experiment it is obvious that serum (consequently blood or other albuminous fluid,) may absorb carbonic acid, and so retain it as not to be separable by the mere removal of the pressure of the air.

The air-pump has hitherto been used almost exclusively by the experimenters on venous blood; and those who deny the existence of carbonic acid in it, do so almost entirely on such evidence. The fact of their not having thus obtained this gas is correct; but there is an error in the conclusion drawn from it, which is the chief cause of the difference of opinion on this subject; for such experiments only afford a positive proof that carbonic acid cannot always be obtained from blood placed under the exhausted receiver of the air-pump: but, with respect to the existence or non-existence of such acid in the blood, the proof is merely negative; for in experiment 4. the pump did not separate carbonic acid from serum previously impregnated with it: consequently such experiments are inconclusive.

5. Carbonic acid was introduced into an empty bladder that had been previously well moistened with warm water. When the bladder was distended about one third, its neck was firmly tied with a waxed thread, by which it was suspended in the centre of a receiver of an air-pump. When the pump was worked, the bladder increased in volume, and in a few seconds was much distended. Nearly the whole of the atmospheric air was exhausted from the receiver, but the bladder, though apparently very tense, did not burst, neither did it decrease in size. A shallow glass vessel containing barytic water had been placed under the same receiver, but the transparency of this was not affected. Hydrogen was then transmitted into the receiver, and the bladder was reduced to the same size as when first suspended under it; but, after an interval of four hours, it had become perfectly flaccid. In fact, there was scarcely a particle of carbonic acid left in it, and the barytic water within the receiver contained a quantity of carbonate of baryta.

6. The double-necked bottle was carefully filled with pure hydrogen, and about five ounces of blood were drawn into it from a vein in the arm, in the same manner as in experiment 3. Both the orifices of the bottle were then closed, and the blood and the hydrogen well agitated together. After this the lower orifice was immersed in distilled water, and the bottle left undisturbed for nearly an hour, to allow the hydrogen to act on the blood. The orifice of the descending tube was then immersed in barytic water, the lower valve was opened, and the whole apparatus put under the receiver of the air-pump. When the pump was used, the gas which was over the blood passed through the barytic water, and immediately rendered it turbid. This experiment seems to prove that venous blood does contain carbonic acid; and as the only difference between experiments 3 and 6 was, that in the former the pump was used *immediately*, and before the hydrogen had time to act on the blood, whilst in the latter the hydrogen was allowed to act nearly an hour, it would appear that the hydrogen has some power of removing the carbonic acid, and that this removal may even take place through a membrane. In the last experiment, the blood which was

used had been carefully excluded from atmospheric air, and the hydrogen was pure; consequently the carbonic acid could have been derived from no other source than the venous blood itself.

7. A few ounces of venous blood were drawn into a double-necked bottle previously filled with hydrogen. After having been gently heated, the hydrogen was found to contain carbonic acid. This experiment was made at Copenhagen in the beginning of 1833, by Professor FORCHAMMER and myself; but the conclusion which we drew from it respecting the existence of *carbonic acid* in the blood was by some objected to, in consequence of the interference of *heat*: the air-pump experiments, however, remove all such objections.

Dr. EDWARDS confined some animals in an atmosphere of hydrogen, and they continued to live for a considerable period, during which it was found that the hydrogen had acquired a portion of carbonic acid, which in some cases was equal in bulk to the size of the animals. By some these experiments were considered as conclusive of the evolution of carbonic acid from venous blood; but others maintained that there might have been a sufficient quantity of oxygen in the pulmonary cells to account for the formation of the carbonic acid. This objection is also removed by the above experiment.

Dr. MITCHELL of Philadelphia made an experiment in 1830 with hydrogen and venous blood, but without obtaining any carbonic acid. Mr. G. H. HOFFMAN of Margate made a similar experiment in 1832, and obtained a sufficient quantity of carnic acid, not merely to render lime water turbid, but even to render the hydrogen uninflammable. These contrary results seem to me to have arisen from Dr. MITCHELL having used the air-pump *immediately*, and before the hydrogen had time to act on the blood so as to displace its carbonic acid; whereas Mr. HOFFMAN agitated the hydrogen with the blood, and probably allowed a sufficient time for their mutual action.

8. A small quantity of venous blood was drawn into the double-necked bottle, containing atmospheric air, the valve at the orifice of the ascending tube was closed, and the orifice of the descending tube was immersed in barytic water. The bottle was put under the receiver and the pump immediately used. But in this experiment the barytic water was not more affected than it would have been by a similar quantity of common air. This proves that when the blood is exposed even to common air, carbonic acid cannot be obtained, when the pump is used immediately, and before any change of colour in the blood has taken place; that is, before the air has had time to act upon it.

9. A small quantity of blood was drawn into the double-necked bottle containing atmospheric air, as in the last experiment. Both of the valves were closed, and *after agitation, the bottle was allowed to stand about an hour, during which the colour of the blood changed from venous to arterial.* The lower orifice was then immersed in barytic water, the apparatus was put under the receiver of the air-pump, and when the pump was used, the gas which escaped *gave a milky appearance to the barytic*

water. In the eighth experiment the pump was used immediately, and before the air had time to act on the blood, or the blood on the air. In the last, one hour was allowed for the action of these agents upon each other; during which the blood on the surface changed from venous to arterial, and the air over the blood received the addition of carbonic acid.

Those who maintain that carbonic acid is *formed* in the lungs will say, that in the last experiment the carbon of the blood attracted the oxygen of the air, and that the carbonic acid so formed was then evolved. But there is one circumstance which is I think fatal to such an explanation, for all the acids blacken the blood, and carbonic acid possesses this blackening property in a remarkable degree. When we agitate a small quantity of carbonic acid gas with arterial blood, the colour immediately changes to venous, and when we add carbonic acid to venous blood it becomes almost black. Now, if the carbon of the blood attracted the oxygen of the air, and if the carbonic acid were thus formed in the blood itself, it is evident that the *first* effect of the air on the blood would be to make this fluid blacker than it had been before; but the opposite of this is the fact, for the *first* effect of the air is, not to blacken, but to brighten the blood; consequently, from this alone, we may infer, that the acid is not formed during the experiment, but that it exists ready formed in the blood, and that it is only removed, and not produced or formed, by the atmospheric air.

I have already observed that one class of experimenters have obtained results by means of the air-pump which are in direct opposition to those obtained by others. May we not now, from the above experiments, explain this difference by supposing that those who could not obtain carbonic acid made their experiments before the air had had time to act on the blood, whilst the others had allowed some time to elapse?

From the preceding statement we may, I think, conclude,

1st, That venous blood contains carbonic acid;

2nd, That the mere effect of diminished pressure upon the surface of the blood is not necessarily followed by the escape of its carbonic acid*.

We have seen in some of the above experiments that atmospheric air possesses a property of removing carbonic acid from venous blood; it becomes therefore a question how this effect is produced. I have ascertained that nitrogen is ineffective; we may therefore infer that the oxygen is the principal agent; and that such is the fact is proved by the following experiments.

10. A piece of moist bladder was tied firmly over the mouth of a tumbler containing pure oxygen gas. This was introduced into a large bell glass filled with carbonic acid. In a short period the membrane which had been tied over the glass became convex, and so tense that it appeared to be on the point of bursting. On examining the air contained in the tumbler, it was found that the oxygen had drawn in a large

* In performing the above experiments I was assisted by Mr. Squire, to whom I feel under great obligation for the zealous and able manner in which he aided me in the whole of the present investigation.

quantity of the carbonic acid; but no oxygen appeared to have passed out of the tumbler.

11. A piece of moist bladder was tied over the mouth of a tumbler containing carbonic acid; this was introduced into a bell glass filled with pure oxygen. In a short period the membrane became concave, and the oxygen in the larger vessel was found to be mixed with carbonic acid.

These two experiments prove that oxygen possesses the power of attracting carbonic acid, even through the medium of a membrane which is much denser than that interposed in the lungs betwixt the air and the blood; consequently, the extreme delicacy of the pulmonary membrane can be little impediment to the transmission of carbonic acid in the process of respiration.

I have ascertained that such transmission, or, in other words, the peculiar power by which oxygen abstracts carbonic acid from the blood, is more energetic in a high than in a low temperature. Hence venous blood drawn in a warm room changes colour more rapidly than blood drawn in a cold atmosphere.

12. A few ounces of venous blood were drawn into the double-necked bottle which had been previously filled with pure oxygen; the valves were closed, the blood was well agitated with the gas, and the colour immediately changed from venous to arterial. The bottle was allowed to stand about half an hour, and was then placed under the receiver of an air-pump. When the pump was used the oxygen was found to be strongly impregnated with carbonic acid; in fact, the first bubbles of air which passed through the barytic water rendered it milky.

From the rapidity of the change of colour from venous to arterial, or from dark to florid, in this experiment, it seems very improbable that any carbonic acid should have been *formed* in the blood, but that, on the contrary, it had previously existed in the blood, and that the whole of this blackening gas had been instantly removed by the oxygen.

13. A piece of the intestine of a rabbit that had been recently killed was filled with *carbonic acid*, and suspended in a bell glass containing *oxygen*. In a short period the acid escaped, and the intestine became quite flaccid.

14. A piece of intestine, similar to that used in the last experiment, was filled with *oxygen*, and suspended in a bell glass of *carbonic acid*; the intestine began to swell almost immediately, and in three minutes it burst.

15. The *lung* of a rabbit was filled with oxygen, and suspended in a bell glass of carbonic acid; it began to swell almost instantly, and in one minute it burst.

16. The *lung* of a rabbit was carefully inflated with carbonic acid, and was then suspended in a bell glass of oxygen. In a very short period it became flaccid, and the external oxygen was impregnated with carbonic acid.

These experiments show how admirably the structure of the lungs is adapted for the action of oxygen on carbonic acid. Vitality may have some share in accelerating this process in the pulmonary organs, but we know that by the agency of oxygen

dead blood may be changed from venous to arterial, even through a dead membrane.

This power which gases possess of acting on each other is in some respects similar to that which takes place in fluids, and which has been described by DUTROCHET under the name of *endosmosis* and *exosmosis*. In the experiments detailed by this philosopher the intervening septum is supposed to contribute materially to the phenomena. In the experiments with gases the intervening membrane does not prevent, but it does not contribute to, the change. But independent of this, the existence of this power in gases was not known to DUTROCHET until after the fact had been fully ascertained by others. Mr. DALTON many years ago proved that hydrogen possessed the power of penetrating or mixing with carbonic acid in opposition to gravity; that oxygen possesses the same property, but in a higher degree, I ascertained in the island of St. Thomas in 1827. I afterwards made experiments on a larger scale, in 1830, at the high rock of Saratoga, where there is an atmosphere of natural carbonic acid; and the result was communicated to many physicians in America previously to any of the American publications on the subject.

It is now more than probable that the changes which LAVOISIER believed to occur in the lungs take place in reality in the extreme circulation. Some later writers have assumed that the elements of carbonic acid exist in the blood, and that its formation commenced in the large vessels as they leave the left side of the heart, and was not finally completed until the blood arrives in the right auricle. But that this opinion is erroneous, is evident from the fact, that the blood even in the smallest arteries is as completely *arterial* as that in the left side of the heart, whilst the blood in the smallest veins is equally *venous*.

There is in the capillary system, over the whole body, an intermediate structure which connects the arterial with the venous circulation, and it is in this structure that the blood is changed from arterial to venous. When the arterial blood leaves the minute arteries, it is no longer confined in actual vessels, but in cells that are formed by the surrounding tissue. When this cellular structure is examined in living animals, with the assistance of a good microscope, minute globules are seen to leave the cells and penetrate the surrounding solids, whilst other globules are seen to return and mix with the blood in the cells. Now as we know that it is in these cells that the blood becomes dark and venous, from the addition of carbonic acid, may we not suppose that the globules which leave the blood are minute particles of oxygen, attracted perhaps from the arterial blood by the fixed carbon of the solids, and that the globules which return are minute particles of carbonic acid? This cannot be easily proved; but as carbon is a principal ingredient in the solids when these are converted into fluids, previous to their removal by the lymphatics, it appears to me not improbable that a part of their carbon may be liberated, perhaps for the purpose of evolving heat. We have seen that the blood which receives oxygen in the lungs passes unchanged through the arteries into the capillary system; and it is ap-

parently there that animal heat is evolved. When carbonic acid, which is the result of the above process, is added to the venous blood, it not only blackens the colour, but renders it incapable of supporting life. For this reason warm-blooded animals have a double circulation, one for circulating the arterial, and another for purifying the venous blood*.

When we obtain *hæmatosine*, or the colouring matter of the blood, in a pure state, it is black; but a solution of any neutral salt possesses the peculiar property of striking a beautiful scarlet or arterial colour with it. When we make an incision into a clot of blood which has just coagulated, we find that the clot is then all equally red: when we cut out a thin slice of this red clot, and immerse it in distilled water, the salt serum oozes into the water. In proportion as this takes place the clot becomes darker; and when the whole of the serum is removed, perfectly black. When in this state neither atmospheric air, nor even pure oxygen, is capable of changing its colour; but when we immerse this black clot in a clear saline solution, it instantly changes from jet black to a scarlet, or arterial colour. From these facts we may conclude, that oxygen is a secondary agent in the change of colour from venous to arterial; and that if the scarlet colour of the blood be essential to life, it is produced, not by oxygen, but by another cause. The mere removal of the carbonic acid from venous blood would not produce any change of colour, were it not that there is in the blood itself another agent which produces the arterial tint, the moment that the blackening effect of the carbonic acid is removed: this is effected by the action of the natural saline ingredients of the blood on the colouring matter. When oxygen is added to blood it may have a slight share in brightening the colour, but it can only perfectly effect this when the colouring matter is in contact with a saline fluid. Oxygen is so far from being the sole cause of the arterial colour, that even pure oxygen is *of itself* inert as a colouring agent; whilst a saline fluid changes the colour of the blood from venous to arterial even in an atmosphere of carbonic acid.

Many authors describe the changes which occur in respiration, by asserting "that oxygen disappears, and carbonic acid is emitted." But from some of the experiments which I have detailed, it is evident that the removal of the acid is the first part of the process, and the addition of oxygen the last. Others have maintained that when

* It is well known that cold-blooded animals use very little food. If a rattlesnake gets one good meal in three months, it is all that he requires: but even this is not actually necessary; for I have seen one of these animals that had not tasted food or water for twelve months, as plump, active, and venomous as those in the wild state. On the other hand, all those animals that have warm blood require an immense quantity of food, and if they do not receive this they soon perish; but nineteen twentieths of this appears to be taken into the system for the evolution of animal heat. The carbon is ultimately derived from the nourishment that we use, and the oxygen is directly derived from the arterial blood: a constant supply of nourishment is therefore necessary in warm-blooded animals, but a very small part of the blood which is formed from this is required for nutrition, and if the whole of it were expended in this way, it is very clear that there would be none left to return by the veins.

two gases act upon each other, the one penetrates the membrane at the same moment that the other is removed. But if this were the case, why did the membrane become *convex* in experiment 10, and *concave* in experiment 11? If the action were equal, the membrane would remain unchanged; but this is so far from being the case, that in some experiments the membranes became distended to such an extent that they actually burst.

It was supposed by SPALLANZANI, and afterwards by Dr. EDWARDS, that in the process of respiration the carbonic acid was merely exhaled from the lungs. We have seen, however, that venous blood so retains that acid that it cannot be removed, even with the aid of an air-pump; consequently, were there not an active agent for the purpose of removing the carbonic acid from the venous blood as it circulates through the lungs, it would remain unchanged, and almost instantly cause death. It is the power which oxygen possesses of attracting carbonic acid, which renders oxygen essential to life. As hydrogen also possesses this power, it supports life for a longer period than most of the other gases; but hydrogen has a deleterious effect on the blood; and when animals are forced to breathe it, though the carbonic acid is removed, a part of the hydrogen is at the same time absorbed, which blackens the blood, and the animals soon die.

The property which oxygen possesses of attracting carbonic acid, furnishes the following explanation of the process of respiration. When the venous blood arrives in the lungs, the oxygen of the atmosphere is, in the first instance, the active or attracting agent. It removes the carbonic acid, which had been the cause of the dark colour of the blood. When this is removed, or perhaps in proportion as it is removed, the blood becomes the attracting agent, and a portion of oxygen is attracted into the blood, and takes the place of the carbonic acid. From the peculiar structure of the lungs, these changes are rapidly effected, particularly at the high temperature of 98°; and when the process is fully completed, we know from the great discovery of HARVEY, that the blood which has received the pure air passes rapidly on to the arterial, and from this again to the capillary system.

If the above theory be correct, it follows that the blood is converted from arterial to venous in the extreme circulation, by the loss of oxygen and the addition of carbonic acid; whilst the venous blood is converted into arterial, by the loss of carbonic acid, and the addition of oxygen; consequently, the essential difference betwixt venous and arterial blood is, that the former contains carbonic acid, and the latter oxygen.

I have said that black is the natural colour of the colouring matter; but when this agent is diffused in a saline fluid, such as the serum, it is of a bright scarlet tint, which is, in fact, the natural colour of arterial blood. When carbonic acid is added to this blood in the extreme circulation, it becomes dark red; but when this acid is removed in the pulmonary organs, the blood then resumes its natural scarlet or arterial colour; and this, as I have said, is produced not directly by oxygen, but chiefly, if not entirely, by the action of the salts of the blood on the colouring matter.

Oxygen, it is true, changes the colour from venous to arterial; this, however, is effected not by any specific action, but by the removal of the carbonic acid, which had been the cause of the dark colour in the venous circulation.

Many objections have been made to the above theory, some of which are frivolous; but there are two which are worthy of notice. The first was made by Mr. PRATER of Edinburgh, who stated, that according to this theory the blood ought to become *arterial* under the exhausted receiver of the air-pump. This objection is removed by the foregoing experiments, which prove that the mere removal of the air's pressure is insufficient to overcome the attraction that subsists between the carbonic acid and the blood. The second objection was made by Dr. GREGORY and Mr. IRVINE of Edinburgh. These gentlemen admit that if the blood were a stronger saline fluid than it is, the salts would be capable of producing all the effects described; but they conceive that the blood is not sufficiently impregnated with saline matter to account for the whole of the phenomena. This objection, even if proved, would only require a modification of the theory; but that there was a fallacy in their experiments which neutralized their conclusion, has been proved by a paper in the Medical Gazette of the 12th of April, 1834.

XX. *Discovery of the Metamorphosis in the second type of the* Cirripedes, *viz. the* Lepades, *completing the Natural History of these singular Animals, and confirming their affinity with the* Crustacea. *By* J. V. Thompson, *F.L.S. Deputy Inspector-General of Hospitals. Communicated by Sir* James Macgrigor, *Bart. M.D. F.R.S.*

Received January 3,—Read March 5, 1835.

THE Fourth Memoir, published in my Zoological Researches and Illustrations, No. III. page 69, &c., having first made known the real nature of the *Cirripedes*, the key of which remained concealed in their metamorphosis, it might have been expected that some naturalist favourably situated to investigate the oceanic tribe of these animals, would have been the first to make the same discovery in regard to these, and thereby complete their natural history. It was scarcely to be expected that the honour of this discovery also should be reserved for the author, fixed to one spot, where none of them naturally exist, and are but casually thrown upon our shores by the waves of the Atlantic, attached to pieces of wreck, or brought into port fixed to the bottoms of ships returning from distant voyages. Fortunately, however, two ships of this description came into this harbour (Cork), one from the Mediterranean, the other from North America, which, not being sheathed with copper, had their bottoms literally covered with Barnacles of the three genera of *Lepas*, *Cineras*, and *Otion*; and having persons employed expressly for the purpose, numbers of these were brought alive in sea water, amongst which were many with the ova in various stages of their progress, and some ready to hatch, which they eventually did in prodigious numbers, so as to enable him to add the proof of their being, like the *Balani*, *natatory* Crustacea *in their first stage*, but of a totally different facies and structure; a circumstance which determines the propriety of the separation of the *Cirripedes* into two tribes, and evinces the sagacity of Mr. MacLeay in being the first to indicate that these two tribes, the *Balani* and *Lepades*, were not so closely related as generally supposed *.

The larvæ of the *Balani*, described in Memoir IV. under the external appearance of the *bivalve Monoculi* (*Astracoda*), have a pair of pedunculated eyes, more numerous and more completely developed members, approximating to those of *Cyclops*, and of the perfect *Triton*; while, in the present type, or *Lepades*, the larva resembles somewhat that of the *Cyclops*, which Müller, mistaking for a perfect animal, named *Amymone*, and which can be shown to be common to a great many of the *Entomostraca*;

* See Horæ Entomologicæ.

or the resemblance is still more striking to that of the *Argulus Armiger* of LATREILLE, which, in fact, is but an *Amymone* furnished with a tricuspidate shield at the back.

The genus *Cineras* was the first in which the larvæ were observed to hatch, July 27, three days after the arrival of the ship; then in *Lepas anserifera*, August 19; and a few days later in *Lepas dentata*; in all of which there is a perfect accordance, with very slight differences, which probably resulted from the more or less perfect development of the larva. The very remarkable and beautiful one of *Lepas anserifera* may be regarded as the perfect type to which all the others are to be referred (fig. 5.).

In the whole of this tribe of the *Cirripedes*, the ova, after expulsion from the ovarium, appear to be conveyed by the ovipositor into the cellular texture of the pedicle, just beneath the body of the animal, which they fill to the distance of about an inch. When first placed in this situation they seem to be amorphous, and inseparable from the pulpy substance in which they are imbedded; but as they approach to maturity, they become of an oval shape, pointed at both ends, and are easily detached. Sir EVERARD HOME has given a very good representation of them, at this stage of their progress, in his Lectures on Comparative Anatomy, from the elegant pencil of Mr. BAUER.

During the stay of the ova in the pedicle, they render this part more opake, and of a blueish tint; the ova themselves, and the cellular texture with which they are surrounded, being of a pale or azure blue colour. It is difficult to conceive in what manner the ova are extricated from the situation above indicated; but it is certainly not by the means suggested by Sir EVERARD HOME in the above-mentioned Lecture, viz. by piercing outwards through the membranes of the pedicle, for the ova are subsequently found forming a pair of leaf-like expansions, placed between either side of the body of the animal and the lining membrane of the shells in *Lepas* (fig. 1.), or of the leathery internal tunic in *Cineras*. These leaves have each a separate attachment at the sides of the animal to the septum, which divides the cavity occupied by the animal from that of the pedicle: they are at first comparatively small, have a rounded outline, and possess the same blueish colour which the ova had in the pedicle; but as the ova advance in progress these leaves extend in every dimension, and lap over each other on the back, passing through various lighter shades of colour into pale pink, and finally, when ready to hatch, become nearly white (fig. 2.). These leaves appear to be composed of a layer of ova irregularly placed, and imbedded in a kind of parenchymatous texture, out of which they readily fall when about to hatch, on its substance being torn asunder; indeed, it appears at length to become so tender as to fall entirely away, so that after the period of gestation is past, no vestige of these leafy conceptacles is to be found.

When the larvæ, barely visible to the naked eye, burst forth from the ova, their development goes on with such rapidity that they seem to grow sensibly while under observation. These changes have been depicted in *Cineras* at figg. 6. 7. & 8., which

last probably did not possess sufficient vitality to pass into the next stage, such as we see that of *Lepas anserifera* (fig. 5.).

The larva of the *Lepades*, then, is a tailed *Monoculus*, with three pairs of members, the most anterior of which are simple, the others bifid, having its back covered by an ample shield, terminating anteriorly in two extended horns, and posteriorly in a single elongated spinous process*.

It must ever remain uncertain how long the larvæ of the *Lepades* remain in their first or free state, but it is probably for a longer or shorter period of time, according as they sooner or later meet with a substance adapted to their respective habits: thus, those of the *Lepas fascicularis* attach themselves in preference to Gulf-weed or floating *Fuci*; *Lepas minima* to slender species of *Antipathes*; *Lepas anserifera*† and *dentata*, *Cineras vittatus*, and *Otion*, to the bottoms of ships; *Lepas anatifera*‡ to floating timber, and to one another; while *Lepas sulcata* seems to prefer the backs of Turtles and the shell of the *Ianthina*: the species, however, have not been sufficiently discriminated, nor observations of this kind made with the requisite care, to enable us to prosecute further this part of their natural history. It is evident that they possess locomotive powers which enable them at every instant to change their situation, and a conspicuous eye to guide them in their choice.

These remarkable and important discoveries, connected as they are with those relating to the *Crustacea*§, complete the natural history of this tribe, and lead us to the following important results, viz.

I. That the *Cirripedes* do not constitute *a distinct class* of animals, as they have been considered by all late naturalists, Dr. LEACH, LATREILLE, LAMARCK, CUVIER, &c., being connected with the *Crustacea decapoda* through the *Balani*, and with the *Entomostraca* by means of the *Lepades*.

II. That they have no relation or affinity whatever with the *Testacea*, as supposed by LINNÆUS and all the older systematists.

III. That the *Crustacea* now therefore furnish examples of a class in which we have animals free and fixed, with eyes and eyeless, and with the sexes separated in some and united in others, all of which are characters to which naturalists have attached the greatest importance as regards classification.

IV. That the proof of metamorphosis being fully and satisfactorily established, tends still to maintain the affinity so long recognised between the *Crustacea* and *Insecta*.

Note.—The same economy in regard to the disposal of the ova has been observed in *Otion*, but hitherto no individual has been found with the ova on the point of hatching.

* Compare with the larva of *Artemis* (Brine Shrimp). Zoological Researches, No. v. Plate II. f. 7, 8. (herewith sent).

† Philosophical Transactions, vol. i. Plate xxxiv. fig. 5.

‡ Philosophical Transactions, vol. i. Plate xxxiv. fig. 6.

§ Zoological Researches, Memoir I. and Addenda, p. 63.

PLATE ~~V.~~ VI

Fig. 1. *Lepas anserifera*, opened from behind to show the first stage of the leaf-like conceptacles of the ova (*m*).
- *a*. Animal.
- *o*. Ovipositor.
- *v*. Valves.
- *p*. Pedicle.

Fig. 2. Another individual, showing the conceptacles in an advanced stage, the right conceptacle being turned back (*m*).

Fig. 3. A portion of the conceptacle magnified.

Fig. 4. An ovum ready to hatch.

Fig. 5. Fully developed larva of *Lepas anserifera*, viewed in front, highly magnified.

Fig. 6. Larva of *Cineras vittatus*, just excluded from the ovum.

Fig. 7. The same more developed.

Fig. 8. The fully developed larva of the same, viewed in front and highly magnified.

Fig. 5

Fig. 2

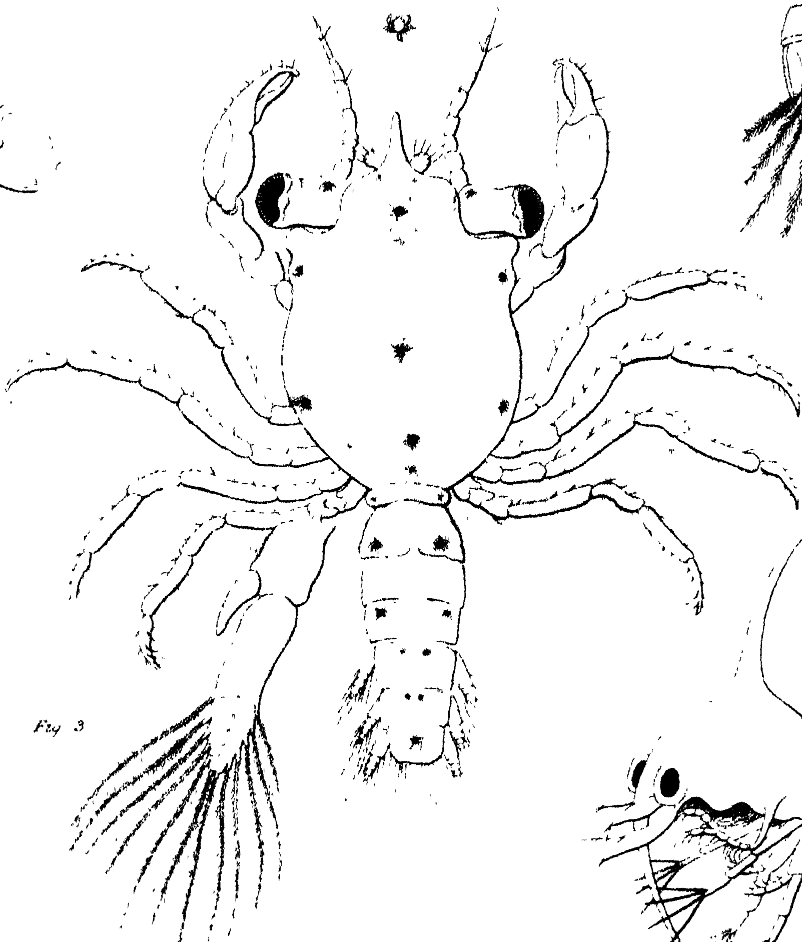


Fig. 4.

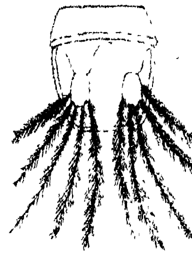


Fig. 3

Fig. 1

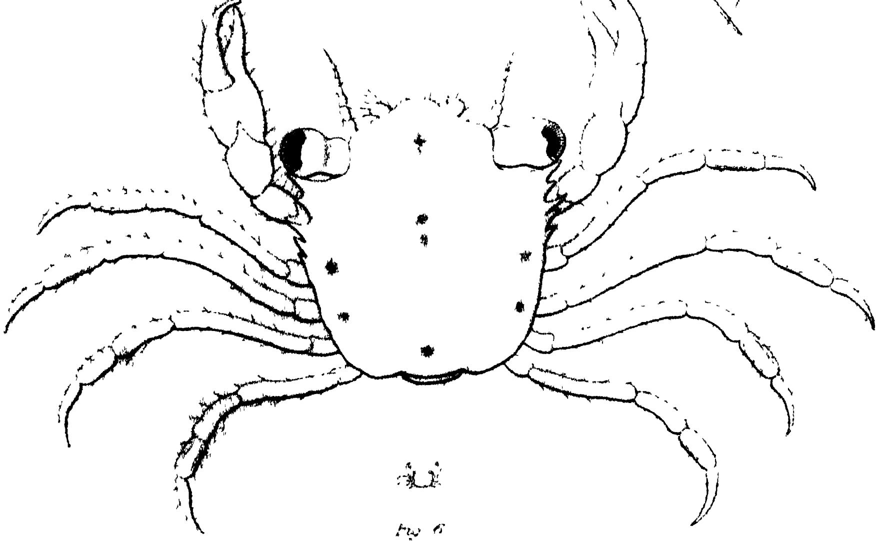


Fig. 6

XXI. *On the Double Metamorphosis in the Decapodous* Crustacea, *exemplified in* Cancer Mænas, *Linn.* *By* J. V. Thompson, *F.L.S. Deputy Inspector-General of Hospitals. Communicated by Sir* James Macgrigor, *Bart. M.D. F.R.S.*

Received May 21,—Read June 4, 1835.

In the Memoir published in my Zoological Researches, p. 1, with its sequel, p. 63, having first made known the fact of the *Brachyura* of the *Decapoda* (Crabs) passing through the intermediate form of *Zoea*; I have now to announce that they undergo another metamorphose, no less singular and unlooked for, in which they assume the form of the genus designated by the name of *Megalopa* by Dr. Leach, from the disproportionate size of the eyes. This second stage we may therefore consider analogous to that of pupa in the class *Insecta*.

By the former memoir it appears that the young of *Cancer Pagurus*, the common market Crab, first presents itself as a *Zoe* *, and that a full-grown *Zoe* was observed passing into some other more perfect form †, which at that time was considered to be that of some species of Crab: the discovery now first detailed, however, shows that it must have been only passing into that of a *Megalope*.

The first proof I had of this new and extraordinary fact, which cancels another anomalous genus of the *Crustacea*, was obtained by keeping in regularly renewed sea-water a number of individuals of a *Megalope* ‡ which makes its appearance in the river Lee, just below the city of Cork, in considerable abundance every summer: these, to my very great surprise, began, after a short time, to change into a minute Crab §, until the whole of them, to the amount of about two dozen, were so metamorphosed. I have frequently since observed the same circumstance, and came to the conclusion that these must be the progeny of the only Crab that is ever found in the higher parts of the river, where these *Megalopæ* were taken, viz. *Carcinus Mænas*, our common Shore Crab. The young Crab, it will be noticed, has not the distinctive characters of its parent, which it probably acquires only after several casts of its shelly covering.

To complete the series of metamorphoses in this species of Crab now became a matter of research; and I have been so fortunate as to succeed in hatching its mature spawn, so as to be enabled to give a representation of its *Zoe* ||, or first stage, and thereby render complete its natural history. In this stage it does not appear to differ materially from that of *Cancer Pagurus*, formerly figured in Zoological Researches

* Pages 9 and 64. † Page 8. ‡ Fig. 2. § Fig. 6. || Fig. 1.

Pl. VIII. fig. 1. It is, however, certainly much smaller, and of a greenish tinge, with a few darker spots.

If the above facts do not warrant the conclusion I have drawn, what other proof can be required? Is it necessary that the young Crab should be traced through its subsequent changes until the character of the species becomes more apparent? or that the grown *Zoe* should be actually seen to change into a *Megalope*? Neither of these is impracticable, but may yet for a long time elude the most zealous and scrutinizing observers.

It appears, then, that the animals of this division of the *Crustacea* not only undergo metamorphosis, as formerly stated, but that they even undergo a double metamorphosis, being hatched from the ova under the singular and grotesque form of *Zoea*, then assume that of *Megalopæ*, and finally that of their parent Crab. How long they remain in each of these two intermediate states it may be difficult to determine with exactitude; but judging from the very considerable size of the *Zoe* I observed about changing its condition *, compared with their very minute size when first hatched, and also from *Megalopæ* not appearing before May or June, while *Zoea* are seen so early as March and April, I think a month may be assigned as the probable duration of the *Zoe* stage. The other, or *Megalope* stage, is less within the scope of observation; but as there is not that very great disparity of size between young and full-grown *Megalopæ*, it is likely that it does not exceed half that of the former.

If further proof of this double metamorphosis be desired, I have been so fortunate as to trace it, but not in quite so satisfactory a manner, in one of the Swimming Crabs, or *Portuni*, and also in *Inachus*, belonging to the section of Triangular Crabs. These examples, derived from some of the principal groups of the *Brachyura*, may be supposed quite sufficient to satisfy the most scrupulous, as to their metamorphosis; I propose, however, in future memoirs, to bring under the notice of the yet sceptical, proofs of the same thing in the following genera, viz. *Eriphia*, *Thelphusa*, *Gegarcinus*, and *Pinnotheres* †. The three former genera, it may be observed, are foreign, which friends in the East and West Indies have enabled me to add to the first proofs of metamorphosis, by having females with ova on the point of hatching, sent home in spirits: the larvæ of these, consequently, have not been seen in the living state; but by examining such as have burst from their envelopes, without being completely developed, it is quite evident that they are *Zoea*.

With regard to the other great division of the *Decapoda*, viz. the *Macroura*, or those with extended tails, I shall only now say, that as far as my observations have gone they also undergo metamorphosis, being *Ipizopoda* when first hatched, and during the whole of their progress to the perfect animal; such is the case in *Astacus marinus*, *Palinurus*, *Palæmon Squilla*, *Crangor*, *Galathea*, *Pagurus*, and *Porcellana*.

To return to the immediate subject of the present memoir, the *Carcinus Mænas*. In its first or *Zoe* stage it is wholly natatory from structure, while in its second it

* Zoological Researches, p. 8.

† This has since been published in the Entomological Magazine.

occasionally walks by means of its thoracic members, now become simple, but more commonly swims by the motion of its subabdominal fins, which are greatly developed for this purpose*. In both stages it is therefore a *Macroura,* but only in the latter evidently related to the *Decapoda.*

It will be quite superfluous to enter into a minute detail of the structure of this *Megalope,* further than may be collected by a reference to the figure and its accompanying explanation.

It must certainly be considered surprising that so many curious facts should have remained until the present time undiscovered; but still more, that from the first announcement of metamorphosis no person has attempted to follow it up; so that I have not only the honour of the discovery, but also the entire merit of having rendered this interesting part of the natural history of the *Crustacea* nearly complete, as the announcements in the previous part of this memoir testify, and my subsequent memoirs will prove.

The facts connected with the metamorphosis in the *Crustacea* and the *Cirripedès* are indeed so much at variance with our previous knowledge, with the dicta of some of our leading naturalists, and of so very extraordinary a nature, that the scepticism which still exists with regard to them may admit of some excuse. The approaching summer I hope will put it in my power to remove all doubts upon the subject, by submitting such of them as offer themselves to the scrutiny of *other observers,* a circumstance which never occurred to me as necessary beyond the circle of my own family; had there been any zoologists in my neighbourhood the case would have been different, but in respect of this branch of science I here unfortunately stand alone.

Whatever indifference may be charged to our own zoologists in regard to these important discoveries, we must do our scientific neighbours the French the justice of noting, that they immediately took up the subject, and two naturalists were selected and deputed to spend a summer at Isle Ré, to make their observations. However, by a subsequent report of one of these gentlemen, M. Milne-Edwards, to the French Institute, it appears that so far from verifying the metamorphosis in *Crustacea,* he pronounced that they were hatched with the form and structure of their adult parent! The observations upon which this decision was based I have not seen stated; but whatever they may have been, they are completely invalidated by the positive proofs I have given and enumerated in the present memoir.

The animals of this class are so recondite in their habits, so difficult to preserve alive for any time, so little known to naturalists beyond the more common species, that the investigation is necessarily attended with great difficulty and frequent disappointment. It must be allowed that I have been peculiarly fortunate; and I am so sensible of the obligation I owe to that Source from whence springs all our

* Fig. 3.

knowledge and intelligence, that I hasten to acquit myself of so sacred and valuable a trust with all the ability I am yet permitted to retain.

PLATE ~~VI~~. V

Fig. 1. *Zoe* of *Carcinus Mænas*, magnified, and also of its natural size.

Fig. 2. *Megalope* of the same, magnified, and also of its natural size. The terminations of three pair of the subabdominal fins are only seen in the figure.

Fig. 3. One of its subabdominal fins more highly magnified.

Fig. 4. The posterior pair of fins magnified.

Fig. 5. One of its inner antennæ, highly magnified.

Fig. 6. The Young Crab, resulting from the above *Megalope*, magnified, and also of its natural size.

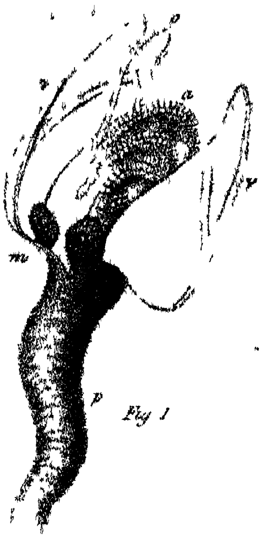


Fig 1

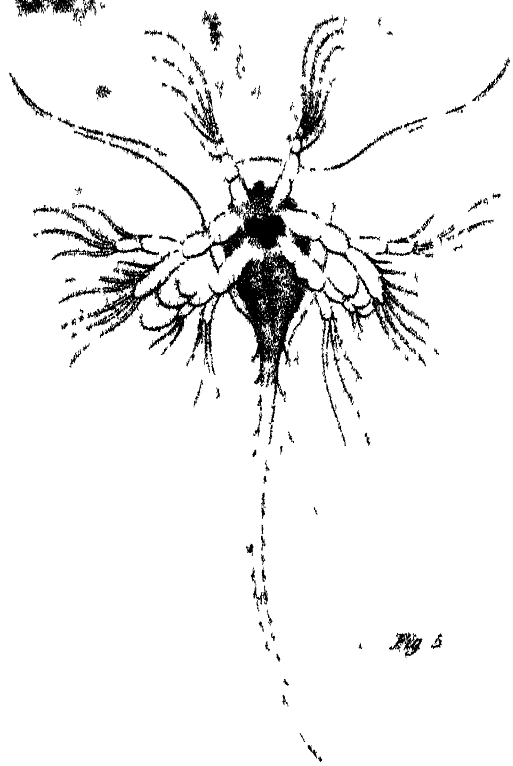


Fig 5

Fig 2

Fig 3

Fig 4

Fig 6

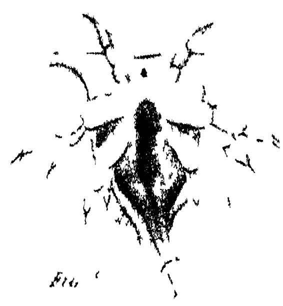


Fig 8

Fig 7

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METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY,

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF

THE PRESIDENT AND COUNCIL.

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge..............=83 feet 2¼ in.

——————————— above the mean level of the Sea (presumed about) ....=95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House................=79 feet.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR JANUARY AND FEBRUARY, 1835.

1835.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	External Thermometer. Fahrenheit. 3 P.M.	External Thermometer. Self-registering. Lowest.	External Thermometer. Self-registering. Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
JANUARY												
T 1	30.107	51.8	30.398	51.9	48	47.9	46.0	46.3	48.6		S	A.M. Light fog and rain. P.M. Lightly cloudy—light brisk wind.
F 2	30.794	46.4	30.819	47.3	35	40.2	42.4	38.9	41.8		N	Light fog and wind.—A.M. Overcast—light haze. P.M. Fine and clear.
S 3	30.782	44.7	30.725	45.4	32	38.9	40.6	36.8	40.5		NE	Fine light haze and wind. P.M. Fine light clouds and wind.
☉ 4	30.622	40.9	30.570	41.3	29	32.9	35.2	30.2	34.5		NE	Fine and cloudless—light wind. Evening, light fog.
M 5	30.564	38.2	30.556	40.4	32	34.3	40.3	30.6	39.4		E	Hoar frost—light fog.—P.M. Light clouds and wind. Evening, fine and clear.
T 6	30.552	37.3	30.475	37.9	32	31.6	34.7	29.0	33.3		E	Thick fog.—P.M. Light haze.
W 7	30.307	35.0	30.212	34.3	27	27.9	29.9	25.8	28.5		E	Thick fog.—P.M. Light rain. Evening, frosty.
T 8	30.128	33.2	30.063	34.6	29	30.0	33.7	25.0	35.8		ENE	Sharp frost—light fog and wind.
F 9	29.875	36.7	29.703	38.6	32	37.2	44.0	28.4	47.5		S	Lightly cloudy—thaw.—A.M. Overcast. P.M. Light rain.
S 10	29.825	40.6	29.841	42.2	36	38.9	43.8	35.8	46.9	.113	SSW	A.M. Fine—light clouds with light brisk wind. P.M. Overcast—drizzling rain.
☉ 11	29.800	44.6	29.853	46.2	42	46.8	50.3	37.2	49.6		S var.	A.M. Lightly overcast—light unsteady wind. P.M. Light rain.
M 12	29.930	46.4	29.918	47.8	44	47.8	49.0	45.7	49.7		SW	Overcast—deposition.—A.M. Fine light clouds. P.M. Lightly cloudy—light wind.
T 13	29.707	46.3	29.594	46.7	40	40.2	45.4	39.8	45.3		ESE	A.M. Fine—light clouds and wind. P.M. Lightly overcast.
● W 14	29.479	47.2	29.473	48.8	42	45.7	49.4	39.9	48.8		SE	Overcast—deposition.—P.M. Overcast—very light rain.
T 15	29.656	48.7	29.693	49.8	43	46.6	48.3	44.2	47.8		SW	A.M. Lightly cloudy. P.M. Fine light clouds and wind.—High wind throughout the night.
F 16	29.091	48.8	29.197	49.6	44	47.2	44.2	44.2	49.6		SW var.	Overcast—very light rain.—A.M. Fine light clouds. P.M. Fine and cloudless—light haze.
S 17	29.598	43.2	29.685	44.6	38	35.2	40.3	31.9	39.3		SW	Fine—light clouds and wind. Evening, sharp frost.
☉ 18	29.808	39.2	29.604	40.3	27	31.5	36.6	27.8	42.2		ESE	Hoar frost—light haze.—P.M. Lightly overcast. Evening, light snow and rain.
M 19	29.101	40.7	29.172	41.6	35	39.9	38.8	29.5	42.3		SW var.	Lightly cloudy.—A.M. Overcast—light unsteady wind. P.M. Cloudy—deposition.
T 20	30.083	37.3	30.091	38.0	26	32.0	33.5	30.5	32.4		NE var.	Fine & frosty—light brisk wind. Even. fine & clear—sharp frost.
W 21	30.328	34.7	30.253	35.7	17	27.4	31.9	24.9	35.7		SW	A.M. Fine and frosty—light fog. P.M. Light fog. Evening, snow—sharp frost.
T 22	30.134	36.6	30.220	38.4	33	37.2	39.2	25.3	38.6		NE	Overcast—light wind.
F 23	30.344	38.6	30.301	39.9	32	38.8	41.4	33.8	44.2		SW	Light fog.—Evening, overcast—light rain.
S 24	30.115	41.7	30.154	43.8	40	45.4	47.6	37.2	47.6	.069	SW	Overcast—deposition.—A.M. Fine light clouds and wind. P.M. Overcast—very light rain.
☉ 25	30.200	43.6	30.214	45.4	39	45.3	49.9	37.2	49.7		S	Overcast—deposition—light wind.
M 26	30.332	45.6	30.332	47.4	41	47.7	48 7	42.8	49.7		SW	Fine—light clouds.
T 27	30.425	46.2	30.406	47.3	37	44.5	46.6	40.6	45.9		SW	Overcast—light fog. P.M. Light clouds and wind.
○ W 28	30.362	45.3	30.279	46.2	37	40.9	47.0	38.6	41.7		SW	Overcast—deposition.
T 29	30 188	45.0	30.128	46.9	38	42.0	48.2	39.8	47.7		S var.	Fine—lightly cloudy—light haze.—A.M. Fine and cloudless—light haze. P.M. Fine light clouds.
F 30	30.087	46.3	30.035	48.3	41	44.2	46.4	40.8	47.6		S var.	A.M. Light fog and wind. P.M. Fine and cloudless—light haze. Evening, cloudy.
S 31	30.000	44.0	30 047	45.5	36	38.6	42.0	35.0	43.6		S	Overcast—light fog and wind.
MEANS..	30.075	42.4	30.065	43.6	35.6	39.5	42.4	35.3	43.1	Sum. .182		Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr. .... 9 A.M. 30.048; 3 P.M. 30.034
FEBRUARY												
☉ 1	30.275	46.0	30.269	48.2	41	44.7	50.0	36.9	49.6		S var.	Overcast—light rain and wind.—P.M. Fine—lightly cloudy. Evening, overcast.
M 2	30.142	48.4	30.178	50.3	45	50.2	53.2	43.3	53.2		SW	A.M. Overcast—light rain and wind. P.M. Lightly overcast.
T 3	30.352	47.7	30.376	50.0	43	45.0	51.2	42.2	50.6		SW	Fine—light clouds and wind.
W 4	30.459	48.0	30.441	50.2	40	43.2	50.3	41.0	50.2		SW	A.M. Fine—light clouds and fog. P.M. Fine and clear—light clouds and wind.
T 5	30.200	48.0	30.033	50.3	42	45.6	50.9	40.7	50.8		SW var.	Fine—light clouds with light brisk wind.—A.M. Cloudy. P.M. Fine and clear—light wind.
F 6	30.172	44.6	30.259	46.2	30	38.0	43.7	35.2	45.4		S var.	A.M. Fine & clear—light haze & wind. P.M. Fine lt. clds. & wind.
S 7	30.067	46.2	29.918	48.3	39	46.6	50.9	36.8	50.5		SW	Lightly overcast—light wind. Evening, cloudy.
☉ 8	29.697	47.0	29.661	48.3	32	42.6	44.7	40.8	46.2		WSW	Fine and clear—light haze & wind. Evening, cloudy—light rain.
M 9	29.849	43.0	29 922	44.3	27	36.7	41.0	33.8	39.8	.061	SW	A.M. Fine and cloudless—light haze and wind. P.M. Fine and cloudless—light brisk wind. Evening, fine and frosty.
T 10	30.091	38.6	30.255	40.6	22	32.4	39.5	29.8	38.2		NE var.	Fine—sharp frost—light brisk wind—snow during the night. Evening, fine and clear—sharp frost.
W 11	30.435	37.8	30.372	41.3	24	33.3	41.9	27.9	44.7		S	Cloudy—light frost and wind.
T 12	30.132	43.2	30.168	44.5	35	46.0	45.2	31.8	46.3		SW	Overcast—light rain and wind. Evening, fine and clear.
○ F 13	30.336	41.9	30.247	45.2	32	39.3	46.3	35.0	46.8	.125	SW	A.M. Fine and cloudless—light haze and wind. P.M. Overcast.
S 14	30.014	41.9	29.917	46.6	40	43.4	49.4	37.5	49.8		SW	Overcast—light rain.
☉ 15	29.701	48.2	29.956	50.2	42	47.6	51.0	42.3	51.4		SW	Overcast—deposition.—A.M. Fine—light clds. Even. overcast.
M 16	29.530	46.3	29.592	48.0	40	43.2	46.2	40.0	47.3		NW var.	A.M. Lightly cloudy. P.M. Overcast—light brisk wind.
T 17	29.711	45.9	29.728	48.0	40	42.4	45.8	38.6	46.7		S var.	Overcast—drizzling rain.—A.M. Fine and clear. P.M. Cloudy.
W 18	29.469	46.3	29.461	48.4	40	44.6	49.2	38.8	49.3		S var.	Overcast—light rain—brisk wind. Evening, cloudy.
T 19	29.465	45.0	29.396	48.0	38	41.3	47.5	35.7	48.3		SW	Fine—nearly cloudless.—A.M. Overcast. P.M. Fine—light clds.—brisk wind. Eve. overcast—heavy rain—high wind.
F 20	29.273	45.0	29.271	47.6	34	39.9	47.6	37.0	48.2	.061	WSW	Fine & cloudless.—A.M. Lightly overcast. P.M. Fine—light clouds & wind. Eve. overcast—light rain—high wind.
S 21	29.295	43.3	29.346	46.4	34	38.4	43.7	33.2	45.8	.133	WSW	Fine and cloudless—light wind. Evening, cloudy.
☉ 22	29.758	42.7	29.614	45.9	35	38.8	43.8	34.2	19.5		WSW	Fine & cloudless—lt. haze & wind.—A.M. Cldy. P.M. Overcast—light rain. Eve. rain—high wind throughout the night.
M 23	29.267	45.6	29.483	48.2	39	46.3	48.6	37.0	49.6	.033	SW var.	A.M. Fine—light clouds—high wind. P.M. Fine—nearly cloudless. Evening, cloudy—high wind.
T 24	29.780	43.2	29.827	45.8	32	38.4	45.4	33.9	45.3		WSW	Fine—light clouds and haze. P.M. Lightly cloudy—high wind.
W 25	29.604	44.8	29.412	46.9	40	46.2	49.4	36.0	49.3		SE var.	A.M. Cloudy—high wind. P.M. Overcast—light drizzling rain and wind.
T 26	29.287	46.5	29.459	49.0	40	44.6	48.5	42.8	49.7		W	Fine—light clouds and wind. Eve. very light rain—high wind.
● F 27	29.227	45.9	29.279	48.5	40	44.4	49.8	38.4	49.3		SW var.	A.M. Overcast—strong wind. P.M. Fine—light clouds & wind.
S 28	29.685	44.8	29.817	46.6	31	41.2	45.2	37.2	44.7		S var.	Fine and clear—light clouds and wind.—Evening, cloudy, light brisk wind.
MEANS..	29.831	45.0	29.845	47.2	36.3	42.3	47.1	37.1	47.7	Sum. .413		Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr. ...... 9 A.M. 29.796; 3 P.M. 29.803

METEOROLOGICAL JOURNAL FOR MARCH AND APRIL, 1835.

1835.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	External Thermometer. Fahrenheit. 3 P.M.	External Thermometer. Self-registering. Lowest.	External Thermometer. Self-registering. Highest.	Rain, in inches. Read off at 9 A.M	Direction of the Wind at 9 A.M.	REMARKS.
MARCH ⊙ 1	29.257	42.9	29.364	43.6	32	36.8	38.7	34.8	38.3		SW	Overcast—light rain and snow.
M 2	30.150	40.6	30.079	42.7	33	35.5	41.6	32.7	47.9		WSW	Overcast—deposition—light wind. Evening, cloudy.
T 3	29.677	45.2	29.780	46.4	34	47.3	45.9	34.6	47.2	.027	SW var.	A.M. Fine—light clds.—high wind. P.M. Fine—nearly cloudless.
W 4	29.887	42.5	29.639	45.3	36	40.3	47.8	35.7	46.8		SW	A.M. Fine—light clouds and wind. P.M. Overcast—heavy rain.
T 5	29.974	40.6	29.978	43.9	29	38.6	45.2	33.8	45.8	.101	SW var.	Fine and cloudless—light haze and wind. Evening, Overcast—very light rain.
F 6	29.378	44.7	29.588	47.2	38	43.5	49.2	37.2	49.6		W	Fine—light clds—brisk wind. Ev. overcast—light rain and wind.
S 7	29.011	45.5	28.855	47.3	38	46.2	45.5	40.0	47.8		S var.	A.M. Lightly overcast—brisk wind. P.M. Overcast—rain and hail—brisk wind.
⊙ 8	29.798	42.0	29.827	44.9	32	38.4	44.2	34.2	44.2		SW var	Fine and cloudless—light haze and wind. Evening, overcast—high wind.
M 9	29.261	44.6	28.912	46.0	38	45.2	48.2	37.2	48.4		SE	Cloudy—very light rain and wind. Evening, fine and clear.
T 10	29.495	41.8	29.602	45.0	31	38.2	45.8	32.7	47.6		WSW	Fine and cloudless—light wind. Evening, very high wind.
W 11	29.550	44.8	29.598	47.2	42	48.5	49.9	36.9	52.7		S var.	Overcast—very light rain and unsteady wind.
T 12	29.756	45.6	29.701	48.8	42	46.3	50.7	40.5	53.4		NW var.	Overcast—light rain and wind. Evening, fine and clear.
F 13	30.160	45.6	30.218	48.8	36	41.4	48.8	37.2	49.2	.011	SW	A.M. Fine—light fog. P.M. Fine—light clouds and wind.
○ S 14	29.994	46.5	29.998	49.7	41	45.2	54.6	40.0	54.2		SE var.	A.M. Overcast—deposition—light wind. P.M. Fine and clear.
⊙ 15	29.837	47.8	29.908	50.0	41	46.2	49.3	43.9	49.5		S	A.M. Overcast—light rain. P.M. Fine—light clouds and wind.
M 16	30.067	46.6	30.051	49.5	39	45.0	49.9	38.9	50.6		WSW	A.M. Fine and clear—light clouds. P.M. Lightly overcast.
T 17	29.893	47.5	29.742	48.8	40	45.7	50.2	43.0	49.7		S var.	Overcast—very light rain—unsteady wind.
W 18	29.930	45.9	30.004	48.7	38	43.3	47.7	39.9	48.6		N var.	A.M. Lightly overcast—light brisk wind. P.M. Cloudy—brisk wind. Evening, fine and clear.
T 19	30.259	43.2	30.247	44.2	30	36.6	42.5	34.7	44.7		N	Foggy.
F 20	30.334	45.0	30.307	48.4	38	45.8	52.2	35.0	52.3		S	A.M. Lightly overcast. P.M. Thick haze.
S 21	30.213	47.3	30.208	49.1	42	48.2	51.0	45.2	51.3		SW	Overcast—very light rain.
⊙ 22	30.261	47.8	30.257	48.7	41	45.9	46.4	43.9	47.6		ENE	A.M. Thick fog. P.M. Overcast—light rain—brisk wind.
M 23	30.259	45.5	30.229	48.2	40	43.7	47.4	37.4	47.7		NNE	A.M. Lightly overcast—light wind. P.M. Fine—light clouds. Evening, cloudy.
T 24	30.219	44.8	30.360	47.3	38	42.7	45.0	39.0	46.0		E	Overcast—very light rain—brisk wind. P.M. Fine—light clouds and wind.
W 25	30.540	42.3	30.532	45.5	33	39.8	47.9	32.9	47.3		N	Fine—light clouds and haze. Evening, cloudy.
T 26	30.474	41.8	30.372	44.9	34	36.5	47.6	32.9	46.8		S	A.M. Lightly overcast. P.M. Fine and cloudless—light haze.
F 27	30.344	44.2	30.301	47.0	38	42.5	49.2	35.7	49.3		ENE	A.M. Thick fog. P.M. Cloudy—light wind.
S 28	30.216	44.0	30.130	45.8	35	42.2	43.7	38.7	43.6		E	Cloudy—light brisk wind.
● ⊙ 29	30.090	43.3	30.018	46.2	34	41.4	44.7	34.2	45.2		E	A.M. Fine—light clouds—haze and wind. P.M. Cloudy—light wind.
M 30	29.871	41.4	29.806	45.2	31	39.6	50.5	33.0	50.6		E	A.M. Fine—thick haze. P.M. Fine—nearly cloudless.
T 31	29.816	45.3	29.720	47.7	37	47.8	51.6	36.9	54.2		SSW	A.M. Fine—lightly cloudy. P.M. Overcast—light rain.
MEANS ..	29.936	44.4	29.914	46.8	36.5	42.7	47.5	37.2	48.3	Sum. .139	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr. .. : ..	9 A.M. 29.903 3 P.M. 29.874
APRIL W 1	29.966	50.0	29.952	53.2	48	54.0	61.3	47.2	62.8		SSW	Fine—light clouds and wind.
T 2	29.881	53.8	29.804	56.4	49	55.7	65.3	48.6	66.2		S	A.M. Fine—light fog. P.M. Overcast. Evening, Light rain.
F 3	29.804	56.8	29.823	59.0	50	56.2	59.4	52.7	61.4		SW	Overcast. Evening, Light rain.
S 4	30.130	54.6	30.148	55.4	48	49.0	52.8	48.0	52.6		ENE	Overcast—light rain and wind.
⊙ 5	30.214	52.3	30.219	53.7	47	48.0	49.3	45.3	49.2	.094	E	Overcast—light rain and wind.
M 6	30.328	51.3	30.366	54.5	42	48.4	57.0	40.4	57.2	.033	E	A.M. Fine and cloudless—light haze. P.M. Lightly overcast. Evening, Fine and clear.
T 7	30.410	50.6	30.352	54.5	42	49.3	59.9	41.8	60.4		E	Fine and cloudless—light haze. Evening, Fine and clear.
W 8	30.344	54.9	30.263	57.0	44	54.3	64.7	43.4	61.7		SW	Fine and cloudless—light haze. Evening, Fine and clear.
T 9	30.210	56.1	30.148	58.9	47	51.7	61.2	45.8	62.6		SSW	Fine—light clouds and wind. Evening, Cloudy.
F 10	30.182	56.7	30.212	59.3	51	55.0	58.2	52.8	58.8		WSW	A.M. Overcast—light wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
S 11	30.348	51.0	30.380	54.8	39	47.3	50.0	41.2	60.8		NE var	Fine—light clouds and wind. Evening, Fine and clear.
⊙ 12	30.317	52.2	30.237	54.5	39	46.0	56.2	36.8	55.8		SW	Fine and cloudless—light haze.
○ M 13	30.196	52.7	30.152	55.4	44	49.3	58.7	40.8	61.0		SW	A.M. Fine and cloudless—light haze. P.M. Cloudy. Evening, Fine and clear.
T 14	30.188	55.6	30.091	57.0	38	53.3	61.5	45.0	61.8		S	Fine and cloudless—light haze.
W 15	29.944	54.7	29.911	56.8	43	49.4	51.0	42.0	59.6		WSW	A.M. Thick haze. P.M. Overcast—light brisk wind. Evening, Light rain.
T 16	30.206	48.6	30.130	49.8	26	39.7	46.5	32.8	46.7	.033	NE	A.M. Fine & cldless—lt.haze. P.M. Overcast—snow—hail & rain.
F 17	30.186	46.5	30.176	47.3	29	38.0	40.2	30.0	43.8	.144	NE	A.M. Fine—light clouds and wind. P.M. Snow—light wind. Evening, Fine and clear.
S 18	30.082	46.3	29.964	48.2	36	44.3	48.0	34.8	49.7		SW	Fine—lightly cloudy. Evening, Overcast—light rain.
⊙ 19	30.295	49.6	30.358	50.7	34	45.3	51.4	37.0	51.3	.027	NE var	A.M. Fine—light clouds and wind. P.M. Cloudy. Evening, Fine and clear.
M 20	30.473	48.7	30.467	52.4	42	49.8	56.3	44.3	57.3		SW	A.M. Thick haze. P.M. Cloudy—light wind.
T 21	29.462	51.3	30.425	53.9	43	51.2	56.7	45.5	56.9		SW	Cloudy—light rain and wind.
W 22	30.445	52.5	30.420	55.5	47	52.7	58.6	46.0	59.6	.061	NNE	A.M. Fine—light clouds and wind. P.M. Cloudy.
T 23	30.465	55.6	30.402	55.3	42	53.3	53.4	45.2	55.7		WNW	Overcast—light wind.
F 24	30.388	54.0	30.309	56.8	47	52.8	57.4	48.3	59.4		SW	A.M. Thick haze. P.M. Cloudy—light brisk wind.
S 25	30.148	58.0	30.029	56.6	38	52.0	51.5	46.2	55.4		NW var.	Fine—light clouds and wind. Evening, Overcast—light wind.
⊙ 26	29.718	51.0	29.675	52.3	34	44.4	47.6	40.2	47.8		NW	A.M. Cloudy—brisk wind. P.M. Overcast—hail and light rain.
● M 27	29.631	50.3	29.621	50.6	32	43.4	47.5	34.2	48.2		NE	A.M. Fine and cloudless—light haze & wind. Evening, Cloudy.
T 28	29.835	50.3	29.808	51.4	33	45.4	50.7	34.4	51.2		NE	Fine—light clouds & light brisk wind. Evening, Fine and clear.
W 29	29.679	46.4	29.627	48.8	41	44.4	46.8	39.6	46.6		NNE var	A.M. Overcast—light rain and wind. P.M. Lightly overcast. Evening, Overcast—light steady rain.
T 30	29.528	48.6	29.556	50.6	45	47.2	49.6	41.9	50.2	.677	ENE	A.M. Overcast—light steady rain. P.M. Overcast. Evening, Overcast—light steady rain.
MEANS ..	30.133	52.1	30.102	54.0	41.3	49.1	54.3	42.4	55.8	Sum. 1.069	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 30.077 3 P.M. 30.040

METEOROLOGICAL JOURNAL FOR MAY AND JUNE, 1835.

1835.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	External Thermometer. Fahrenheit. 3 P.M.	External Thermometer. Self-registering. Lowest.	External Thermometer. Self-registering. Highest.	Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
MAY F 1	29.635	50.0	29.649	53.0	46	47.4	52.3	44.6	52.6	.158	SW	Overcast—very light rain and wind.
S 2	29.647	51.7	29.626	54.6	45	51.5	54.5	44.0	57.8		S	A.M. Fine—light clouds. P.M. Overcast—light rain and wind.
☉ 3	29.770	54.2	29.804	55.5	43	51.9	53.6	45.0	53.6	.036	WNW	Overcast—light wind.
M 4	29.883	56.2	29.922	56.3	43	52.5	54.3	44.7	59.2		SW	A.M. Fine—light clouds. P.M. Overcast—light brisk wind.
T 5	30.109	54.6	30.083	57.4	44	49.5	58.5	45.0	59.7		NE	Overcast—light rain and wind.
W 6	29.865	54.7	29.835	58.6	49	53.6	61.4	48.9	62.3	.133	S	A.M. Lightly overcast—very light rain. P.M. Fine—light clouds. Evening, Fine and clear.
T 7	29.968	60.5	30.008	60.3	48	56.6	60.9	45.6	61.2		SW	A.M. Fine—light clouds and wind. P.M. Cloudy. Evening, Fine and clear.
F 8	30.109	60.3	30.079	61.6	51	55.4	63.5	46.0	63.9		SSW	Overcast—Evening, Fine and clear.
S 9	30.004	64.3	29.913	64.7	55	62.6	67.2	54.5	68.4		S	Fine—light clouds.
☉ 10	29.816	63.6	29.794	65.0	49	59.8	64.8	50.0	65.6		SW	A.M. Fine—light clouds. P.M. Cloudy. Evening, Fine and clear.
M 11	29.984	64.5	29.972	64.2	50	57.0	60 6	45.7	63.7		SW	A.M. Fine—lightly cloudy. P.M. Overcast—light rain.
○ T 12	29.833	59.6	29.766	63.6	50	57.2	60 8	52.4	63.6	.033	SW	Fine—light clouds & wind. Even. Overcast—light steady rain.
W 13	29.604	65.3	29.631	64.0	48	59 3	61.8	50.9	63.2	.097	SW	A.M. Lightly overcast—light brisk wind. P.M. Fine—light clouds and wind. Evening, Overcast—heavy rain.
T 14	29.495	55.2	29.558	56.2	47	48.9	48.4	45.4	55.0	1.033	E	A.M. Overcast—light steady rain. P.M. Overcast—heavy rain, with distant thunder and lightning.
F 15	29.720	56.6	29.730	58.8	47	54.5	56.5	45.2	58.6	.369	S	A.M. Fine—light clouds and wind. P.M. Thunder, accompanied with heavy rain. Evening, Cloudy.
S 16	29.774	59.8	29.829	60.7	48	58.5	63.2	47.4	63.2	.183	S	Fine—light clouds. Evening, Fine and clear.
☉ 17	30.004	65.5	29.976	62.2	50	59.8	64.0	46.8	64.5		SE var.	Fine—nearly cloudless.
M 18	29.930	63.9	29.885	64.0	50	65.0	70.0	48.4	72.8		E	A.M. Fine and cloudless—light haze. P.M. Fine—lightly cloudy—light brisk wind.
T 19	29.873	63.2	29.851	65.2	52	63.9	66.6	54.2	68.7		E	Fine—light clouds and wind.
W 20	29.942	59 9	29.976	62.4	53	55.6	60.9	52.8	61.8		E	Overcast—light rain and wind.
T 21	30.208	62.8	30.200	64.2	49	56.7	61.3	50.3	62.3	.047	E	A.M. Fine—light clouds and wind. P.M. Cloudy—light brisk wind.
F 22	30.174	62.8	30.103	63.5	46	55.4	65.5	47.6	66.7		N	A.M. Thick haze. P.M. Fine—light clouds and wind. Evening, Overcast—light rain.
S 23	30.186	64.3	30.128	66.0	53	60.7	68.2	52.6	69.6	.052	NNE	Fine—light clouds. Evening, Cloudy.
☉ 24	30.156	66.5	30.075	66.8	56	66.6	70.4	55.7	71.4		S	Fine—light clouds. Evening, Cloudy.
M 25	29.881	64.6	29.770	66 2	50	60.6	65.2	56 0	66.4		SW	A.M. Cloudy. P.M. Fine—light clouds and wind. Evening, Cloudy—very light rain.
T 26	29.542	62.4	29.456	62.8	50	55.9	58.0	48.6	58.6		SW	Overcast—light rain and wind.
● W 27	29.695	60.6	29.748	62.7	50	55.7	59.5	46.2	61.6	.097	SW	A.M. Cloudy—light wind. P.M. Overcast—thunder and lightning—heavy rain.
T 28	29.656	58.3	29.968	61.2	49	54.8	56 0	46.2	58.7	.241	E	Cloudy—light brisk wind. Evening, Fine and clear.
F 29	30.069	59.4	30 077	61.2	43	54.1	58.8	43.0	59.8		NE	Fine—light clouds and wind. Evening, Cloudy.
S 30	30.103	59.5	30.061	60.0	40	52.2	57.7	41.7	58.0		N	A.M. Cloudy—light brisk wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
☉ 31	30.045	58.3	29.966	60.8	42	51.7	60.4	43.8	64.2		S	A.M. Fine—light clouds and wind. P.M. Cloudy. Evening, Overcast—light rain.
MEANS..	29.893	60.1	29.885	61.4	48.3	56.4	60.8	48.0	62.5	Sum. 2.479	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.813 — 3 P.M. 29.801
JUNE M 1	29.911	56.2	29.954	58.9	48	52.7	56.6	48.6	62.8	.116	S	Overcast—light rain and wind.
T 2	30.047	61.6	30.027	62.0	50	63.0	66.3	46.0	66.7	.063	ESE	A.M. Fine—light clouds. P.M. Overcast—light rain.
W 3	29.901	58.6	29.897	62.4	52	58.8	67.6	52.7	68.4	.200	E	A.M. Overcast—light rain. P.M. Fine—light clouds & wind. Evening, Cloudy.
T 4	30.006	62.7	30.023	64.5	58	61.8	63.8	55.9	65.3		N	Lightly overcast—light rain and wind.
F 5	30.083	59.6	30.081	63.4	53	55.7	61.7	50.5	65.2	.036	S var.	Overcast—light brisk wind. Evening, Fine and clear.
S 6	30.079	65.2	30.071	67.2	59	65.3	75.2	52.8	75.5		NE	Fine and cloudless—light haze and wind.
☉ 7	30.160	67.6	30.130	69.6	60	65.3	75.9	57.7	77.3		N	Fine light clouds and haze, with distant thunder. Evening, Fine and clear.
M 8	30.141	74.3	30.113	72.8	75	74.0	78.6	60.6	80.2		N	Fine and cloudless—light wind.
T 9	30.239	75.8	30.208	74.5	64	75.0	79.3	62.8	80.6		S	Fine—light haze and wind. Evening, Fine and clear.
○ W 10	30.331	76.6	30.321	75.4	63	75.2	81.7	62.0	81.8		E	Fine and cloudless—light brisk wind.
T 11	30.394	77.2	30.378	77.2	66	77.2	82.3	65.4	83.6		N	Fine and cloudless—light haze and wind.
F 12	30.416	77.0	30.370	77.6	65	75.3	81.7	63.4	82.2		N	A.M. Fine and cloudless—light haze and wind. P.M. Fine—light clouds. Evening, Cloudy.
S 13	30.352	68 2	30.299	72.0	58	58.9	69.2	57.0	69.9		N	A.M. Overcast—light wind. P.M. Fine—light clouds. Evening, Cloudy.
☉ 14	30.285	67.3	30.214	69.7	50	59.6	70.6	52.6	71.5		N	Fine—light clouds and wind. Evening, Cloudy.
M 15	30.237	72.4	30.221	72.9	61	68.8	74.7	59.4	75.4		NE	A.M. Cloudy—light brisk wind. P.M. Fine—light clouds. Evening, Cloudy.
T 16	30.295	76.6	30.244	74.2	58	70.2	74.0	60.0	75.8		NW	Fine light clouds and wind. Evening, Overcast—heavy rain during the night.
W 17	30.154	73.0	30.111	74.5	63	71.6	73.7	65.2	75.9	.133	N	Fine—light clouds—brisk wind. Evening, Overcast—light shower, with distant thunder.
T 18	30.115	68.4	30.103	72.6	58	62.6	70.0	57.5	70.8	.027	NE	Cloudy—light wind. Evening, Fine and clear.
F 19	30.202	74.7	30.172	71.6	50	63.2	67.8	54.9	69.2		NE	Fine—light clouds. Evening, Cloudy.
S 20	30.010	67.0	29.980	70.3	53	62.6	70 0	58.2	70.8		SW	A.M. Lightly overcast. P.M. Fine light clouds and wind. Evening, Cloudy.
☉ 21	30.095	74.2	30.010	70.6	52	64.2	71.9	51.8	72.5		SW	A.M. Cloudy—light wind—P.M. Fine—nearly cloudless. Evening, Cloudy.
M 22	29.853	67.7	29.768	70.5	58	65.2	72.4	58.2	73.7		SW	A.M. Cloudy—light brisk wind. P.M. Fine—light clouds and wind. Evening, Overcast—heavy rain and wind.
T 23	29.681	69.2	29.724	69.4	52	62.8	65.3	55.3	67.8	.391	SW var.	Fine—light clouds, and light brisk wind.
W 24	29.542	63.7	29.215	64.3	53	58.6	60.4	49.3	60.8		ESE	A.M. Overcast—light rain—brisk wind. P.M. Heavy rain—very high wind. Evening, Overcast—light continued rain.
T 25	29.447	67.2	29.548	60 5	45	54.7	53.6	46.7	59.6	.352	W	Overcast—light rain throughout the day.
● F 26	29.901	67.5	29.742	62.2	45	59.3	52.3	44.6	62.6	.250	SW	A.M. Fine—light clds. P.M. Overcast—heavy rain & high wind.
S 27	30.012	65.9	30.111	62.2	46	55.2	55.5	45.4	59.8	.630	SW	A.M. Fine—light clouds & wind. P.M. Cloudy—light showers.
☉ 28	30.269	65.2	30.247	62.7	45	58.3	61.8	47.4	62.8		WSW	A.M. Fine—light clouds and wind. P.M. Cloudy.
M 29	30.253	64.3	30.190	63.3	48	58.2	62.7	46.3	63.7		E	A.M. Fine—light clouds and wind. P.M. Cloudy.
T 30	30.162	64.0	30.119	64.9	46	62.3	66.0	50.4	68.3		E	Fine—light clouds and wind.
MEANS..	30.086	68.3	30.053	68.5	55.1	63.9	68.8	54.6	70.7	Sum. 2.198	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr........	9 A.M. 29.981 — 3 P.M. 29.948

Date of issue.	Date of issue.	Date of issue.